Cryptanalysis of branching program obfuscators

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Crypto 2018

European Research Council

Established by the European Commission

What is this talk about

Two partial attacks against some candidate obfuscators built upon the GGH13 multilinear map [GGH13a]

- an attack for specific choices of parameters
- a quantum attack

Main idea of the two attacks

Transform known weaknesses of the GGH13 map into concrete attacks against the candidate obfuscators

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$$\forall C \in \mathcal{C}, \forall x, C(x) = O(C)(x)$$

In this talk, $\mathcal{C} =$ polynomial size circuits

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Many cryptographic constructions from iO: functional encryption, deniable encryption, NIZKs, oblivious transfer, ...

Multilinear maps (mmaps) and iO

Observation

Almost all iO constructions for all circuits rely on multilinear maps (mmap).

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In this talk: we exploit known weaknesses of GGH13 to mount concrete attacks against some iO using it.

History (branching program obfuscators based on GGH13)

Some candidate iO for all circuits and attacks:

2013: [GGH+13b], first candidate

2014-2016: [AGIS14, BGK+14, BR14, MSW14, PST14, BMSZ16], with proofs in idealized models (the mmap is supposed to be somehow ideal)

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2017: [CGH17], attack against [GGH+13b] (in input-partitionable case)

2017: [FRS17], prevent [CGH17] attack

State of the art and contributions

iO (using GGH13) / Attacks	Branching program obfuscators				Circuit obfuscators
	[GGH+13b]	[BR14]	[AGIS14, MSW14] [PST14, BGK+14] [BMSZ16]	[GMM+16]	[Zim15, AB15] [DGG+16]
[MSZ16]		✓	✓		
[CGH17]⋆	✓				
This work 1† [CHKL18]	✓	✓	✓	✓	
This work 2‡ [Pel18]			✓	✓	✓

⋆ for input-partitionable branching programs

† for specific choices of parameters

‡ in the quantum setting

Outline

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- 2ℓ matrices $A_{i,b}$ (for $i \in \{1, \ldots, \ell\}$ and $b \in \{0, 1\}$),
- two vectors A_0 and $A_{\ell+1}$,
- a function $\mathsf{inp} : \{1, \ldots, \ell\} \to \{1, \ldots, r\}$ (where r is the size of the input).

i	1	2	3	4	5	6
$\mathsf{inp}(i)$	1	1	2	1	3	2

$$x = \quad 0 \quad 1 \quad 1$$

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Cryptographic multilinear maps

Definition: κ-multilinear map

Different levels of encodings, from 1 to κ.
Denote by $\mathsf{Enc}(a, i)$ a level-i encoding of the message a.

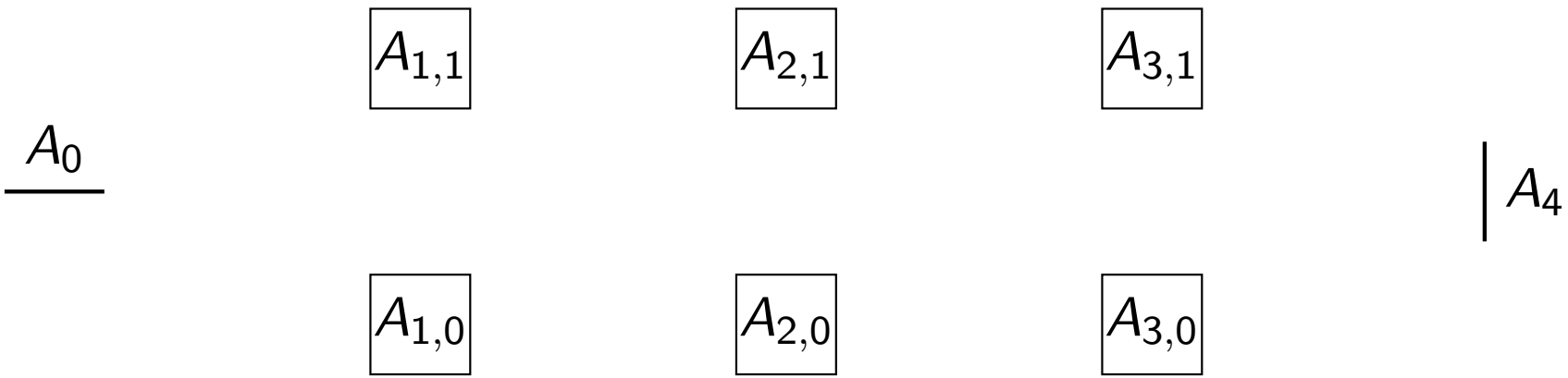
Addition: $\mathsf{Add}(\mathsf{Enc}(a_1, i), \mathsf{Enc}(a_2, i)) = \mathsf{Enc}(a_1 + a_2, i)$.

Multiplication: $\mathsf{Mult}(\mathsf{Enc}(a_1, i), \mathsf{Enc}(a_2, j)) = \mathsf{Enc}(a_1 \cdot a_2, i + j)$.

Zero-test: $\mathsf{Zero\text{-}test}(\mathsf{Enc}(a, \kappa)) = \mathsf{True}$ iff $a = 0$.

Simple obfuscator

- **Input:** A branching program
- Randomize the branching program
  - Add random diagonal blocks
  - Killian's randomization
  - Multiply by random (non zero) bundling scalars
- Encode the matrices using GGH13
- **Output:** The encoded matrices and vectors



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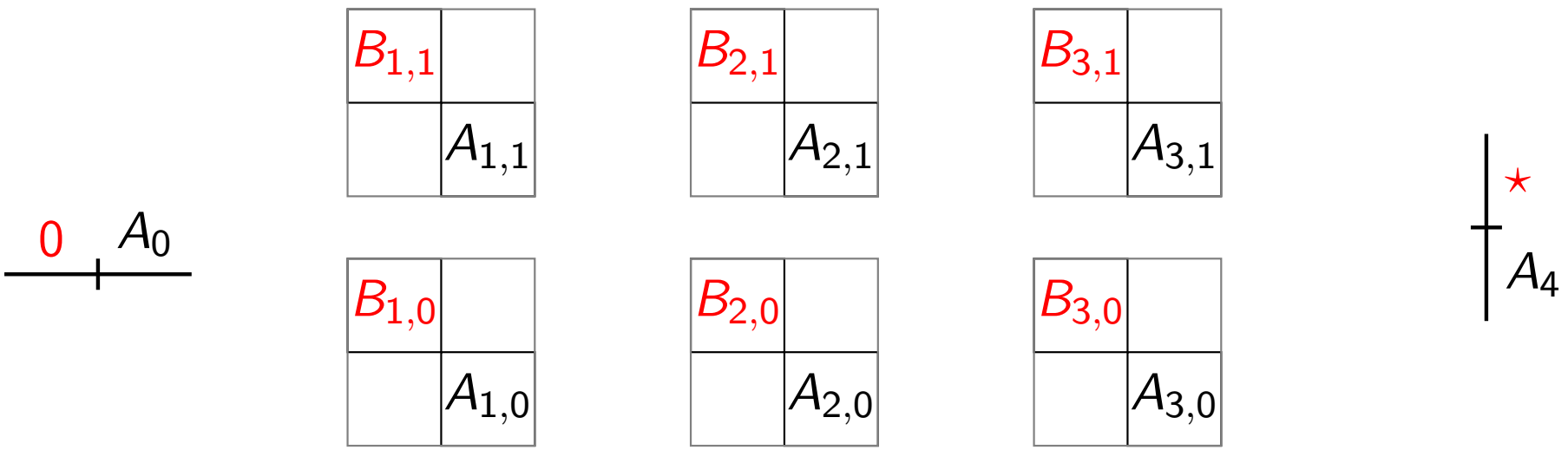

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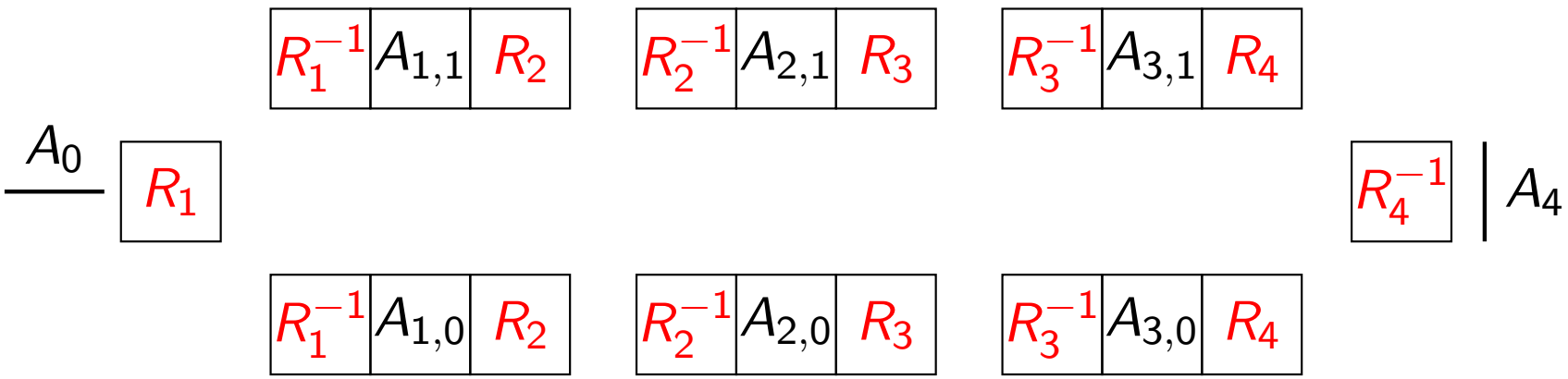

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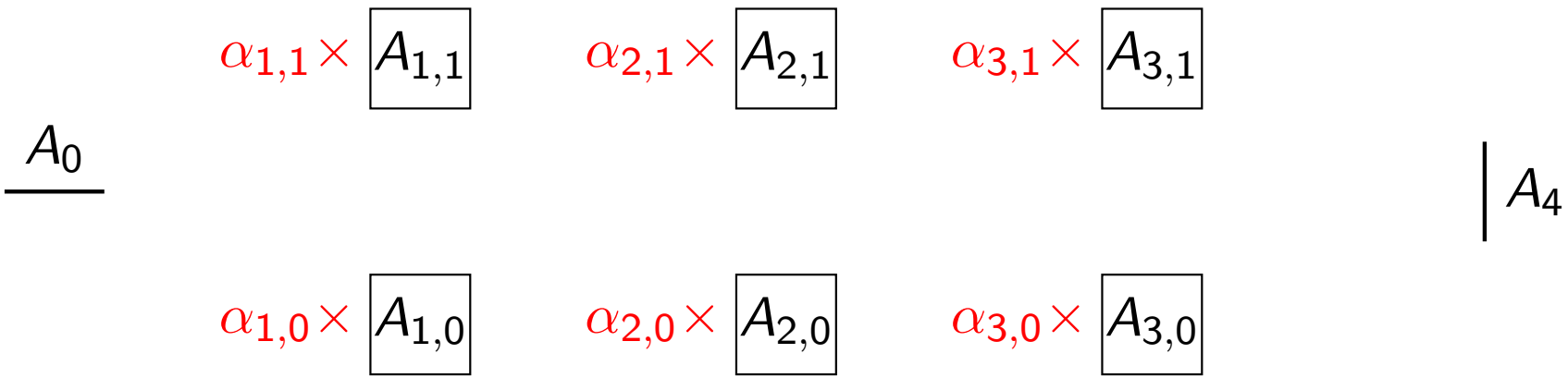

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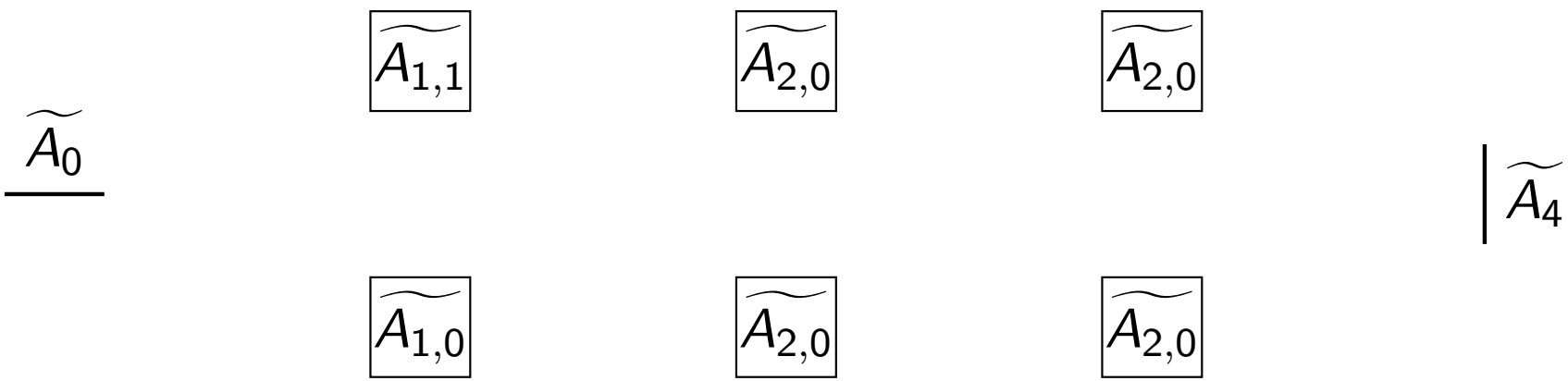

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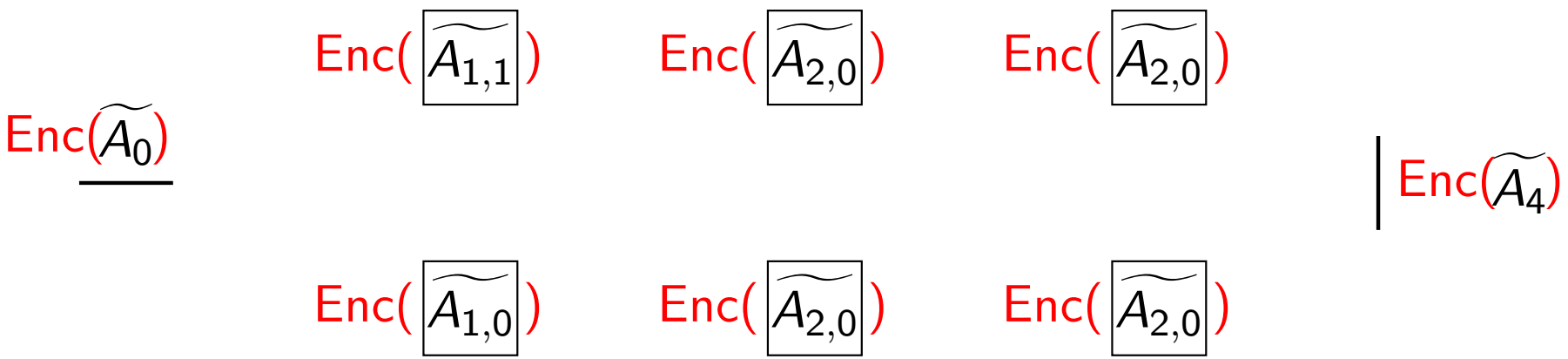

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- The plaintext space is $\mathcal{P} = R/\langle g \rangle$
  for a "small" element g in R.
- The encoding space is $R_q = R/(qR) = \mathbb{Z}_q[X]/(X^n + 1)$
  for a "large" integer q.
  - We write $[x]_q$ the elements in R_q for $x \in R$.

The GGH13 multilinear map: encodings and zero-test

- Sample z uniformly in R_q and h in R of the order of $q^{1/2}$.
- **Encoding:** An encoding of a at level i is

$$u = [(a + rg)z^{-i}]_q$$

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Zero-test

To test if $u = [cz^{-\kappa}]_q$ is an encoding of zero (i.e. $c = 0 \mod g$), compute

$$[u \cdot p_{zt}]_q = [chg^{-1}]_q.$$

This is small iff c is a small multiple of g.

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Global ideas of the two attacks

Main idea

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  - classical polynomial time, for specific choices of parameters
- Attack 2 [Pel18]:
  - short principal ideal problem algorithm [CDPR16]
  - quantum polynomial time [BS16][1]

[1] or classical sub-exponential time [BEF+17] for specific (unused) parameters

Attack 1: Starting point = NTRU

For two encodings $[a_1 \cdot z^{-1}]_q, [a_2 \cdot z^{-1}]_q$ for small a_1, a_2, we can compute

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$$(c \cdot a_1, c \cdot a_2) \in R^2$$

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$$(c \cdot a_1, c \cdot a_2) \in R^2$$

For another encoding $[a_3 \cdot z^{-1}]_q$, compute

$$[a_3 \cdot z^{-1}]_q / [a_1 \cdot z^{-1}]_q \cdot (c \cdot a_1) = c \cdot a_3 \in R.$$

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or, for GGH13 encodings,

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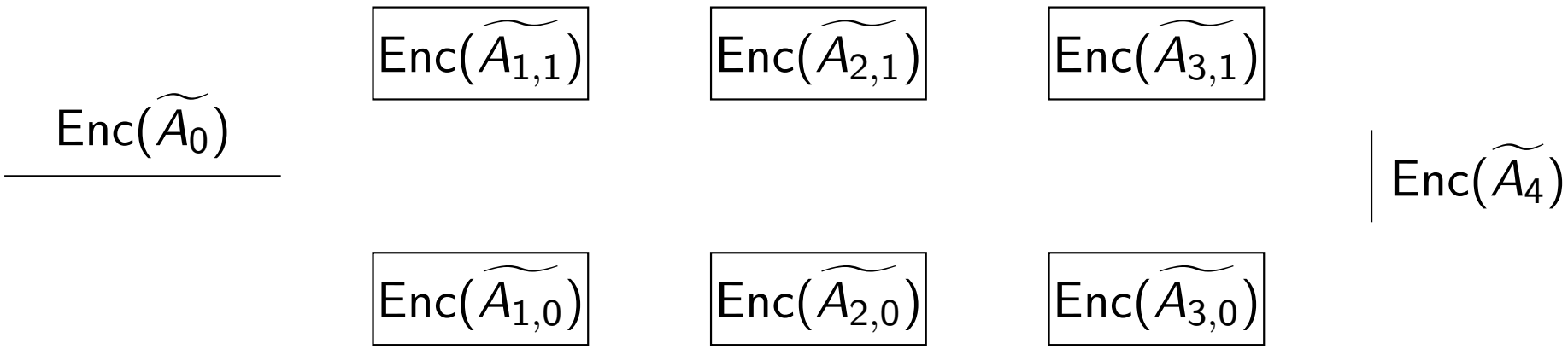

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$$\underline{c_0(\widetilde{A_0} + R_0 g)} \quad \begin{array}{|c|c|c|} \hline c_{1,1}(\widetilde{A_{1,1}} + R_{1,1} g) & c_{2,1}(\widetilde{A_{2,1}} + R_{2,1} g) & c_{3,1}(\widetilde{A_{3,1}} + R_{3,1} g) \\ \hline c_{1,0}(\widetilde{A_{1,0}} + R_{1,0} g) & c_{2,0}(\widetilde{A_{2,0}} + R_{2,0} g) & c_{3,0}(\widetilde{A_{3,0}} + R_{3,0} g) \\ \hline \end{array} \quad \Big| \; c_4(\widetilde{A_4} + R_4 g)$$

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These matrices $\in R$ rather than R_q

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BP matrix $\quad$ Enc$(\widetilde{A})$

output 0 $\quad$ enc$(0) = [rg/z^{\kappa}]_q$

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$$\begin{array}{lll} \text{BP matrix} & \text{Enc}(\widetilde{A}) & c(\widetilde{A} + Rg) \\ & \Rightarrow & \Rightarrow \quad c\widetilde{A} \bmod g \\ \text{output } 0 & \text{enc}(0) = [rg/z^{\kappa}]_q & c'rg \end{array}$$

$c\widetilde{A} \bmod g$ do not contain the randomness r and level parameter z

Attack 1: Mixed-input Attack

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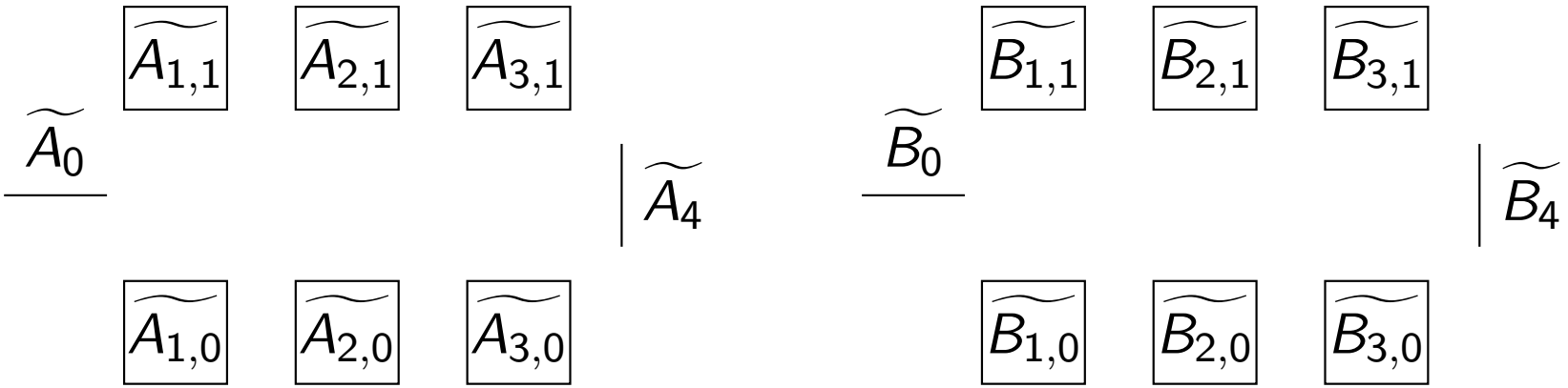

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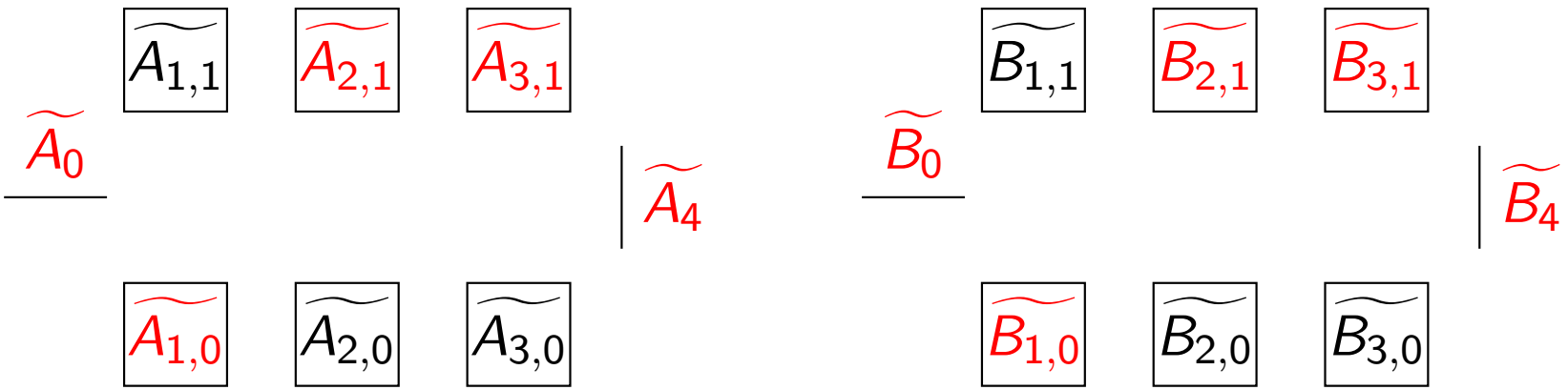

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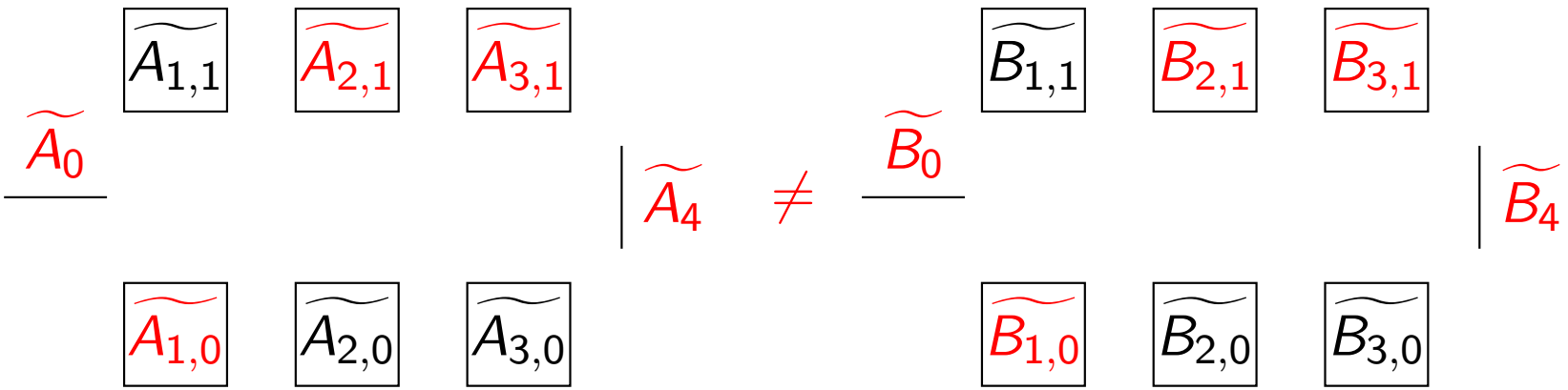

outputs zero

outputs non-zero

Attack 1: Matrix Zeroizing Attack

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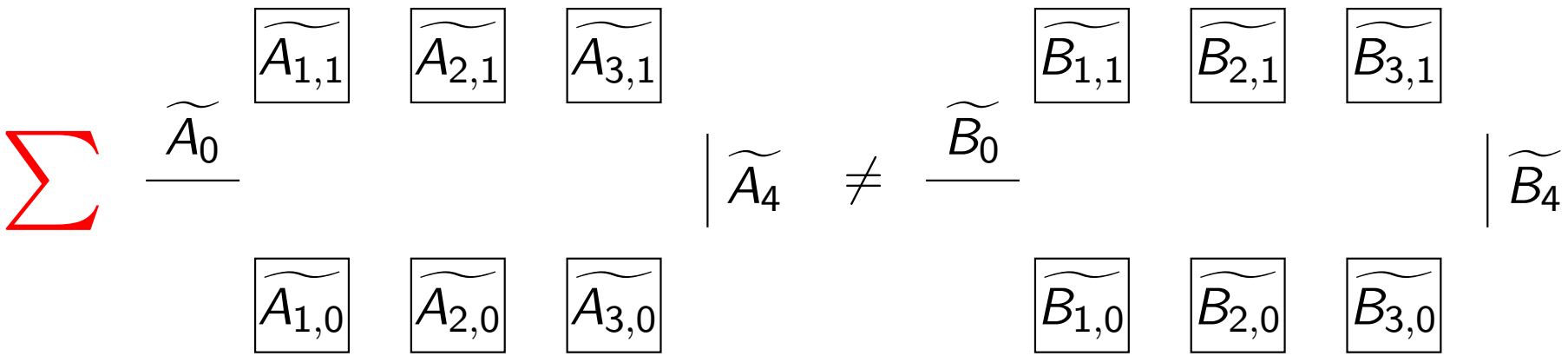
Matrix-zeroizing attack: extended mixed-input attack

- *Invalid* inputs can induce the different outputs of equivalent BPs
- Summation of mixed-input can yield the different outputs of BPs

i	1	2	3
$\mathsf{inp}(i)$	1	1	2

invalid input
010, 011, 100, 101, $\cdots$



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New Zerotesting Procedure

We can run 2κ level zerotest,
or 2κ level obfuscated program

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- Run mixed-input attack on obfuscated program at level κ

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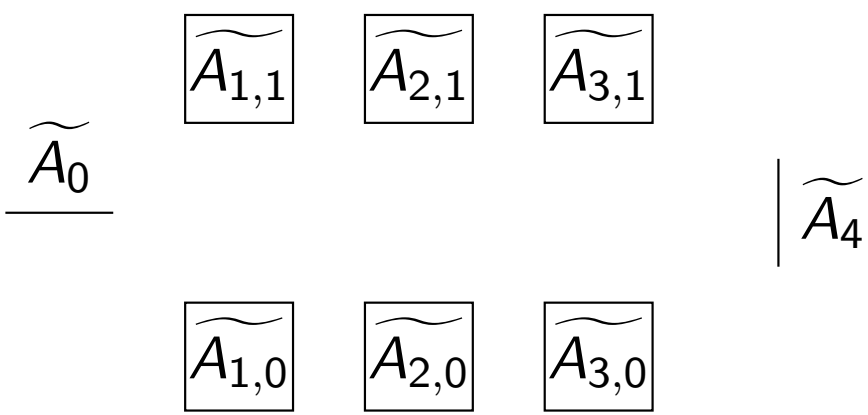

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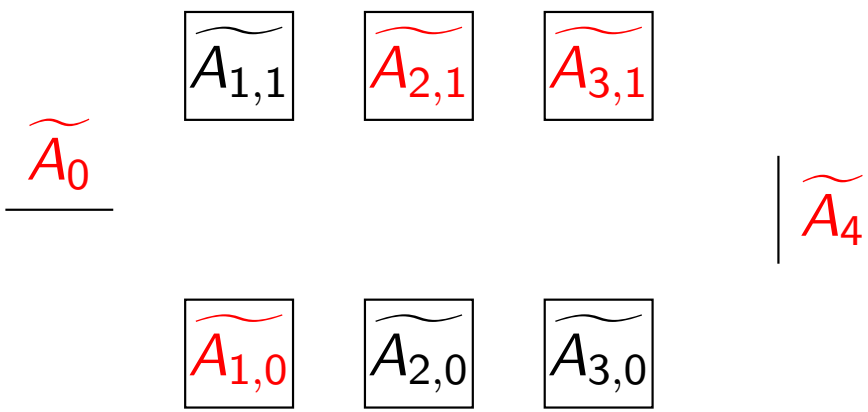

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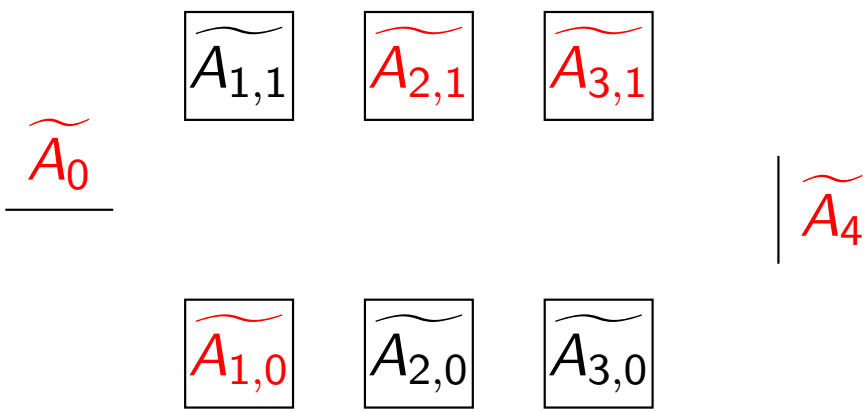

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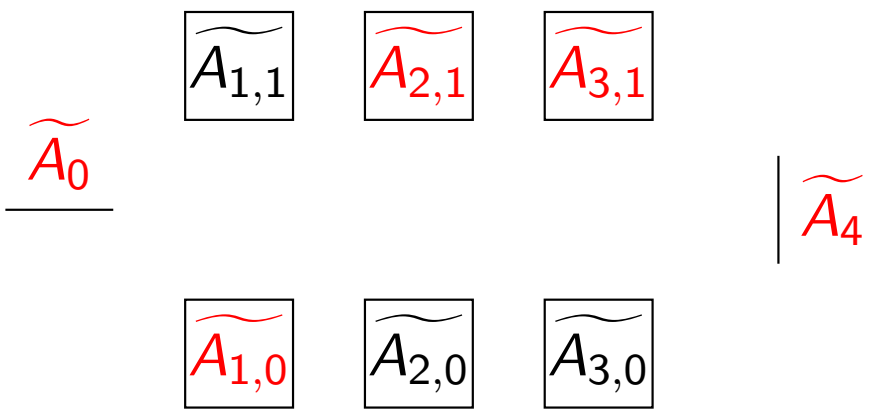

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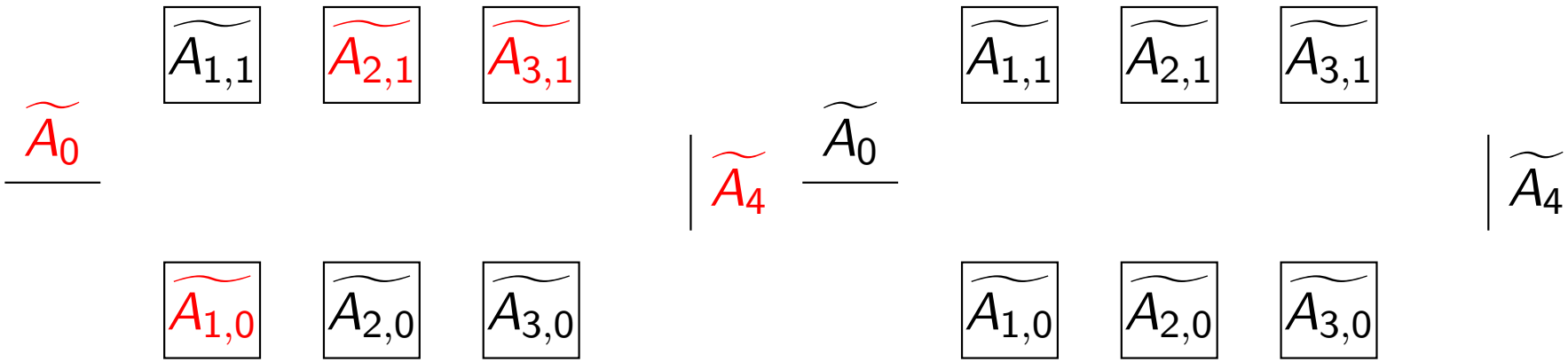

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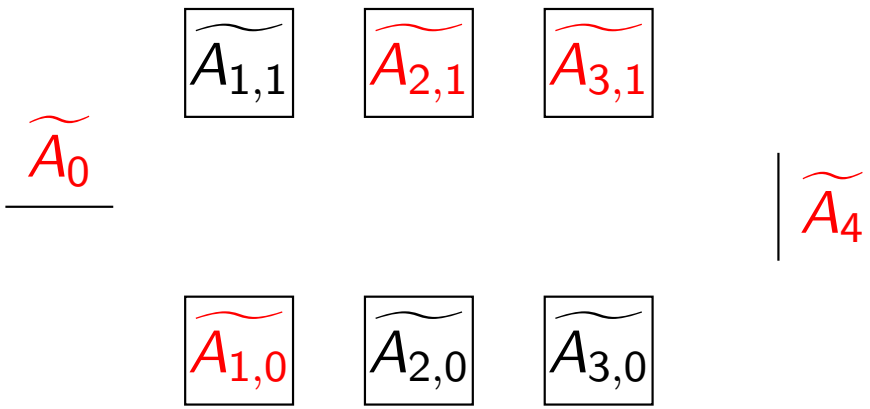

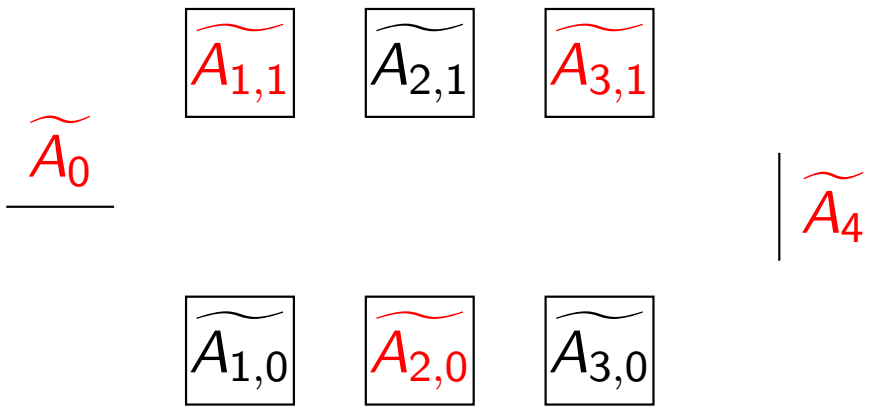

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Summary and work in progress

iO (using GGH13) / Attacks	Branching program obfuscators				Circuit obfuscators
	[GGH+13b]	[BR14]	[AGIS14, MSW14] [PST14, BGK+14] [BMSZ16]	[GMM+16]	[Zim15, AB15] [DGG+16]
[MSZ16]		✓	✓		
[CGH17]⋆	✓				
This work 1† [CHKL18]	✓	✓	✓	✓	
This work 2‡ [Pel18]			✓	✓	✓

⋆ for input-partitionable branching programs

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- [GGH$^+$13b] in quantum world

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- Circuit obfuscations in classical world
  - Extending the NTRU attack!

Perspectives / Open problems

- Obfuscation for evasive functions
- Countermeasure on the attacks
  - Parameter constraints to prevent our classical attack[3]: $n = \tilde{\Omega}(\kappa^2 \lambda)$

[3] n: dimension of space, κ: multilinearity level, λ: security parameter
To prevent classical PIP attack and our attack: $n = \tilde{\Omega}(\max(\kappa^2 \lambda, \lambda^2))$

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- Proofs in idealized models VS Constructions with concrete schemes

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- Many concrete schemes are not fit in the idealized model

$\Rightarrow$ This gap can cause a significant weakness of the concrete scheme!

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To prevent classical PIP attack and our attack: $n = \tilde{\Omega}(\max(\kappa^2\lambda, \lambda^2))$

Thank you!

감사합니다! Merci!

For more details, see papers or eprint reports

- Quantum attack: ia.cr/2018/533
- Classical attack: ia.cr/2018/408
- Combined/Extended work: Coming Soon..?

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