Combiners for Backdoored Random Oracles

Balthazar Bauer, Pooya Farshim, Sogol Mazaheri

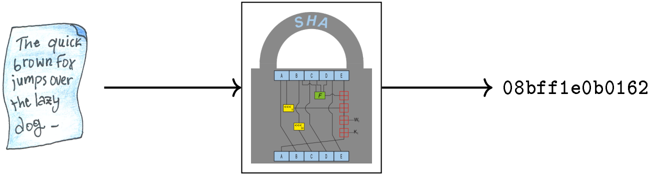
ENS, Paris TU Darmstadt

Backdoors

Backdoors

> It makes more sense to address any security risks by developing intercept solutions during the design phase, rather than resorting to a patchwork solution when law enforcement comes knocking after the fact.
>
> James Comey (former FBI director, Oct. 2014)

Hash Functions



Hash Functions are Everywhere:

KDFs

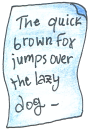
OWFs

FDH

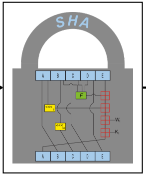
MACs

PoW

Hash Functions



→



→ `08bff1e0b0162`

Hash Functions are Everywhere:

KDFs

OWFs

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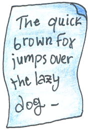
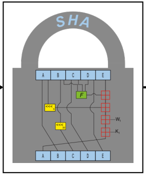
security proofs are not always possible...

Random Oracles





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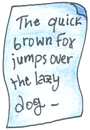
Random Oracles = Ideal Hash Functions



→

ideal hash function

→ `08bff1e0b0162`

Random Oracles = Ideal Hash Functions

ideal hash function

Random Oracles are Practical,

enabling proofs of many practical schemes:

RSA-OAEP

TLS

Identification protocols

FDH

DSA

PSS

Backdoored Random Oracles (BROs)

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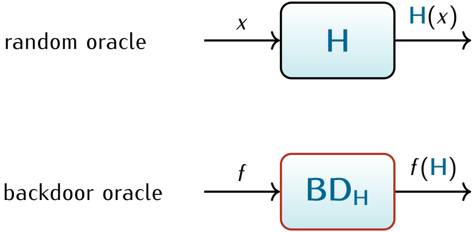
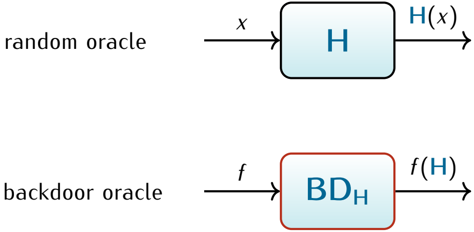
random oracle

Backdoored Random Oracles (BROs)



Backdoored Random Oracles (BROs)

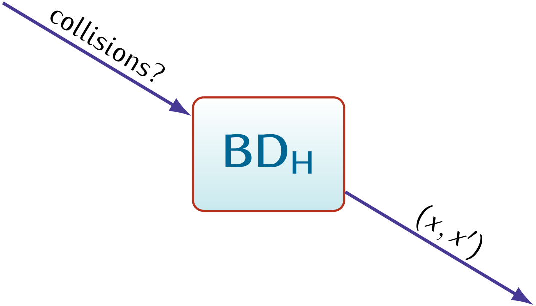


adaptive and unrestricted access to the backdoor oracle

Backdoor Capabilities

BD_H

Backdoor Capabilities



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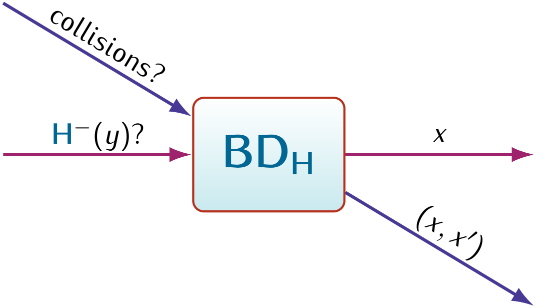

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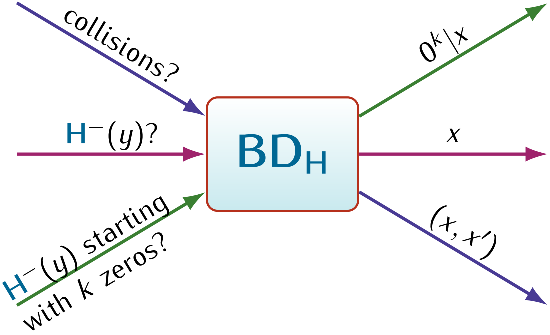

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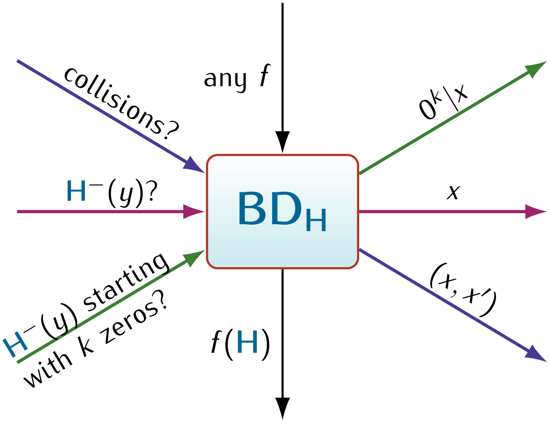

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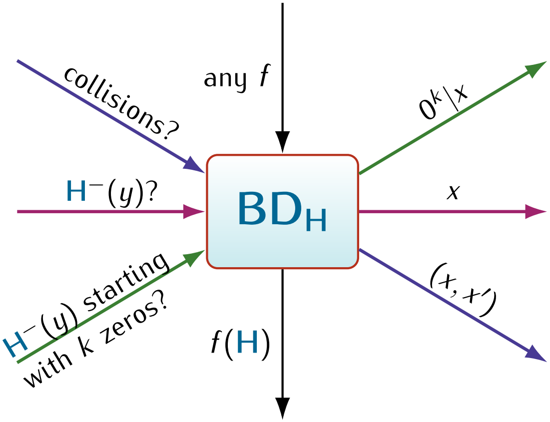
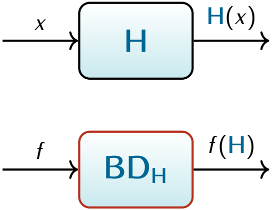


no security is possible...

Combining BROs



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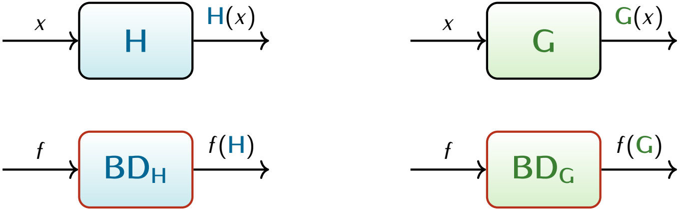

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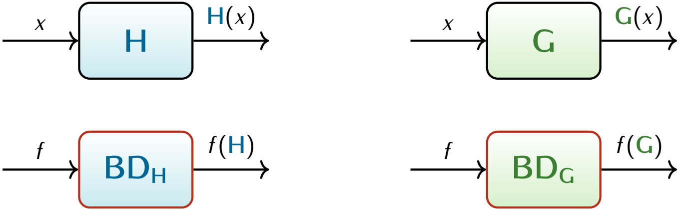

Can we combine two **independent** but **backdoored** hash functions to build one that is secure against adversaries with access to **both** backdoor oracles?

Combiners

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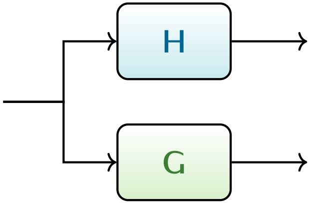
concatenation:



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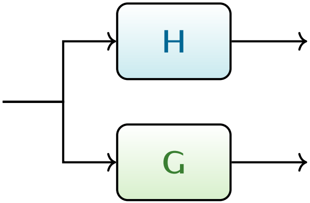
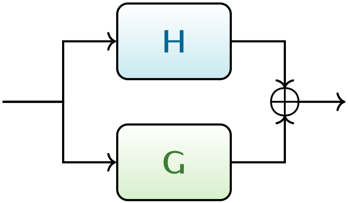


xor:



Combiners

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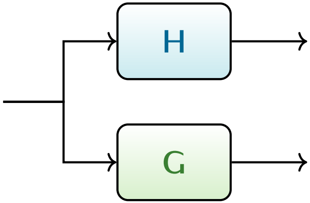

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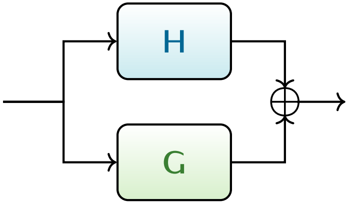
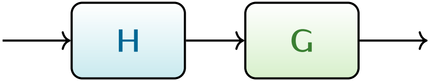


cascade:



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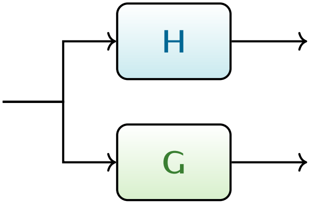
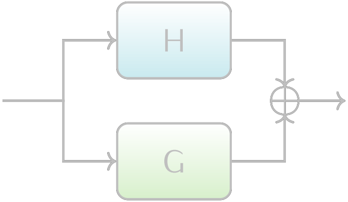


xor:



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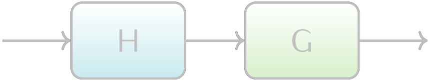
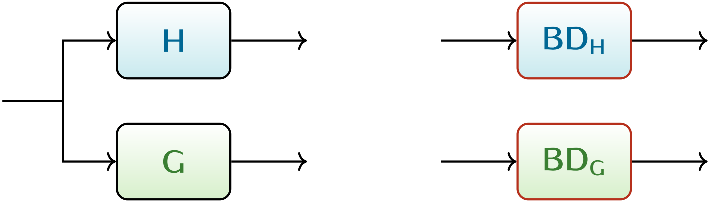


Concatenation in 2-BRO



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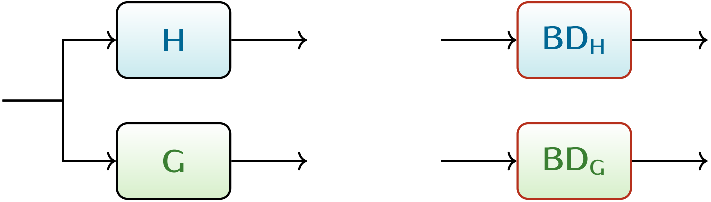
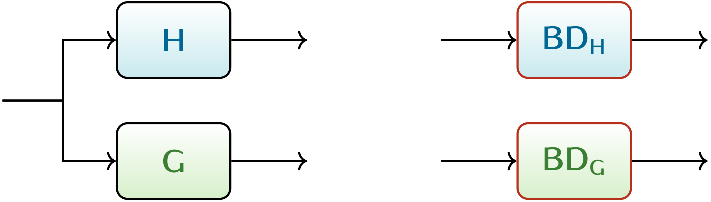


one-way security?

Concatenation in 2-BRO



one-way security? pseudorandomness?

collision-resistance?

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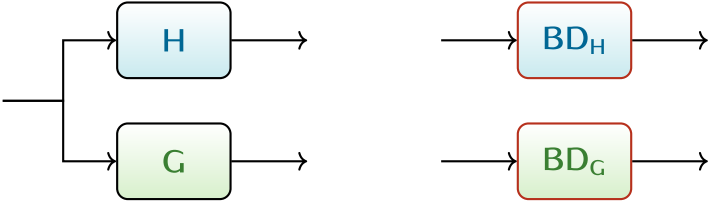

one-way security?   pseudorandomness?

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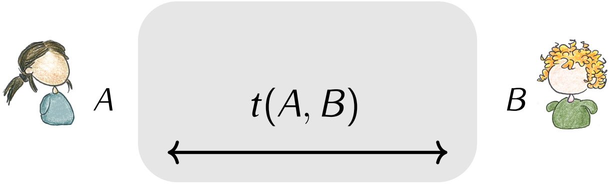
We need results from **communication complexity**...

Communication Complexity



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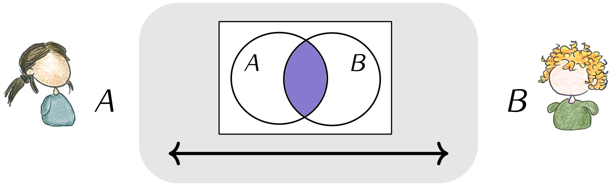

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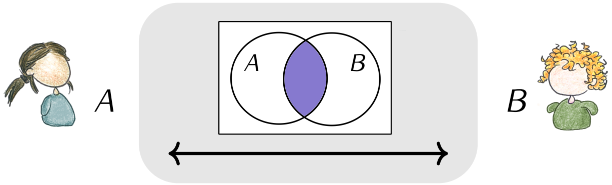

INT: find $x \in A \cap B$.

DISJ: decide $A \cap B = \emptyset$

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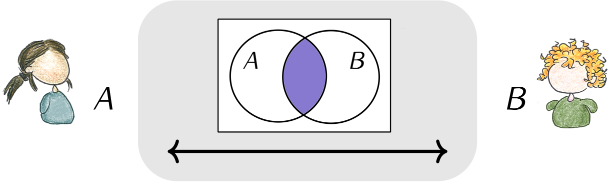

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Theorem ([Babai, Frankl, Simon 86]): For independent random sets $A, B \subseteq [2^n]$ of size $2^{n/2}$, and protocols with 99% correctness, it holds that

$$\mathsf{CC}(\mathsf{DISJ}) \geq \Omega(2^{n/2}).$$

Communication Complexity - Generalized

$\|A\|, \|B\|$	lower-bound	problem	by
$= 2^{n/2}$	$\Omega(2^{n/2})$	DISJ	[Babai, Frankl, Simon 86]
$\approx 2^{n/2}$	$\Omega(2^{n/2})$	DISJ	[Moshkovitz, Barak 12], [Guruswami, Cheraghchi 13]

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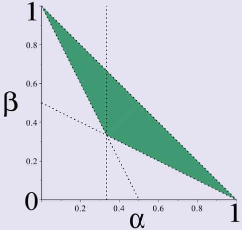
Theorem: For independent random sets $A, B \subseteq [2^n]$ of expected sizes $2^{n(1-\alpha)}$ and $2^{n(1-\beta)}$ respectively,

$$\mathrm{CC}(\mathrm{INT}) \geq \Omega(2^{n(\min(\alpha,\beta)+\alpha+\beta-1)}),$$

for (α, β) in the feasible region.



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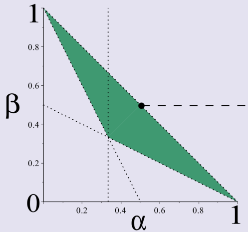

One-Way Security of Concatenation Combiner

Theorem: Inverting a random value $u|v$ under $H|G$ in the 2-BRO model is as hard as the set-intersection problem.

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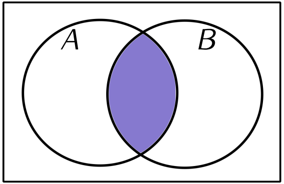

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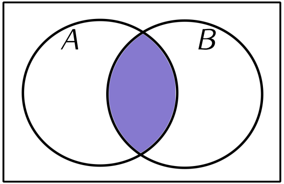

Then, for any pre-image x of $\mathsf{u}|\mathsf{v}$:

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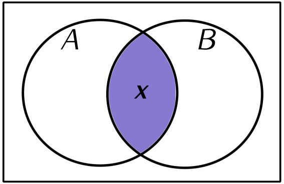

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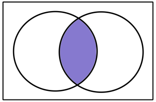
Hence, $x \in A \cap B$.

Security of Concatenation in 2-BRO

One-Way Security

Inverting a random value u|v is as hard as the **set-intersection** problem.



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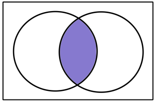
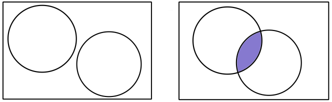


Pseudorandomness

Deciding whether a random value u|v has a pre-image is as hard as the **set-disjointness** problem.



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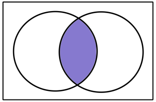

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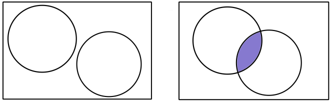
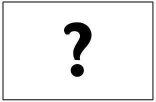


Collision-Resistance

Finding a collision is as hard as ...



Collision-Resistance of Concatenation

Theorem: Finding a collision under $\mathsf{H}|\mathsf{G}$ in the 2-BRO model is as hard as finding 2 sets, given many, and 2 elements in their intersection.

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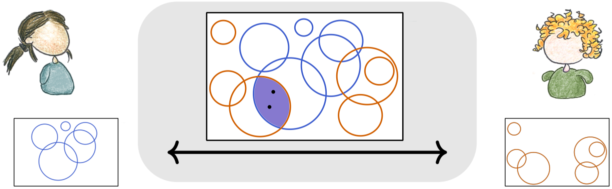

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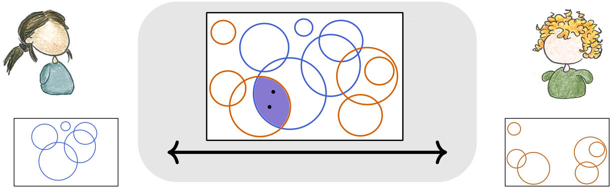
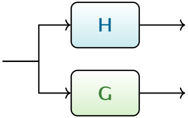
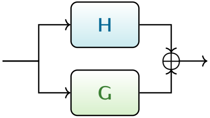
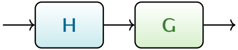


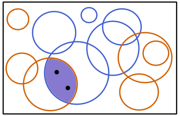
Hardness of the above problem is open.

Combiners and Security Notions

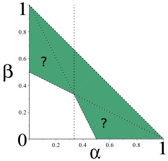
	OW	PRG	CR
	✓	✓	??
	✓	?	??
	✓	✓	??

Open Problems

- lower bound for the multi-INT problem



- extend parameters for DISJ and INT



- combiners for other backdoored primitives

π E

Thank You.

Thanks to Giorgia Marson for drawing Alice, Bob, and the sheet.