GGH15 beyond permutation branching programs proofs, attacks, and candidates

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ENS
ÉCOLE NORMALE SUPÉRIEURE

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> Alice finds a public-key encryption scheme based on Schrodinger's equation.

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> Alice missed the NIST PQC round one. But she find it cool to post it on the blockchain, and offers 100 Bitcoins to whoever breaks it.

> Not only does Alice post on the blockchain, she does it cool by encrypting the 100 Bitcoins using Witness encryption.

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> WitnessEnc(x, m), x = instance, m = message

Functionality: if x = SAT -----> can use the witness to decrypt the msg.

Security: if x = UNSAT -------> msg is hidden.

WitnessEnc(x = "there is an attack to Alice's PKE scheme", msg = 100 Bitcoins)

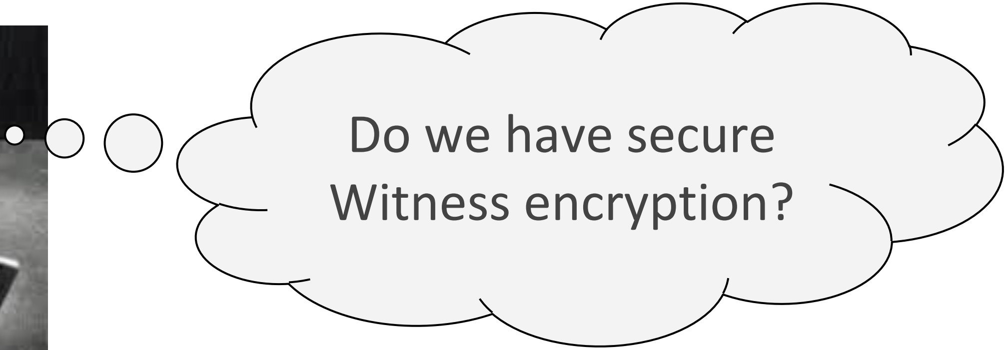
> Current status of witness encryption: there are several candidates (more-or-less based on multilinear maps); none of them are based on established cryptographic assumptions.

> [Garg et al. 13] candidate witness encryption based on GGH13.

> Broken by [Hu, Jia 16]

> [Gentry, Lewko, Waters 14] from multilinear subgroup decision assumption (which is also open)

> Null-iO candidates (there are many) => Witness encryption candidates



I am the title

GGH15 beyond permutation branching programs proofs, attacks, and candidates

Wait, what's the relation of witness encryption and the title??

A candidate multilinear map

GGH15 beyond permutation branching programs
proofs, attacks, and candidates

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applications

Private constrained PRFs

Multi party key agreement

Lockable obfuscation
(Compute-then-Compare obf.)

General purpose
Indistinguishability obfuscation

Security ????

$\boxed{\text{GGH15}}$ beyond permutation branching programs

proofs, attacks, and candidates

(As secure as LWE)

What we knew:

Private constrained PRFs

Multi party key agreement

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Motivation of this work: systematically study GGH15, discover more attacks and safe applications

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Motivation of this work: systematically study GGH15, discover more attacks and safe applications (maybe witness encryption?)

GGH15 beyond permutation branching programs

proofs, attacks, and candidates

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Witness encryption ???

Multi party key agreement

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Summary of the results for GGH15 + non-perm branching programs:

- Proofs (focus of the talk):
  > Introduce new lattice toolkits;
  > New analysis techniques for GGH15.
  > Leads to PCPRFs and lockable obfuscation for general BPs.

- Attacks: New attacks on the iO candidates.

- Candidates: Witness encryption and iO.

Multilinear maps in a nutshell

> Multilinear maps: motivated in [Boneh, Silverberg 2003]

$$g, g^{s_1}, g^{s_2}, g^{s_3}, \ldots \rightarrow g^{\prod s}$$

Can be thought of as homomorphic encryption + public zero-test

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> Bilinear maps from elliptic curves [Miller 1986]
> n-linear maps candidates: (all based on non-standard use of lattices)
>>>> Garg, Gentry, Halevi 2013 [GGH 13]
>>>> Coron, Lepoint, Tibouchi 2013 [CLT 13]
>>>> Gentry, Gorbunov, Halevi 2015 [GGH 15] (LWE-like)

*New: Trilinear maps from abelian varieties [Huang 2018], requires further investigation.

GGH15 in a nutshell

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> (Ring)LWE analogy:

$$A, S_1 A + E_1, \ldots, S_k A + E_k \rightarrow \prod S A + E \mod q$$

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GGH15: "the blockchain in multilinear maps"

(also appear as "cascaded LWE" in [Koppula-Waters 16], [Alamati-Peikert 16])

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$$A_0\ D_1 = S_1A_1+E_1, \quad A_1\ D_2 = S_2A_2+E_2 \mod q$$

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D_i is sampled using the trapdoor of A_{i-1}

Lattice trapdoor 101
[Ajtai 99, Alwen, Peikert 09, Micciancio, Peikert 12]

A with trapdoor

Given Y find D s.t. A x D = Y

GGH15 in a nutshell

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Publish A_0, D_1, D_2 as the encodings of S_1, S_2

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$$\text{Eval} = A_0 D_1 D_2 = (S_1A_1+E_1)D_2 = \underbrace{S_1S_2A_2}_{\text{functionality}} + \underbrace{E_1D_2+S_1E_2}_{\text{small}} \mod q$$

When witness encryption meets multilinear maps ...

[Gentry, Lewko, Waters 14] witness encryption from mmaps subgroup decision assumption, which is instance independent.

[Gentry, Lewko, Waters 14] a special witness encryption from mmaps.

A strawman implementation of GLW14 in GGH15

$$A_0 D_{1,0} = S_{1,0} A_1 + E_{1,0}, \ldots, \quad A_{h-1} D_{h,0} = S_{h,0} A_h + E_{h,0} \quad \bmod q$$

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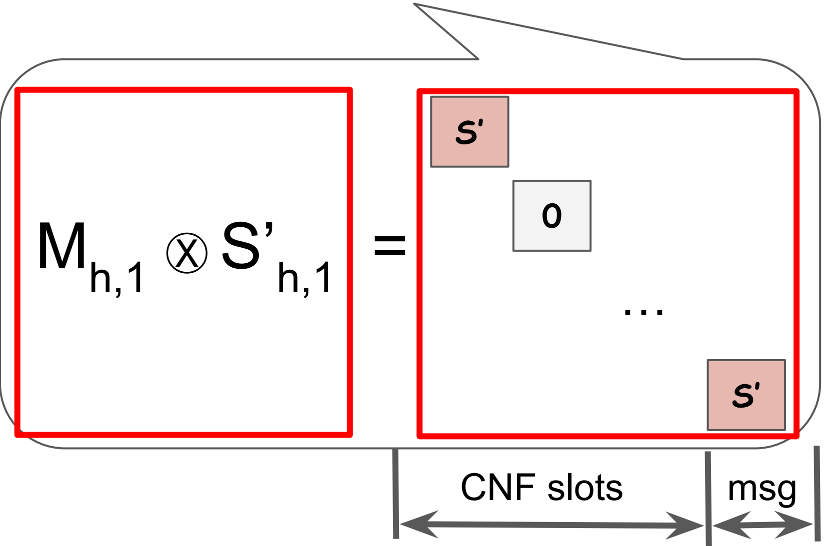

- Low-rank matrices (bad news)
- Read-once BP (good news)

So far: A witness encryption with special structure that uses

GGH15 + low-rank matrix branching program.

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Q: Can we show anything secure for low-rank BP + GGH15?

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Q: Can we show anything secure for low-rank BP + GGH15?

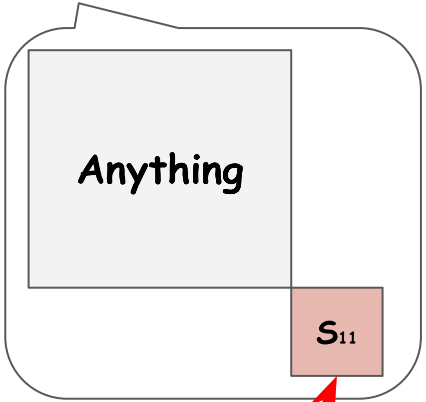
A: Yes! … In some limited cases

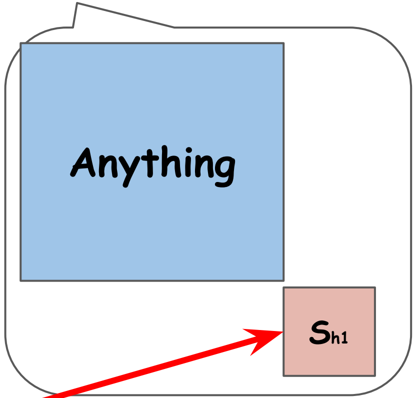
As secure as LWE:

When there is one “slot” that is always random in all the matrices.

$$A_0 D_{1,0} = S_{1,0} A_1 + E_{1,0}, \ldots, \quad A_{h-1} D_{h,0} = S_{h,0} A_h + E_{h,0} \quad \mod q$$

$$A_0 D_{1,1} = \boxed{S_{1,1}} A_1 + E_{1,1}, \ldots, \quad A_{h-1} D_{h,1} = \boxed{S_{h,1}} A_h + E_{h,1} \quad \mod q$$





The “always random” slot

Where can the special type of BP be useful?

$$A_0 D_{1,1} = \boxed{S_{1,1}} A_1 + E_{1,1}, \ \ldots, \quad A_{h-1} D_{h,1} = \boxed{S_{h,1}} A_h + E_{h,1} \mod q$$

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We don’t know how to build a witness encryption or iO from this type of BP :(

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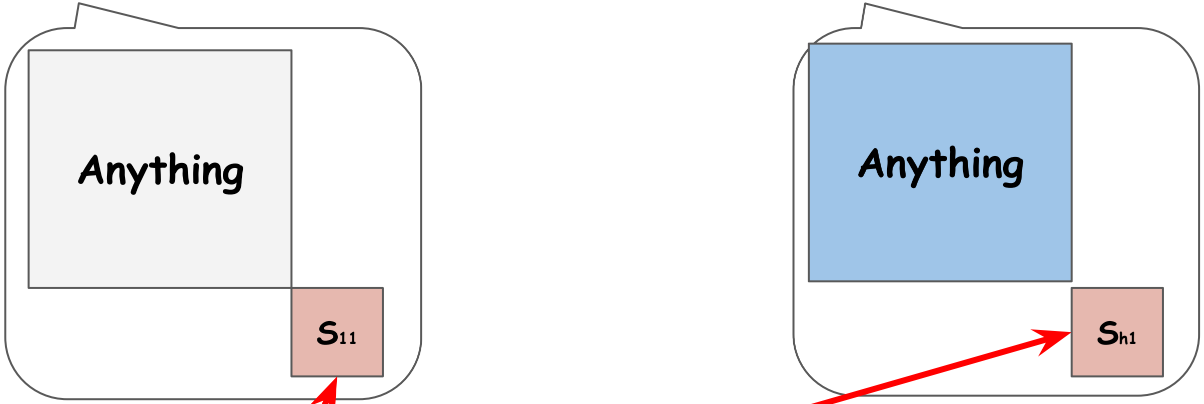

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We can simplify the private constrained PRF, Lockable obfuscation :)

E.g. Instantiate the private puncturable PRF from [Boneh, Lewi, Wu 17] described under the multilinear subgroup decision assumption:

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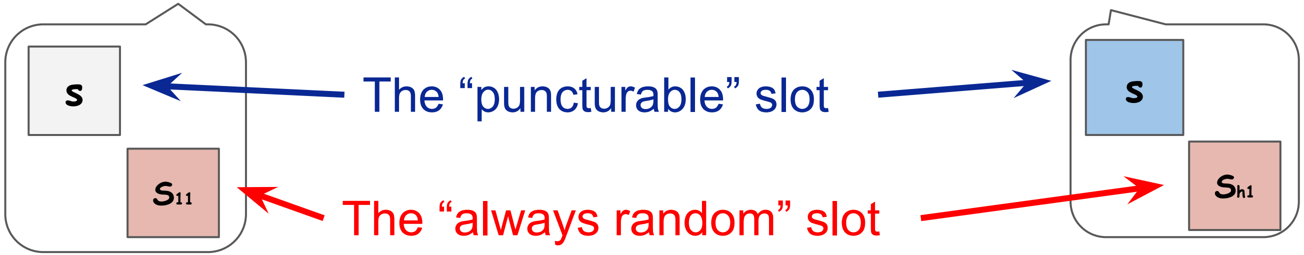

How to prove security for GGH15 + low-rank BPs?

What are you
trying to prove?

How to prove security for GGH15 + low-rank BPs?

Semantic security:

$$A_0\, D_{1,0} = S_{1,0} A_1 + E_{1,0},\ \ldots,\quad A_{h-1}\, D_{h,0} = S_{h,0} A_h + E_{h,0} \quad \bmod q$$

$$A_0\, D_{1,1} = S_{1,1} A_1 + E_{1,1},\ \ldots,\quad A_{h-1}\, D_{h,1} = S_{h,1} A_h + E_{h,1} \quad \bmod q$$

$$\approx \text{ computational}$$

$$A_0\, D_{1,0} = \boxed{U_{1,0}},\ \ldots,\quad A_{h-1}\, D_{h,0} = \boxed{U_{h,0}} \quad \bmod q$$

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"A" matrices: using trapdoors; not using trapdoors

Replay: the proof for GGH15 + permutation BP

[Canetti, Chen 17], [Goyal, Koppula, Waters 17], [Wichs, Zirdelis 17]

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Goal: prove semantic security

For permutation BP [Canetti, Chen 17], [Goyal, Koppula, Waters 17], [Wichs, Zirdelis 17]:

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$$A_0 D_{1,0} = S_{1,0} A_1 + E_{1,0}, \ldots, \quad A_{h-1} D_{h,0} = S_{h,0} A_h + E_{h,0} \quad \text{mod } q$$

$$A_0 D_{1,1} = \boxed{S_{1,1}} A_1 + E_{1,1}, \ldots, \quad A_{h-1} D_{h,1} = \boxed{S_{h,1}} A_h + E_{h,1} \quad \text{mod } q$$

$$\begin{pmatrix} S_{11} & & \\ & S_{11} & \\ & & S_{11} \end{pmatrix} \qquad \begin{pmatrix} S_{h1} & & \\ & & S_{h1} \\ & S_{h1} & \end{pmatrix}$$

LWE 101 [Regev 05]

$$A \,,\quad S \times A + E$$

$$\approx_{\text{computational}} \quad A \,,\quad U$$

LWE 101 [Regev 05]

A , S x A + E

$\approx$ computational A , U

Permutation - LWE:

$$\begin{pmatrix} A(1) \\ A(2) \\ A(3) \end{pmatrix}, \boxed{\begin{matrix} & S & \\ S & & \\ & & S \end{matrix}} \times \begin{pmatrix} A(1) \\ A(2) \\ A(3) \end{pmatrix} + E$$

$\approx$ computational

$$\begin{pmatrix} A(1) \\ A(2) \\ A(3) \end{pmatrix}, U$$

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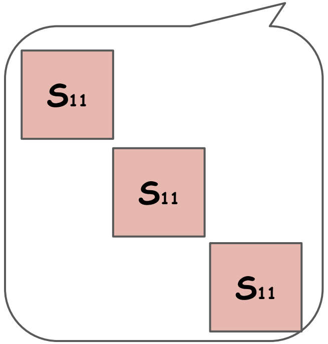

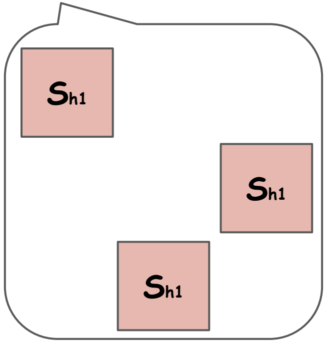

[Step 1] LWE: $A_h, S_{h,0} A_h + E_{h,0}, S_{h,1} A_h + E_{h,1} \approx A_h, U_{h,0}, U_{h,1}$

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$$A_0 D_{1,1} = \boxed{S_{1,1}} A_1 + E_{1,1}, \ \ldots, \quad A_{h-1} D_{h,1} = \; U_{h,1} \quad {}_1 \quad \bmod q$$

$$\begin{pmatrix} s_{11} & & \\ & s_{11} & \\ & & s_{11} \end{pmatrix}$$

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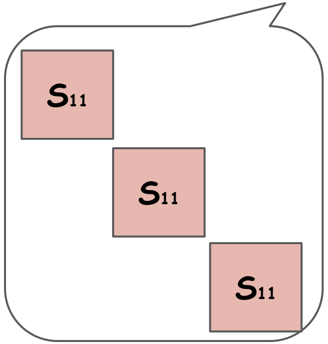
$$A_0 D_{1,1} = \boxed{S_{1,1}} A_1 + E_{1,1}, \ldots, \quad A_{h-1} D_{h,1} = U_{h,1} \;\; 1 \quad \text{mod } q$$

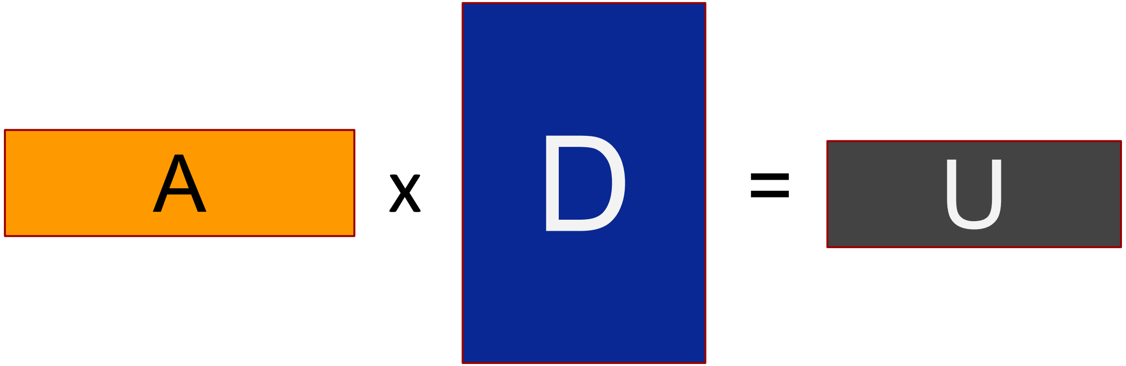


[Step 2] GPV: close the trapdoor of A_{h-1}

[Gentry, Peikert, Vaikuntanathan 08]

U is uniform

A trapdoor is used



[Gentry, Peikert, Vaikuntanathan 08]

U is uniform

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$$A \times D = U$$

$\approx$ statistical

$$A \times D = U$$

close the trapdoor of A

Goal: prove semantic security

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...

[Step …] LWE .... GPV: close the trapdoor of A_1

Goal: prove semantic security

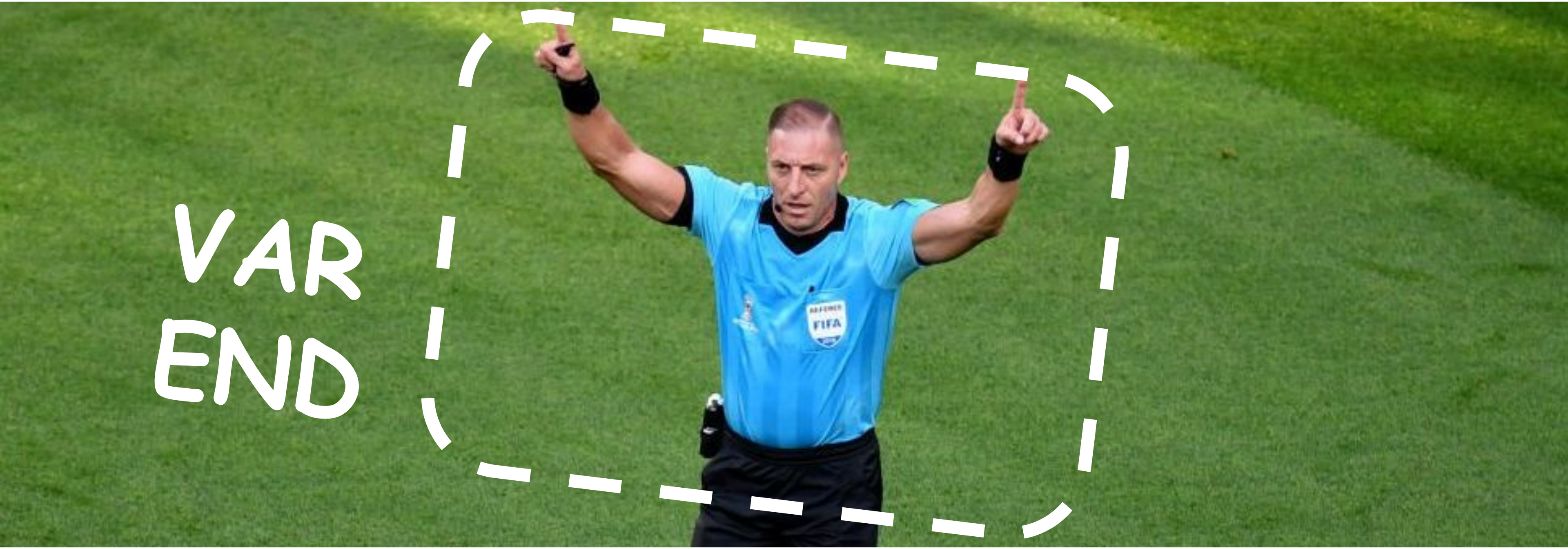
For permutation BP [Canetti, Chen 17], [Goyal, Koppula, Waters 17], [Wichs, Zirdelis 17]:

"A" matrices: using trapdoors; not using trapdoors

$$A_0 D_{1,0} = U_{1,0}, \ldots, A_{h-1} D_{h,0} = U_{h,0} \quad \text{mod } q$$

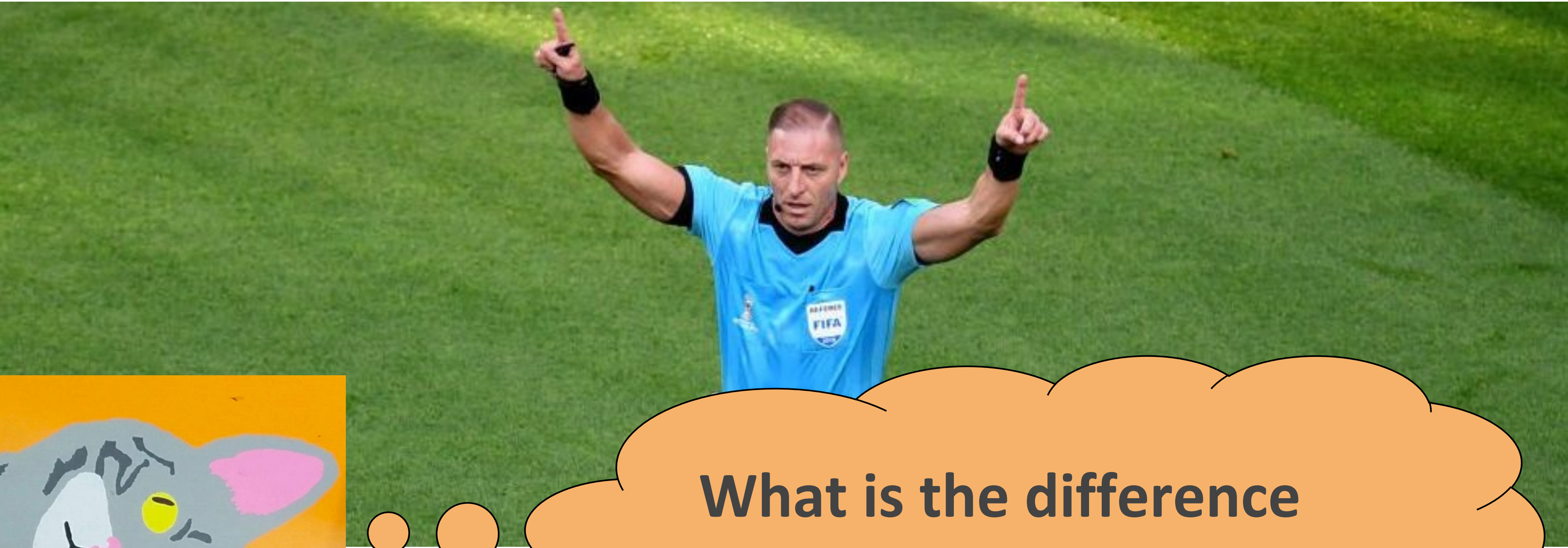
$$A_0 D_{1,1} = U_{1,1}, \ldots, A_{h-1} D_{h,1} = U_{h,1} \quad \text{mod } q$$

[Final Steps] Another LWE + GPV



Replay: the proof for GGH15 + permutation BP

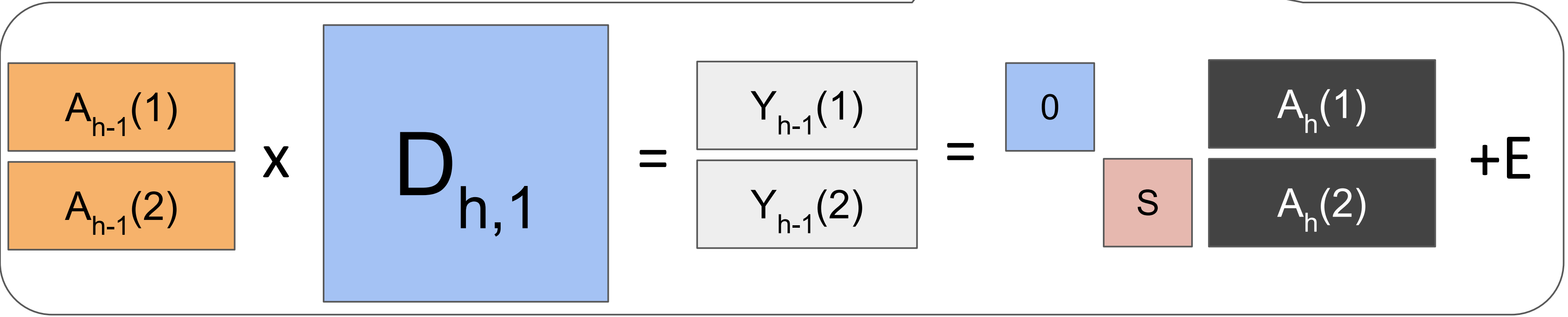
[Canetti, Chen 17], [Goyal, Koppula, Waters 17], [Wichs, Zirdelis 17]



What is the difference for low-rank matrices?

For possibly low-rank secret matrices: helpful to separate the matrices into (1) and (2)

$$A_0 D_{1,1} = S_{1,1} A_1 + E_{1,1}, \ldots, \boxed{A_{h-1} D_{h,1} = S_{h,1} A_h + E_{h,1}} \quad \text{mod } q$$



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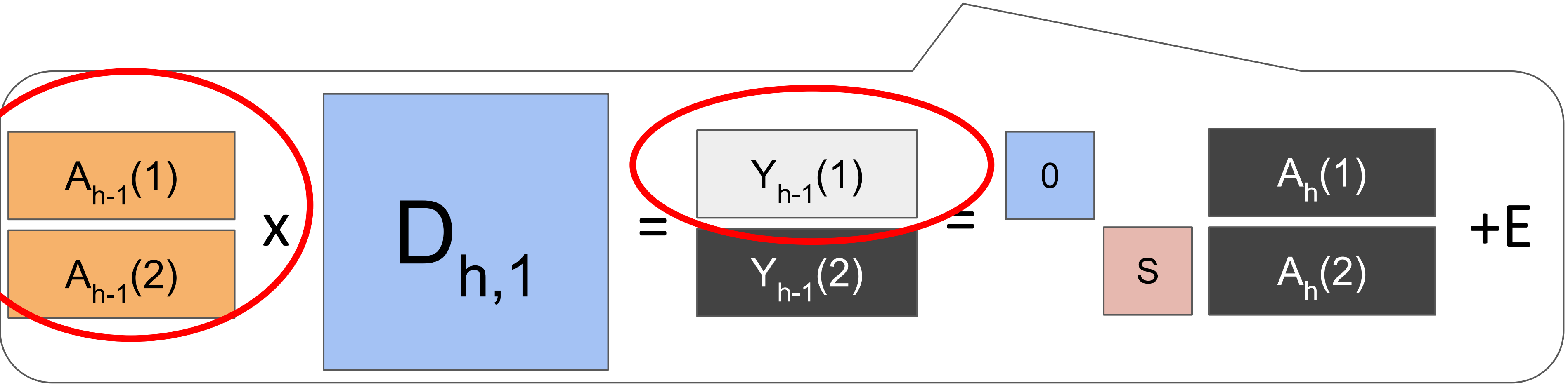


Observation: $Y_{h-1}(1)$ is not random

The problem: How to close the trapdoor of A_{h-1} ?

Lattice trapdoor Lemma 1:

Z is arbitrary

U is uniform

A trapdoor is used

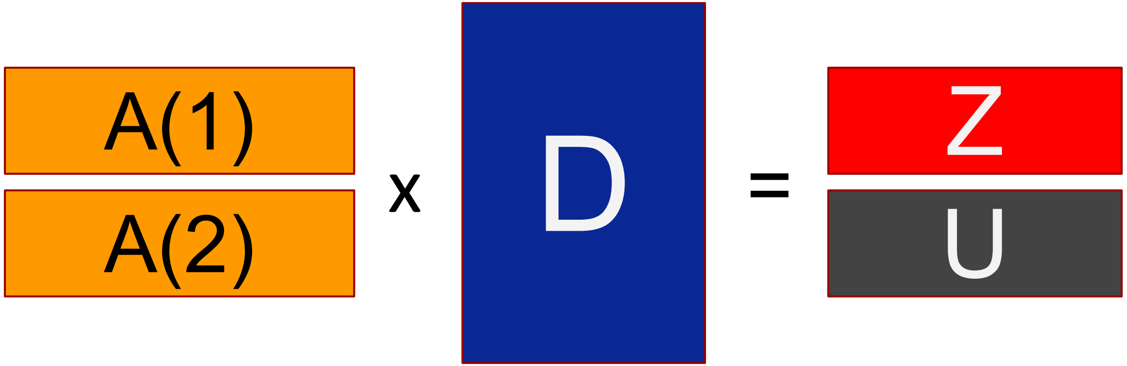
$$\begin{bmatrix} A(1) \\ A(2) \end{bmatrix} \times D = \begin{bmatrix} Z \\ U \end{bmatrix}$$

Lattice trapdoor Lemma 1:

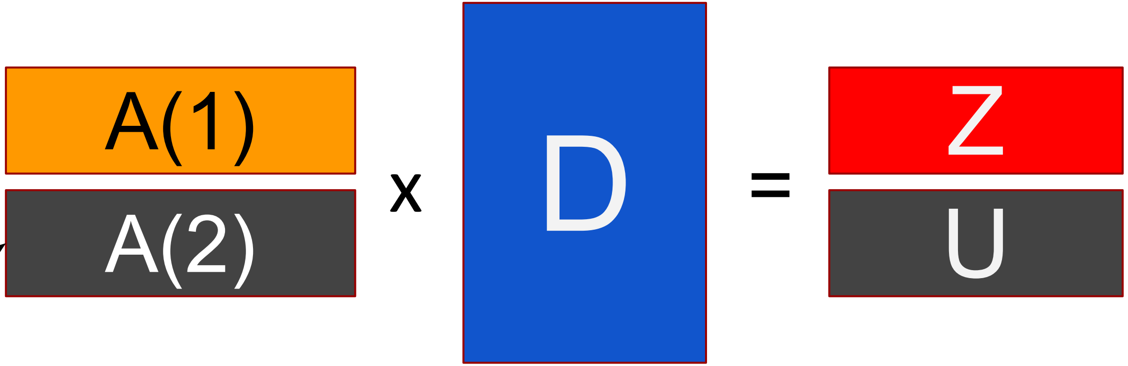
Z is arbitrary

U is uniform

A trapdoor is used



≈ statistical



close the trapdoor of A(2)

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$$\begin{bmatrix} A_{h-1}(1) \\ A_{h-1}(2) \end{bmatrix} \times D_{h,1} = \begin{bmatrix} Y_{h-1}(1) \\ Y_{h-1}(2) \end{bmatrix} = \begin{bmatrix} 0 & \\ & S \end{bmatrix} \begin{bmatrix} A_h(1) \\ A_h(2) \end{bmatrix} + E$$

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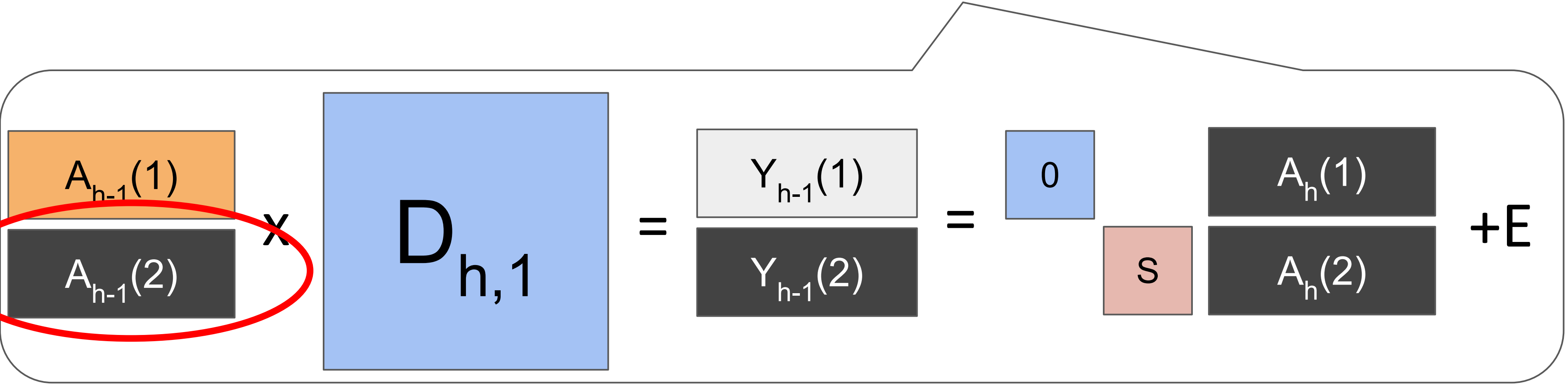

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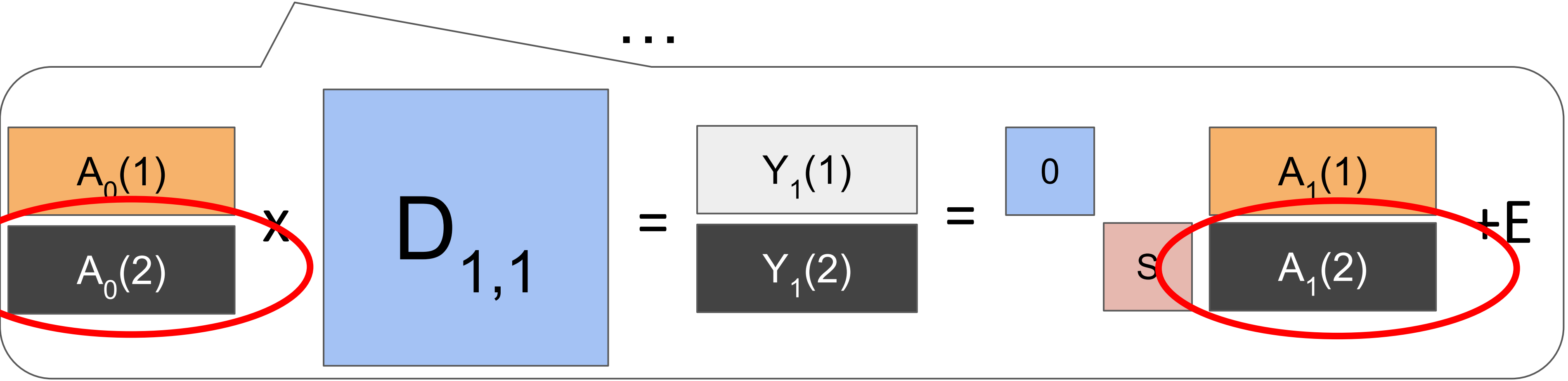

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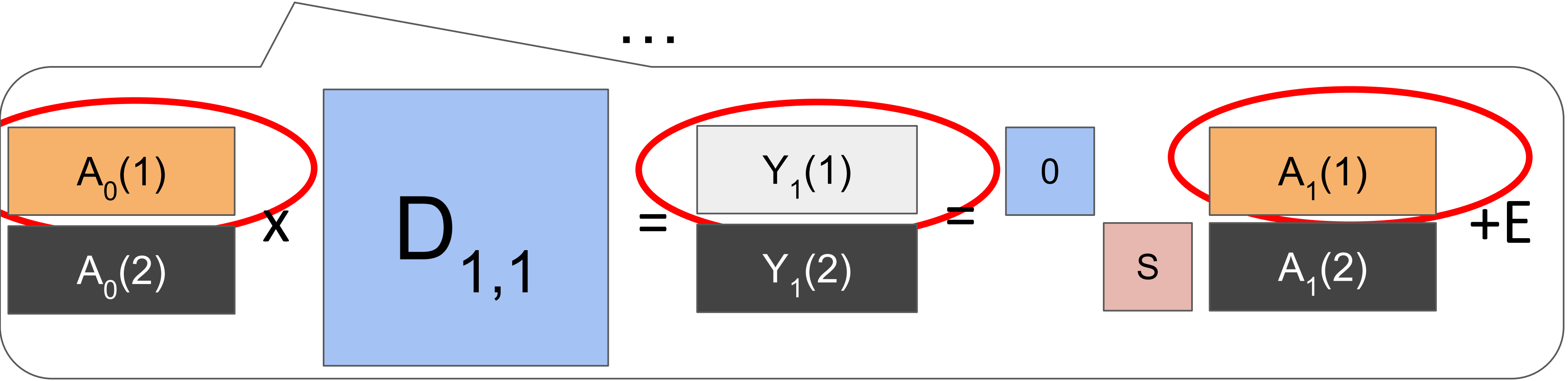

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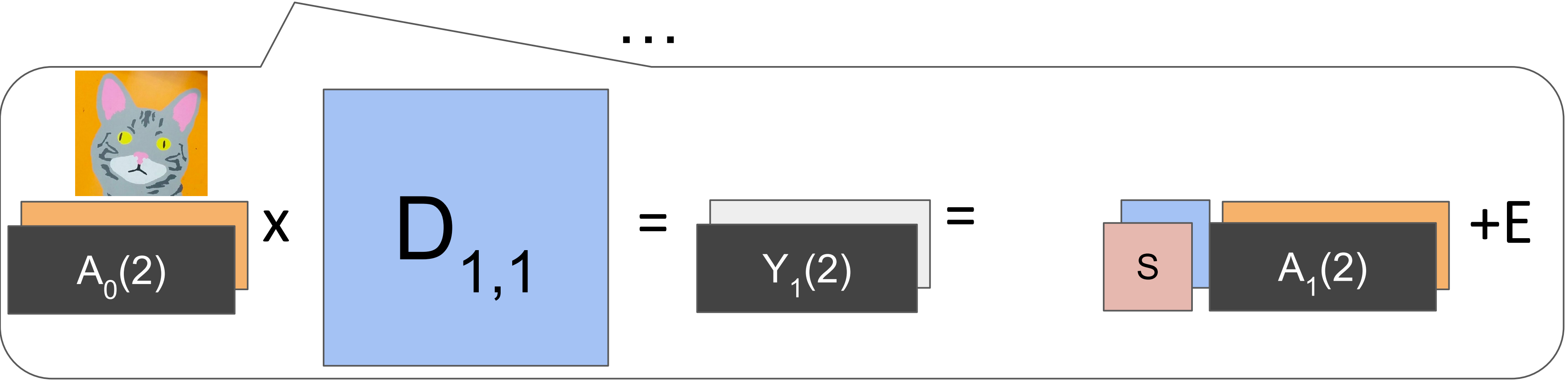

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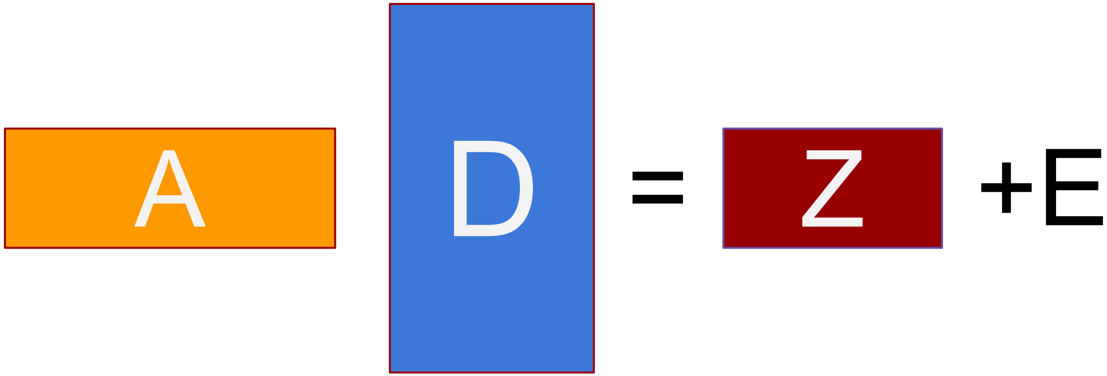
Lattice trapdoor Lemma 2:



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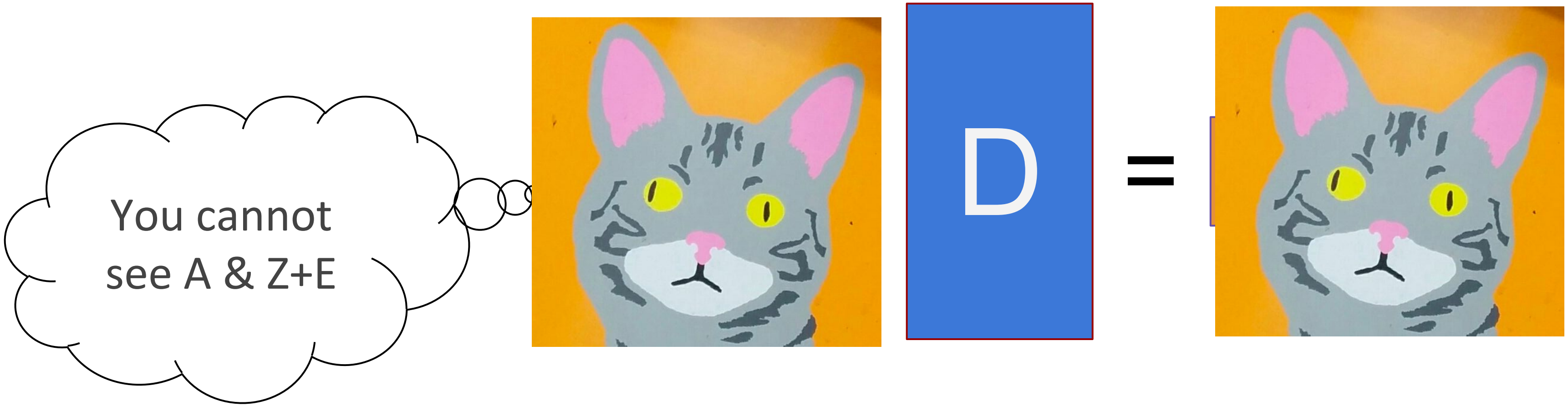

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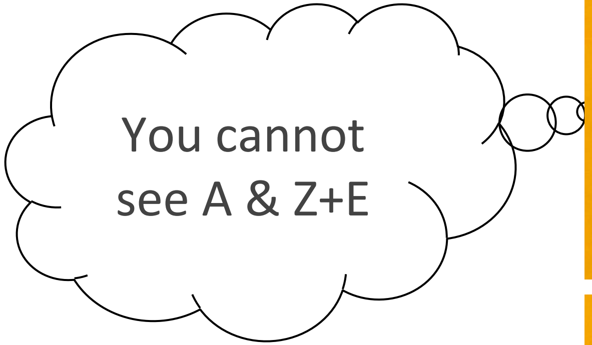
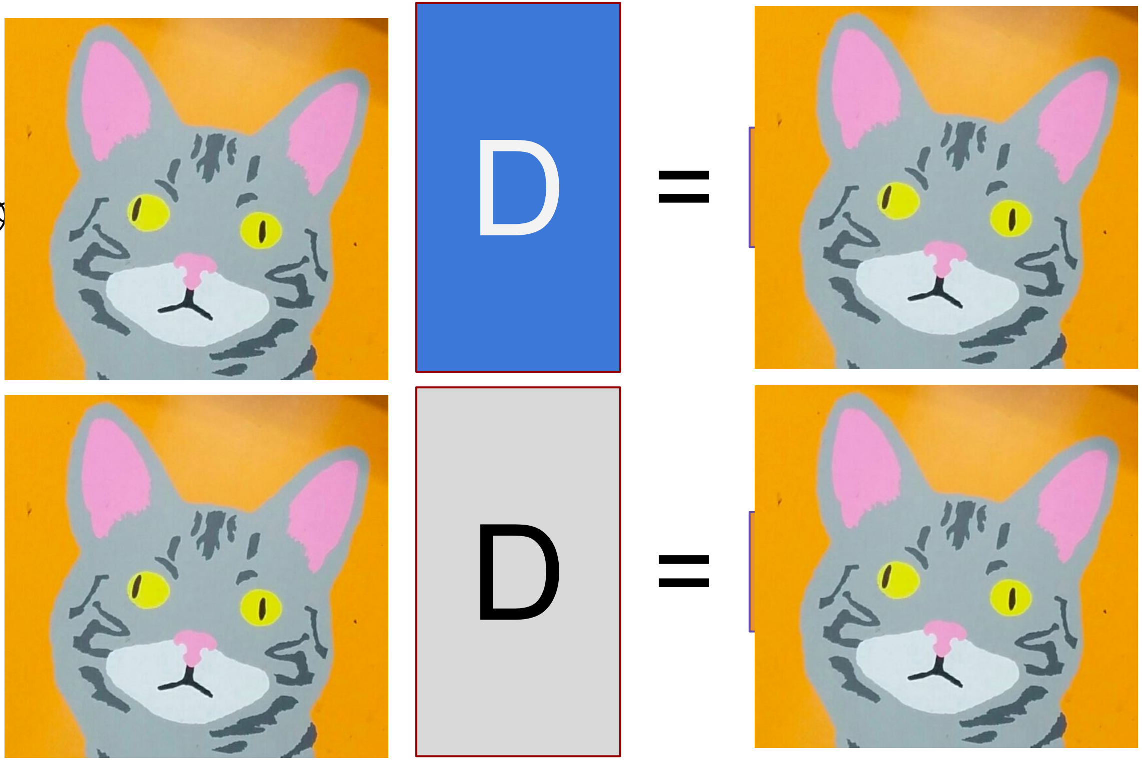


≈ computational



For any Z, for a uniformly random A, D is the preimage of Z+E.

If A & Z+ E is hidden, then D is indistinguishable from random Gaussian.

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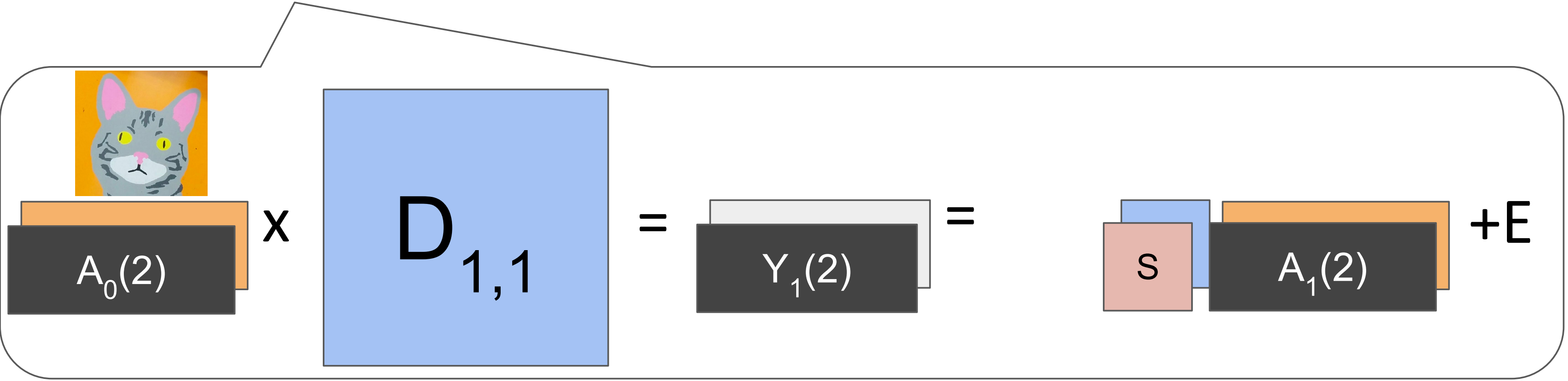

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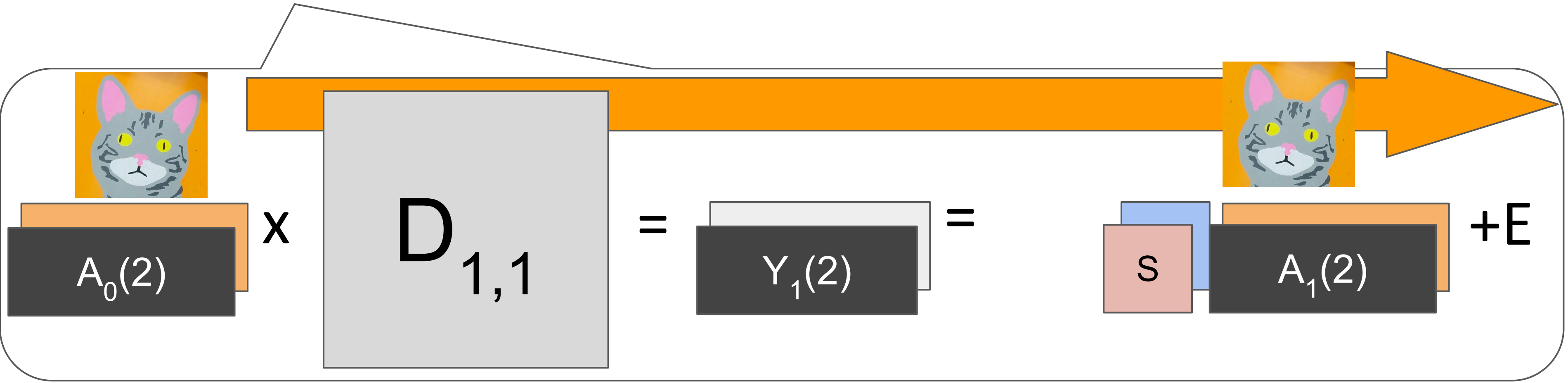

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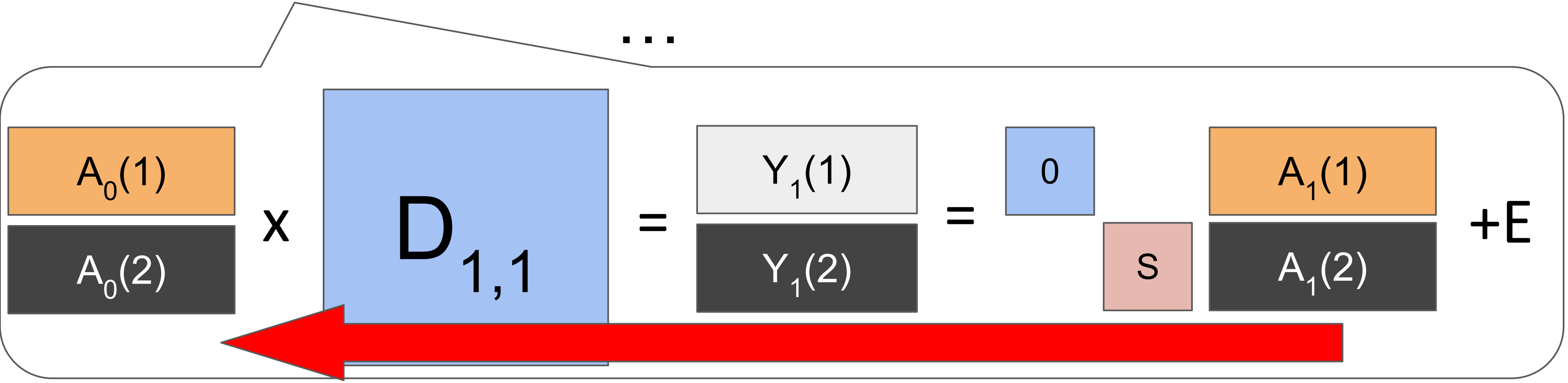

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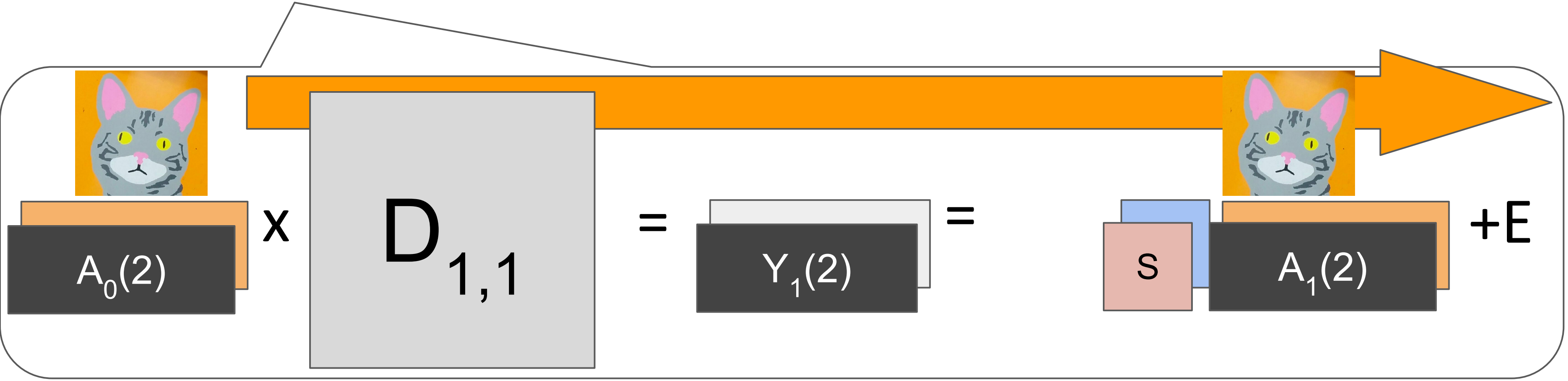

First use the lower level random matrices to come left (need new lemma 1)

Then use the upper level “hidden A at the left” to go right (need new lemma 2)

End of the proof for GGH15 + low-rank BP

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Q: What about the other cases without a proof from LWE?

A: Hmm … some of them can be broken.

New attack on iO candidates based on GGH15.

With a very simple attack algorithm

New attack on iO candidates based on GGH15.

With a very simple attack algorithm:

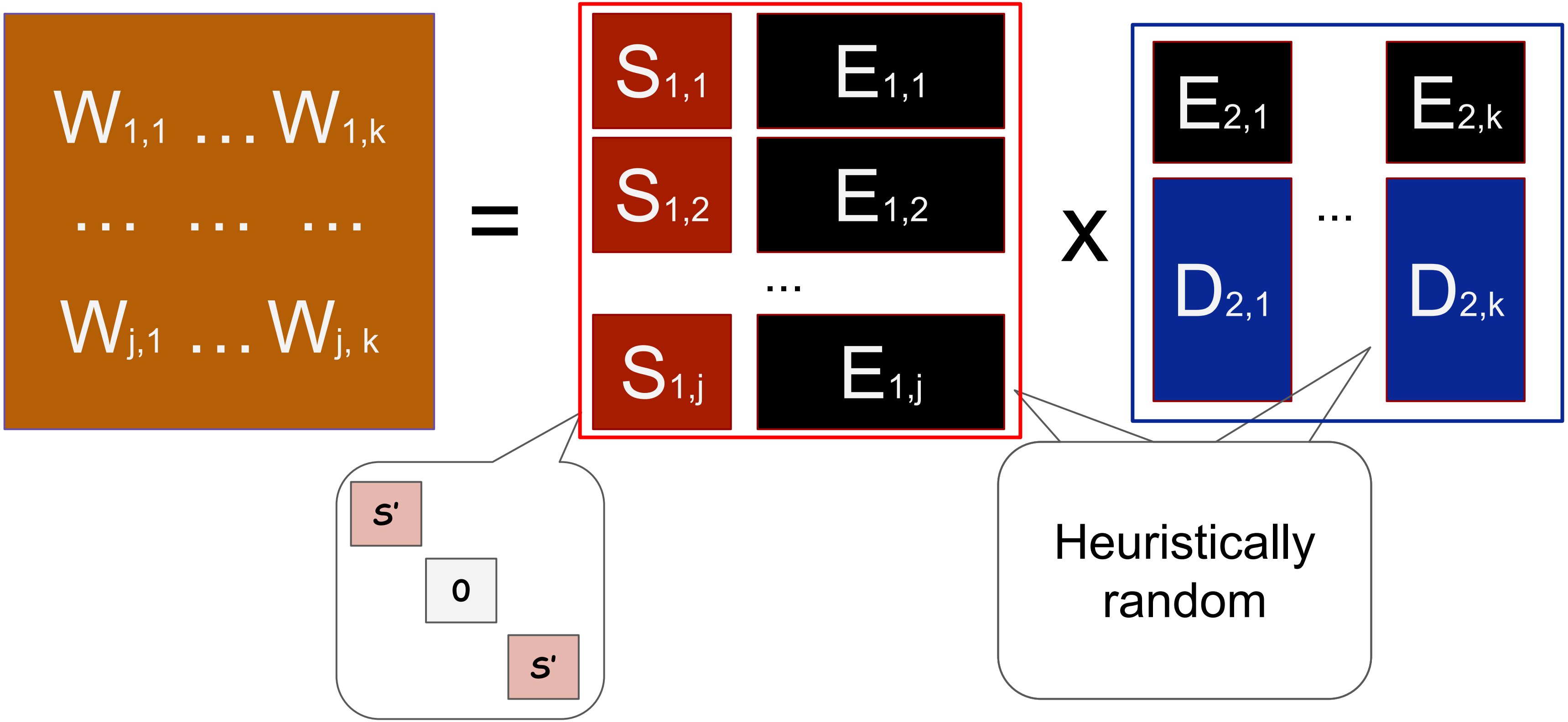
First compute a matrix,

$$\begin{matrix} W_{1,1} & \ldots & W_{1,k} \\ \ldots & \ldots & \ldots \\ W_{j,1} & \ldots & W_{j,k} \end{matrix} = \text{Results on many inputs that eval to small}$$

New attack on iO candidates based on GGH15.

With a very simple attack algorithm:

First compute a matrix, then compute the rank of the matrix.



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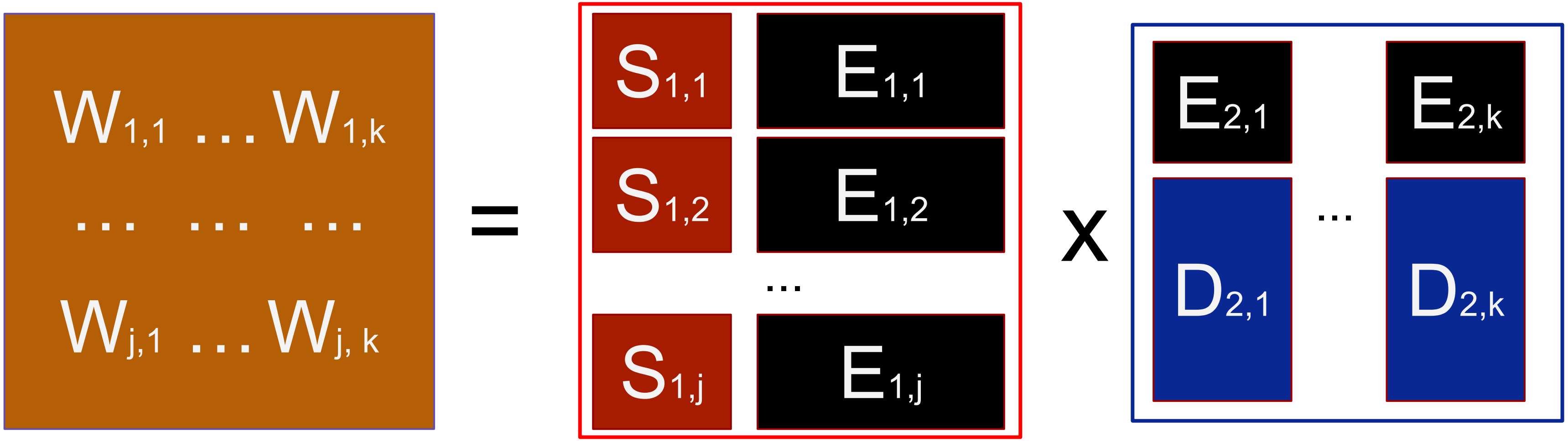

The analysis is quite involved, especially for the extension to non-input-partitioning BPs.

[code] https://github.com/wildstrawberry/cryptanalysesBPobfuscators/blob/master/ggh15analysis.sage

Almost done ...

- Proofs: Introducing new lattice toolkits;
  leads to new PCPRFs and lockable obfuscation for non-perm BPs.

- Attacks: New attacks on the iO candidates.

Wait, what about
witness encryption??

Almost done ...

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  > Witness encryption: read-once BP, the simplest instantiation of GLW14 on GGH15 (removing all the unnecessary parts), *“a stone throw”* from the provable case.

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  > Witness encryption: read-once BP, the simplest instantiation of GLW14 on GGH15 (removing all the unnecessary parts), *"a stone throw"* from the provable case.
  > iO: read super-constant time BP (merely a demonstration of what is not covered by the attack).

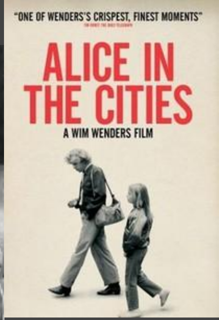
Other related works & Implications

The lattice lemmas appear in the concurrent work of [Goyal, Koppula, Waters 18] that builds traitor tracing from LWE.

[Bartusek, Guan, Ma, Zhandry] limitation of the attacks on GGH15-based iO candidates.

One of the future direction:
Build applications from multilinear maps with “slots”
=> instantiate using GGH15 with diagonal matrices,
see if there is a chance of proving from LWE

Only
25 mins?



Thanks for your time!

GGH15 Beyond Permutation Branching Programs:
Proofs, Attacks, and Candidates

https://eprint.iacr.org/2018/360