On the Local Leakage Resilience of Linear Secret Sharing Schemes

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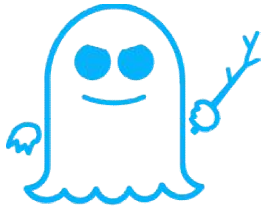
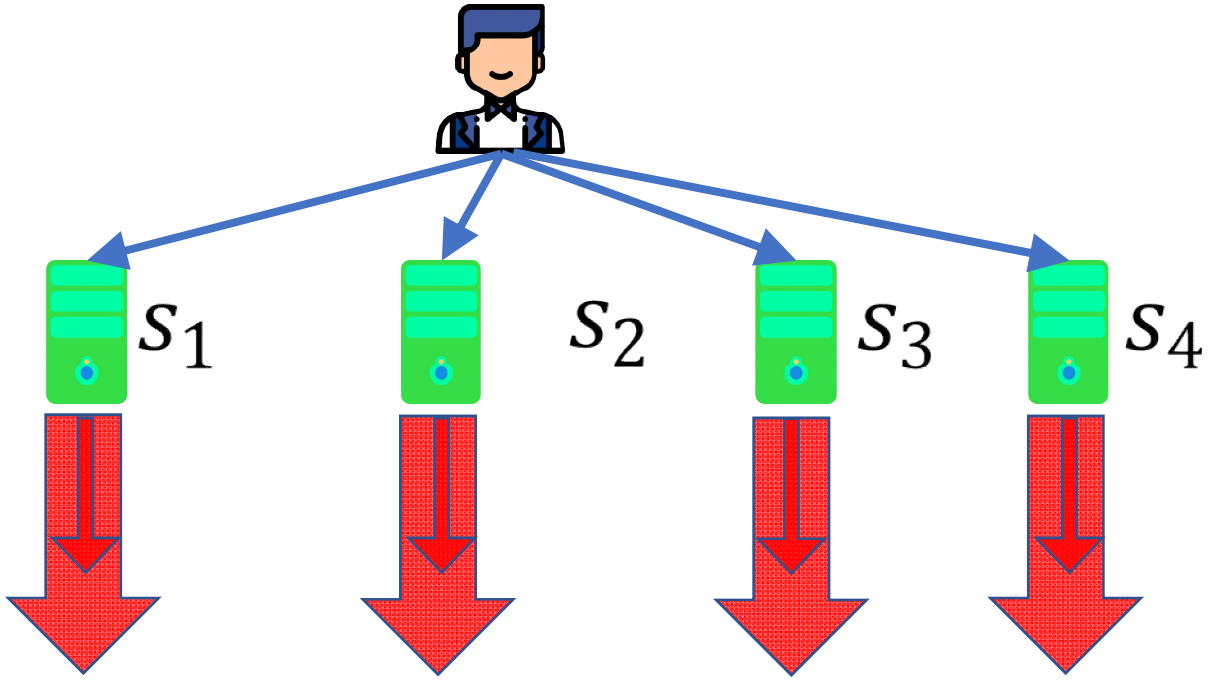


Leakage attacks can be devastating

Proposed Solution: Secret Sharing, MPC



A few full corruptions

All the servers?

Partial leak from all

Leakage Resilient Cryptography

[ISW03, MR04, DP07, DP08, AGV09, NS09, FRR+10, BKKV10, LLW11, BGJK12, DF12, BDL14, BGK14, GR15, DLZ15, GIMSS16 …]

- Strong leakage models
- Specially-designed schemes

Are **standard**

Additive
Shamir

Secret Sharing Schemes

Leakage Resilient?

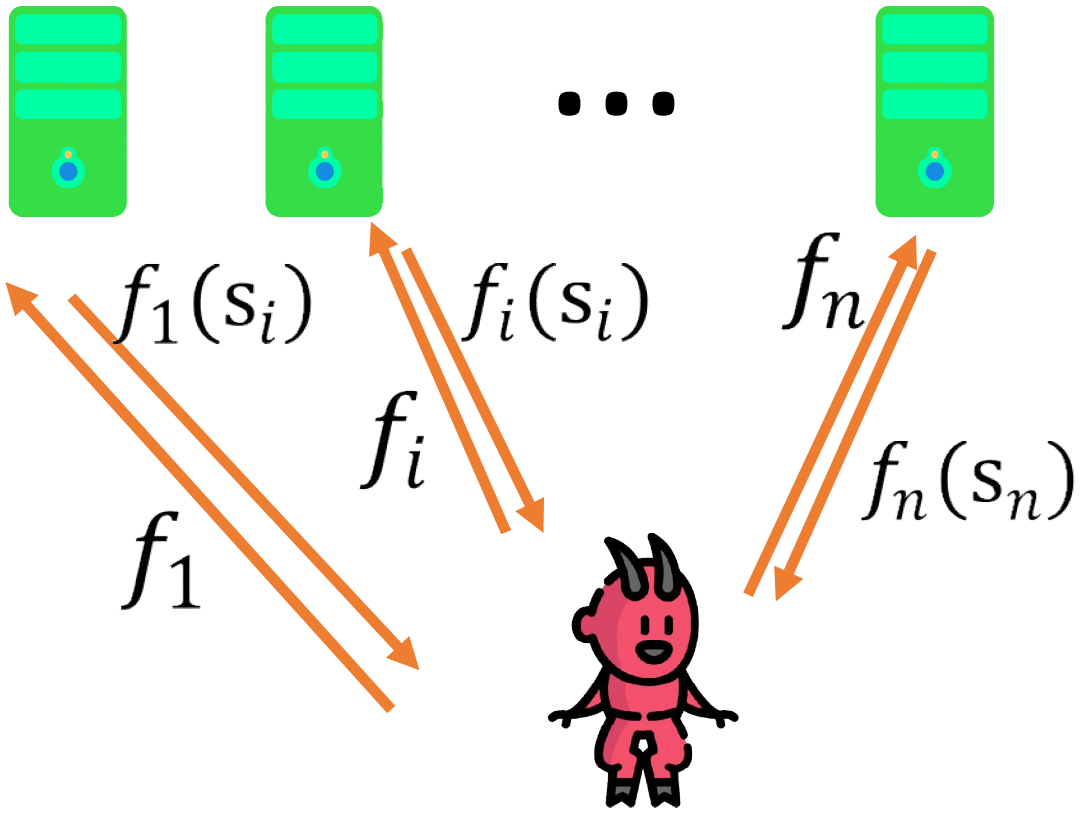
Limited General Results

Secret Sharing generically protects
against weak forms of leakage.

[DDF14] Noisy Leakage

[BIVW16] Low approximate degree leaks

Leakage Model: Local Leakage



Leak **any** partial information about state.

Output of each f_i is short

Restricted form of Only Computation Leaks[Micali-Reyzin04, GR12, BDL14], Bounded Comm. Leakage [GIMSS16]

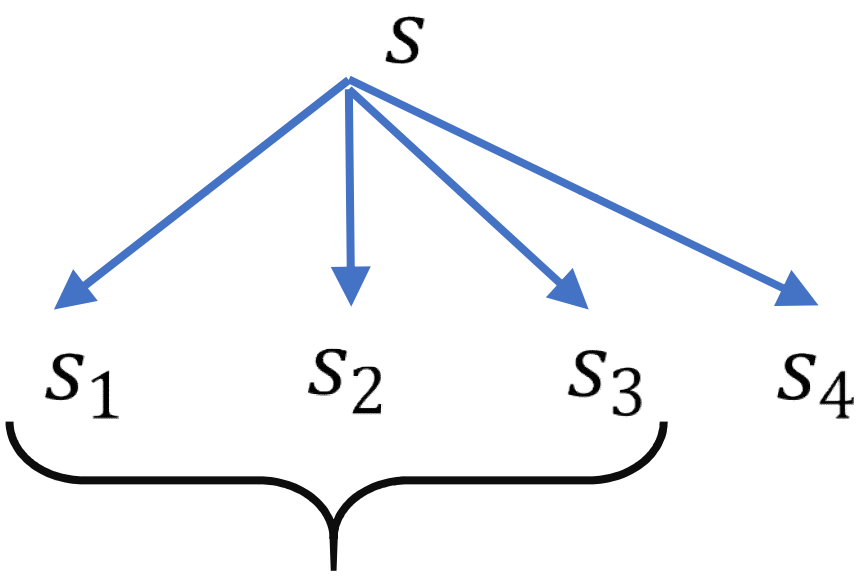
Is Local Leakage reasonable?

Local: Justified by physical separation

Shrinking: Timing, power, selective failures give limited information

Adversarial

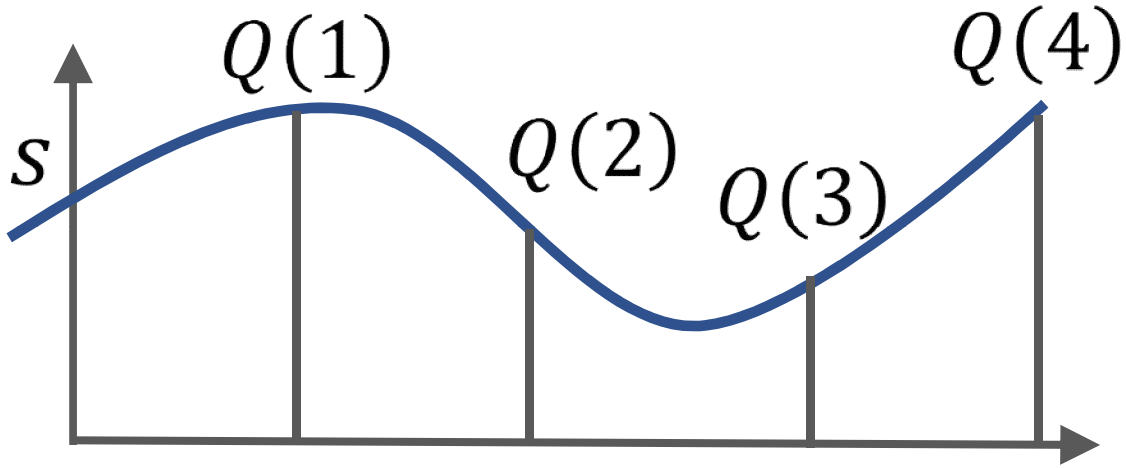
Additive Secret Sharing



Completely random

$$s = s_1 + s_2 + s_3 + s_4$$

Shamir Secret Sharing



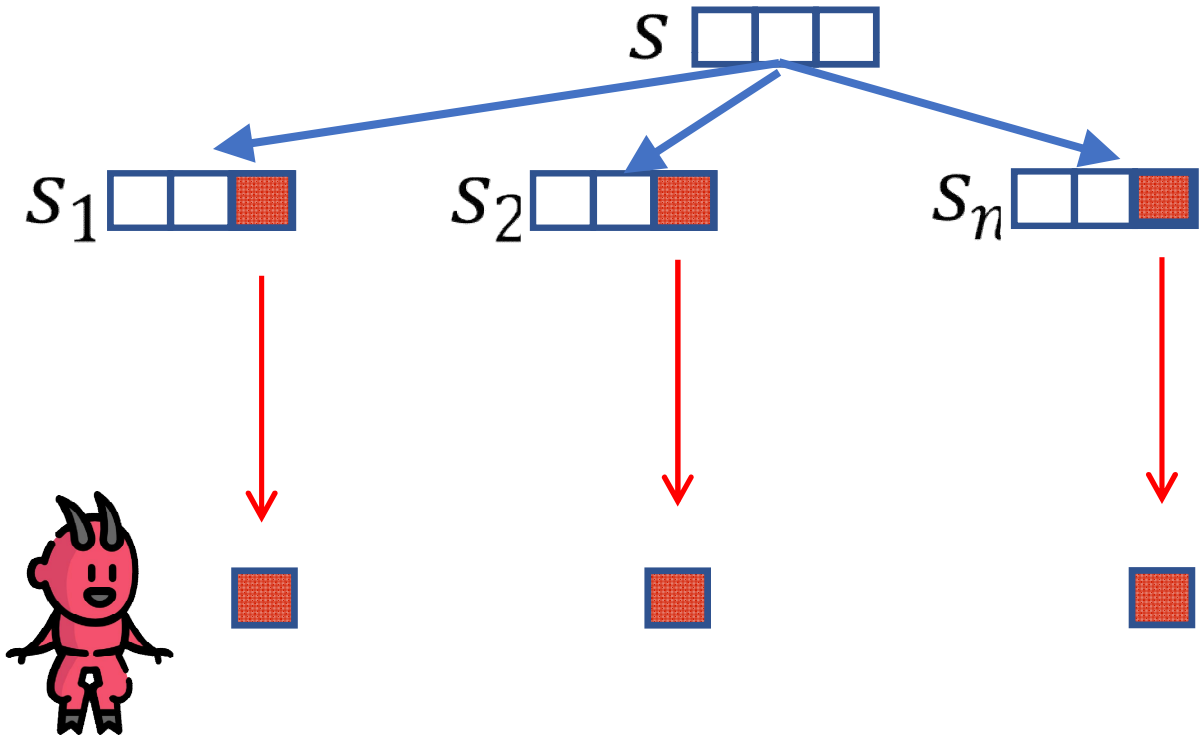
Random poly $Q(0) =$

Shares are evaluations

Threshold: Degree + 1 points to reconstruct

Is Additive Secret Sharing Local Leakage Resilient?

A. Not Necessarily. Field: $\mathbb{F}_{2^k}$



$$s = s_1 + s_2 \ldots + s_n$$

One bit each leaks
one bit of the secret!

How about Shamir ?

A. Not

Guruswami-Wootters 16: One bit per server can reconstruct the whole secret!

Regenerating Codes: When a server goes down, minimum communication you need to reconstruct.

$\mathbb{F}_{2^k}$: small additive subgroups

Other Fields?

Results Overview

Leakage Resilience of Additive &
Shamir Secret Sharing

Application: Leakage Resilience of GMW protocol

Application: Local Share Conversion

Results: Additive Secret Sharing

Prime order fields $\mathbb{F}_p$ are more leakage resilient

Thm.

Over $\mathbb{F}_p$, additive secret sharing is

local leakage-resilient.

$\Omega(\log p)$ bits of leakage, $\varepsilon = 2^{-\Omega(n)}$ distinguishing adv

n: number of servers

Results: Shamir Secret Sharing

Prime order fields $\mathbb{F}_p$ are more leakage resilient

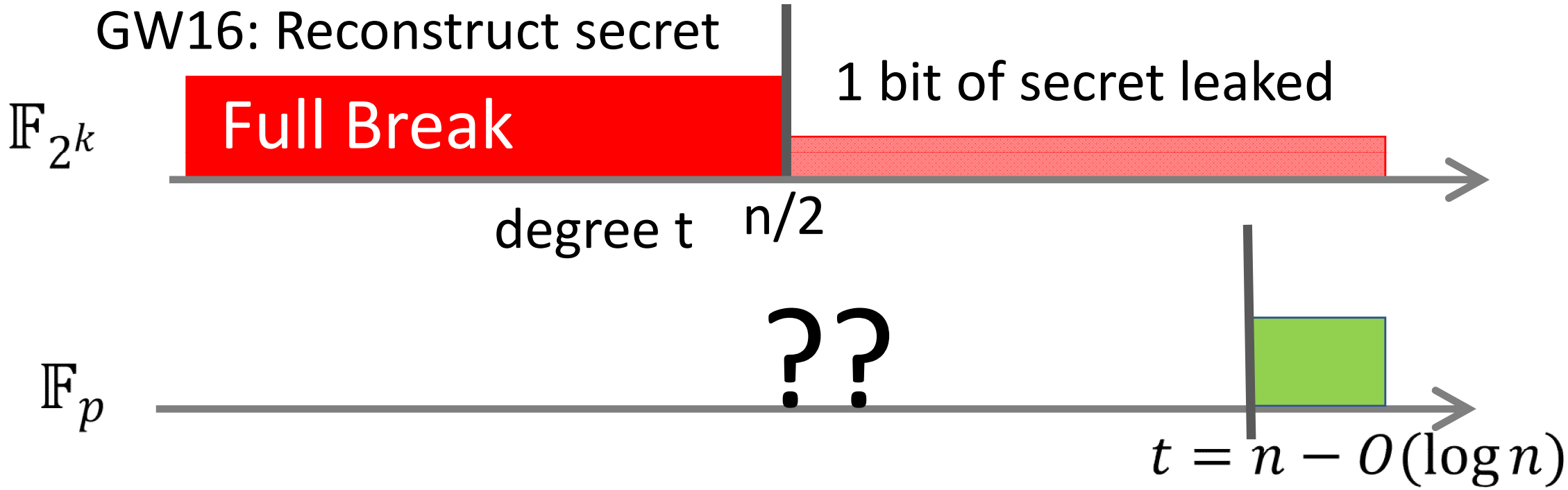
Thm.

Over $\mathbb{F}_p$, high threshold $t = n - O(\log n)$,

t-out-of-n Shamir scheme is leakage-resilient.

$\Omega(\log p)$ bits of leakage, $\varepsilon = 2^{-\Omega(n)}$ distinguishing advantage

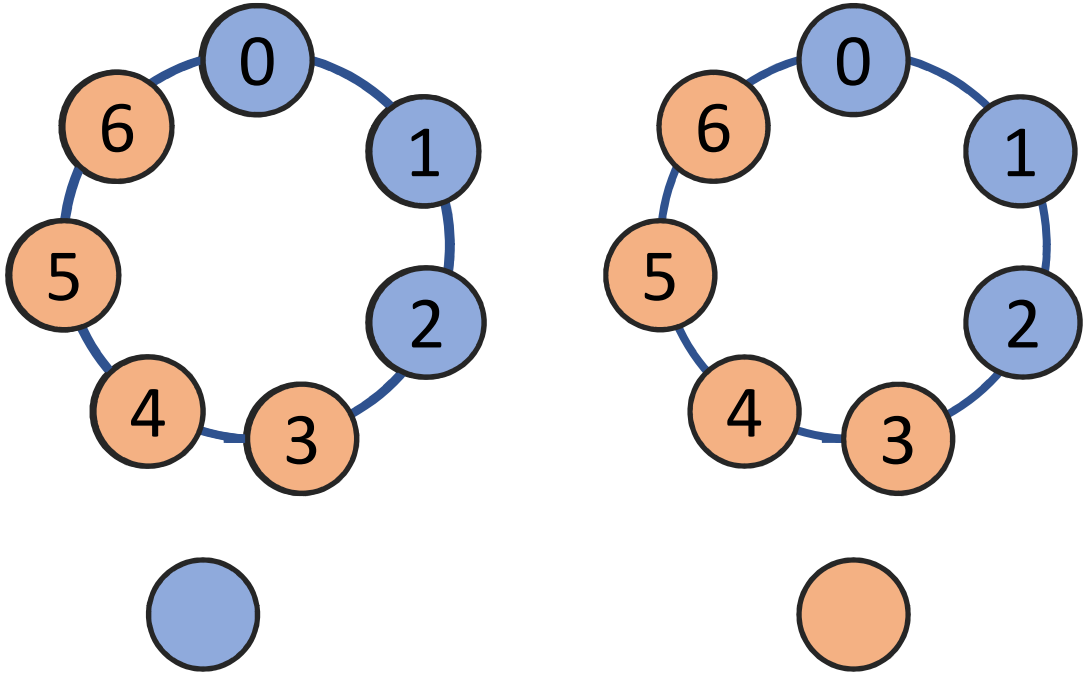
Fast MPC implementations use $\mathbb{F}_{2^k}$



Conjecture. For large n, $\frac{n}{100}$-out-of-n Shamir over $\mathbb{F}_p$ is local leakage-resilient.

Why large n? Approximate Subgroup Attack

$\mathbb{F}_7$

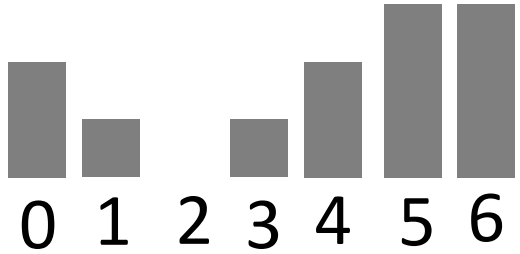


Random Secret

s_1, s_2 completely random

$s = s_1 + s_2$

$s \in \{0,1,2\} + \{3,4,5,6\}$



Generalizes to const. servers

Application: MPC

Honest-but-Curious GMW w/ preprocessed Beaver Triples

Goldreich-Micali-Wigderson87, Beaver91

Preprocessing	Secret Shared Inputs Beaver Triples for product gates
Computation	Addition: Locally Add Shares Multiplication: Use Beaver Triples

Application: MPC

Thm. Over field $\mathbb{F}_p$, GMW Protocol with "Beaver Triple preprocessing" is leakage-resilient:

a. (Corruptions) $n/2$ corruptions

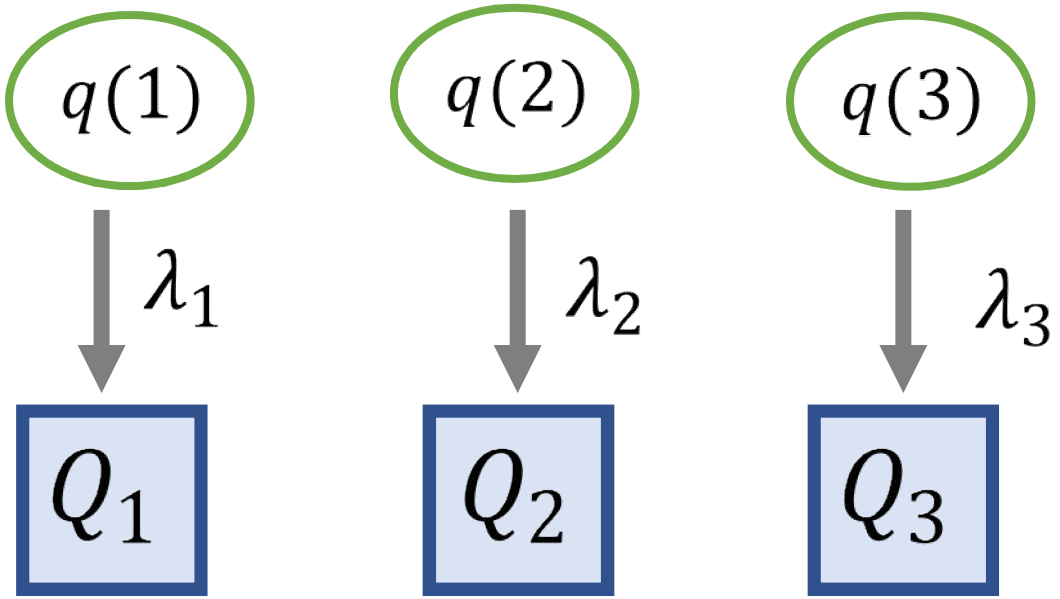
b. (Partial Leakage) $\Omega(\log p)$ bits of leakage.

Byproduct : On Local Share Conversion

[Beimel-Ishai-Kushilevitz-Orlov12]

Locally convert secret under one scheme to related secret under other scheme

Shamir → Additive (Secret unchanged)



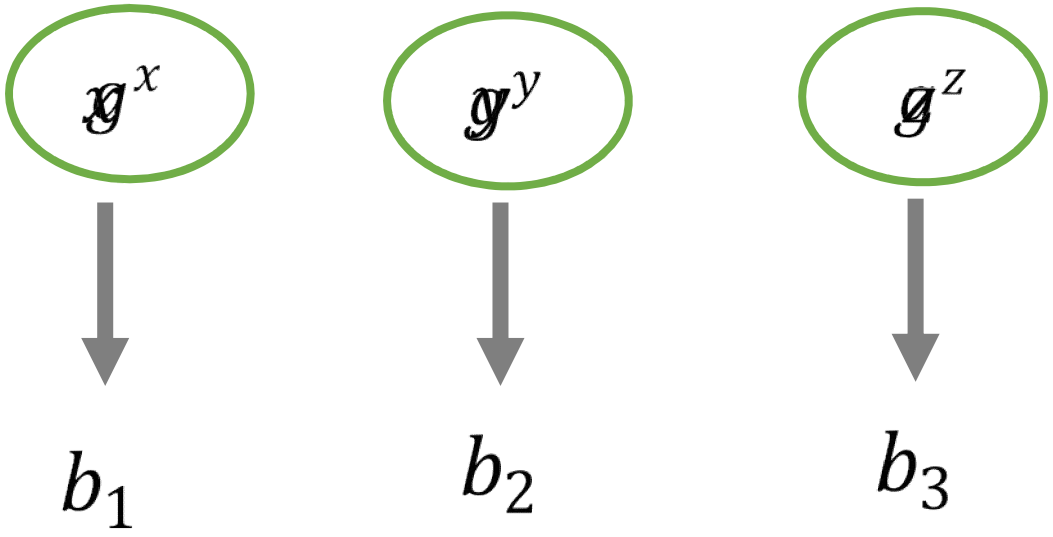
Lagrange Coefficients.

$$s = \lambda_1 q(1) + \lambda_2 q(2) + \lambda_3 q(3)$$

$$s = Q_1 + Q_2 + Q_3$$

Byproduct : On Local Share Conversion

Homomorphic Secret Sharing
(Boyle-Gilboa-Ishai16)



To get 3-server HSS:

$$g^x \cdot g^y \cdot g^z = g^b$$

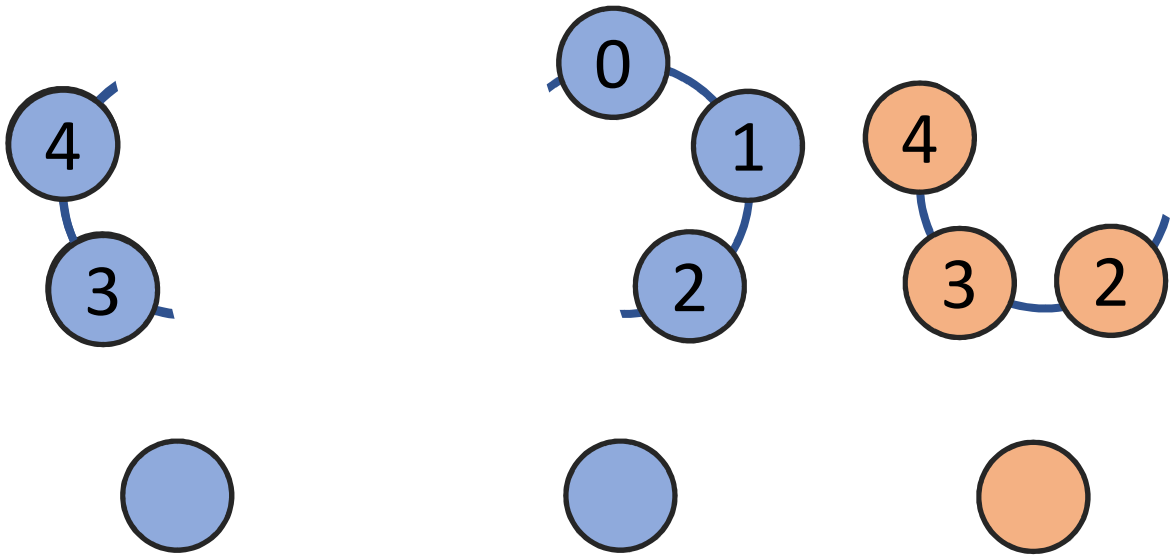
$$b = b_1 + b_2 + b_3$$

Thm: 3-party Additive over $\mathbb{F}_p$ ⇏ Additive over $\mathbb{F}_2$

Techniques

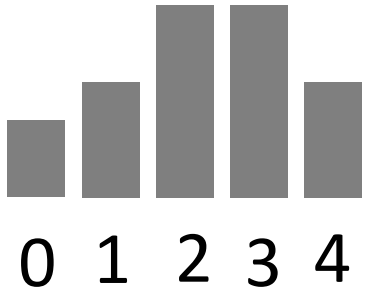
Prime order: $\mathbb{F}_5$

No subgroups



$$s \in \{4,3\} + \{0,1,2\} + \{2,3,4\}$$

Some information.



Want: s is close to random

Techniques $s \in \{4,3\} + \{0,1,2\} + \{2,3,4\}$

Cauchy-Davenport Theorem: Over $\mathbb{F}_p$,

$$|A_1 + A_2| \geq \min(p, |A_1| + |A_2| - 1)$$

Need: $A_1 + A_2$ is close to random on $\mathbb{F}_p$.

To show: For any $A_1, A_2, \ldots, A_n \subseteq \mathbb{F}_p$ where $|A_i| \approx p/2^l$,

$A_1 + A_2 + \cdots + A_n$ is close to uniform on $\mathbb{F}_p$

Number Theory Techniques. Fourier Analysis over $\mathbb{F}_p$.

Summary

Prime order fields $\mathbb{F}_p$ are better against leakage

Thm: Additive & High threshold (t~n) Shamir over prime-order $\mathbb{F}_p$ are local leakage resilient.

Application: Honest-but-Curious GMW is leakage resilient.

Application: Local Share Conversion Impossibility results.

Conjecture: Shamir over $\mathbb{F}_p$ is leakage resilient when $t = {}^{n}/_{2}$


THANK
YOU
return

Usually in Leakage Resilience

Given Leakage Model,
Does ∃ Leakage Resilient Scheme?

[Cite many works on leakage.]

Here: Given existing schemes,
how Leakage Resilient are they?

Results:

Prime order fields $\mathbb{F}_p$ behave differently from $\mathbb{F}_{2^k}$!

Thm. For large n (no of servers), additive secret sharing over $\mathbb{F}_p$ is leakage-resilient to $\log p/4$- bits of leakage.

Thm. For large n, & $t = n - O(\log n)$ t-out-of-n Shamir secret sharing over $\mathbb{F}_p$ is leakage-resilient to $\log p/4$- bits of leakage.

Q: How Leakage Resilient are

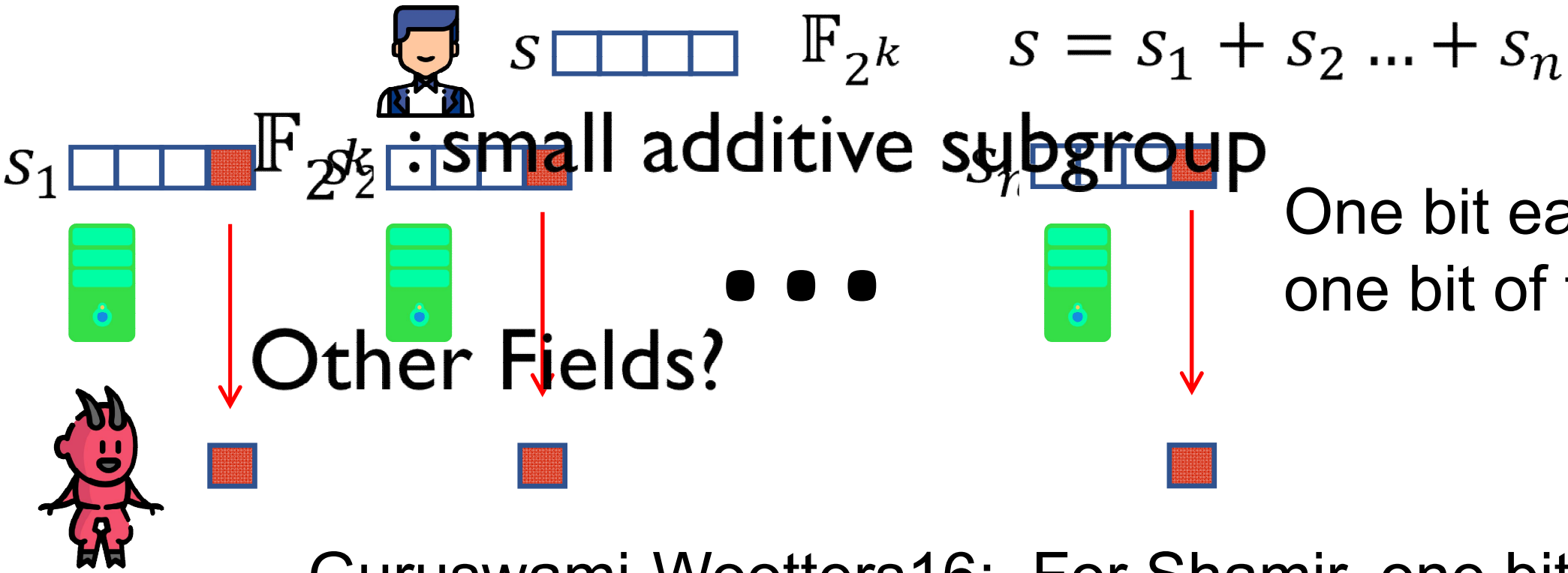
- Shamir & Additive Secret Sharing?
- GMW & BGW style MPC Protocols?

Why?

- Exist and are Used.
- Useful Properties: Homomorphisms.

Is Additive Secret Sharing Local Leakage Resili

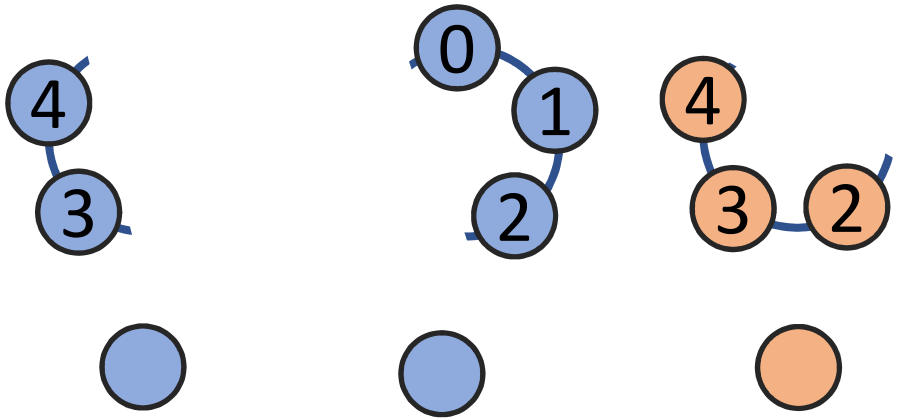
A. Not always.



One bit each leaks one bit of the secret!

Guruswami-Wootters16: For Shamir, one bit each allow full reconstruction of secret.

Techniques



To show:

Pr[ ] ≈ Pr[ ]

secret = 0 secret = 1

Leaks partition $\mathbb{F}_p$ into subsets.

To show: For any $A_1, A_2, \ldots, A_n \subseteq \mathbb{F}_p$ where $|A_i| \approx p/2^l$,

$A_1 + A_2 + \cdots + A_n$ is close to uniform on $\mathbb{F}_p$

Number Theory Techniques. Fourier Analysis over $\mathbb{F}_p$.

Application: Honest-but-Curious GMWs
w/ preprocessed Beaver Triples

$[x]$: Secret Shared

Preprocess

Secret Share Inputs $[x], [y], [z]$

Beaver Triples for product gates: $[a], [b], [ab]$

Compute:

Addition: Locally Add Shares $[w] = [x] + [y]$

Multiplication: $w = x \cdot y$. Use Beaver Triples

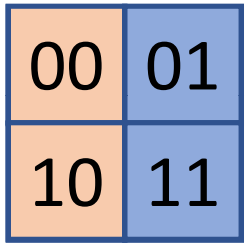
1. Compute and Publish $[x - a], [y - b]$
2. $[x \cdot y] = (x - a)(y - b) + [a](y - b) + (x - a)[b] + [ab]$

Techniques

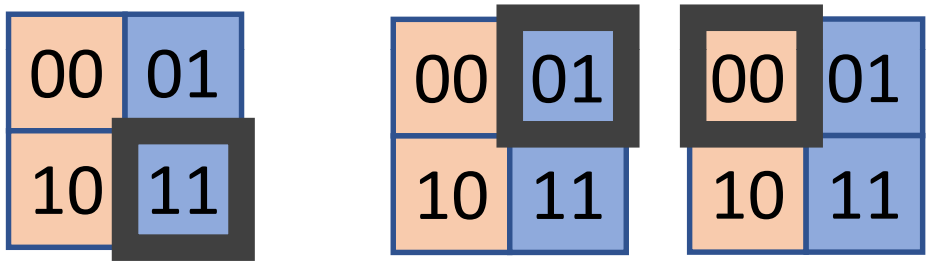
Doesn't work: $\mathbb{F}_4$

Attack: Leak lsb(share)

lsb(secret) = sum of leaks



Secret: 10 = 11 + 01 + 00



Leak reveals: Share's coset

The coset is a group.

Learn secret's coset.