Generic Attacks against Beyond-Birthday-Bound MACs

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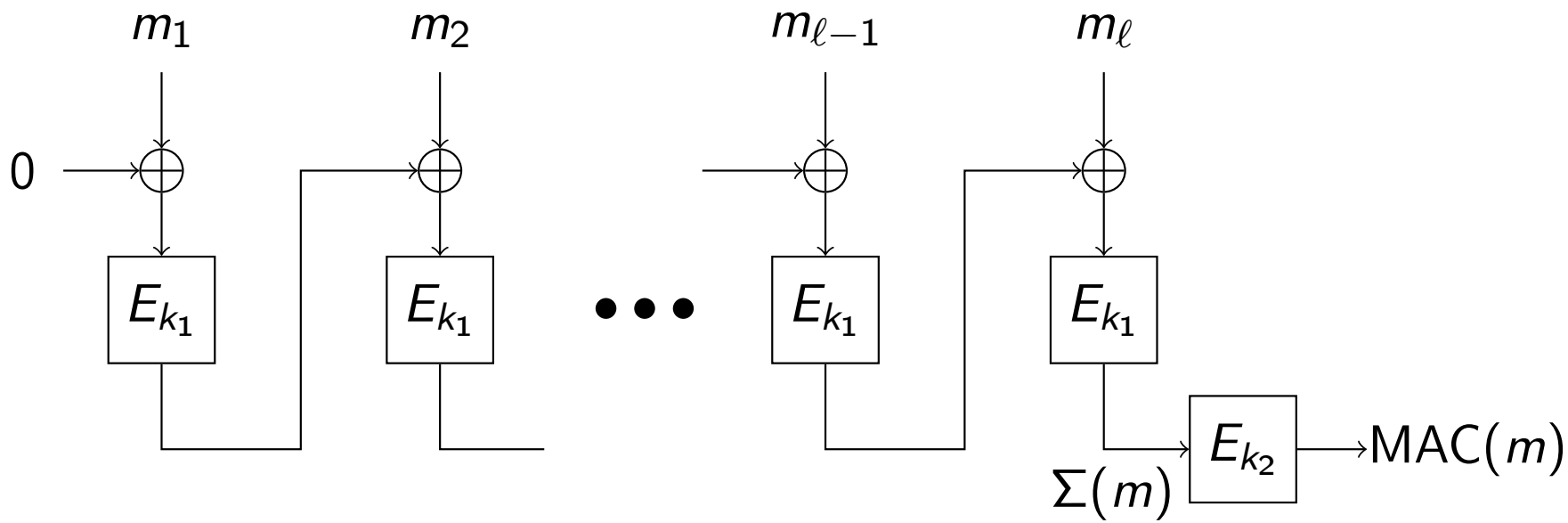
Introduction

- **Symmetric cryptography:** Alice and Bob share the same key.
- **Active attacker:** Eve might intercept and manipulate Alice's messages...
- **Authentication:** Alice computes and appends a keyed MAC or tag T.

ECBC-MAC



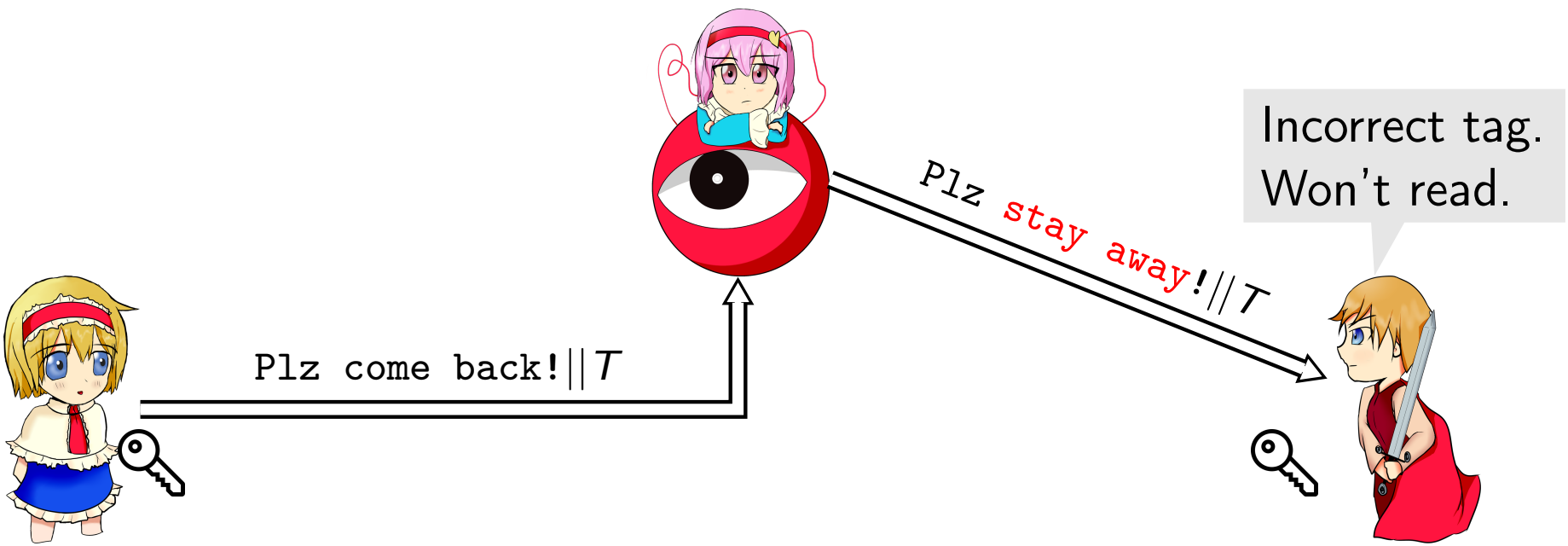
The plaintext m is padded and split into n-bit blocks.

$$\mathrm{MAC}(m) = E_{k_2}\big(\Sigma(m)\big)$$

Alice sends MAC(m) along with m to guarantee authenticity.

Introduction

- **Verifying:** Bob verifies the tag with the shared key and only reads the message if it is correct.
- **Forgery:** Eve cannot modify the message without forging a new and correct tag.



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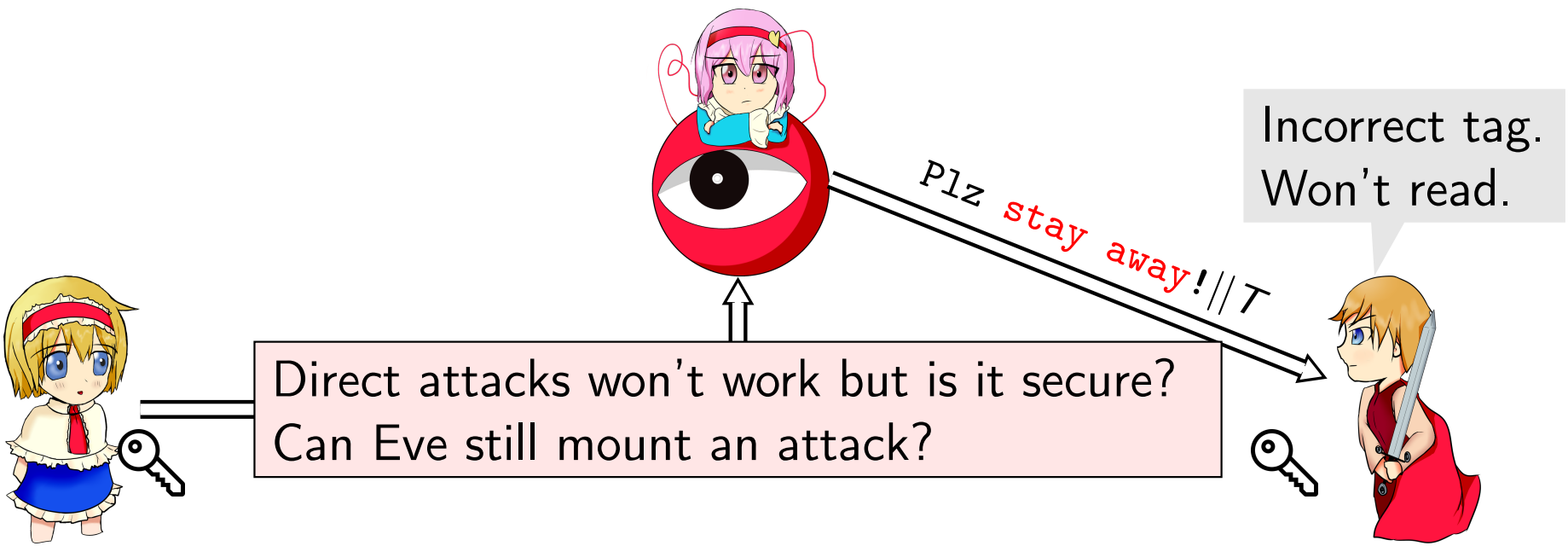

A security game

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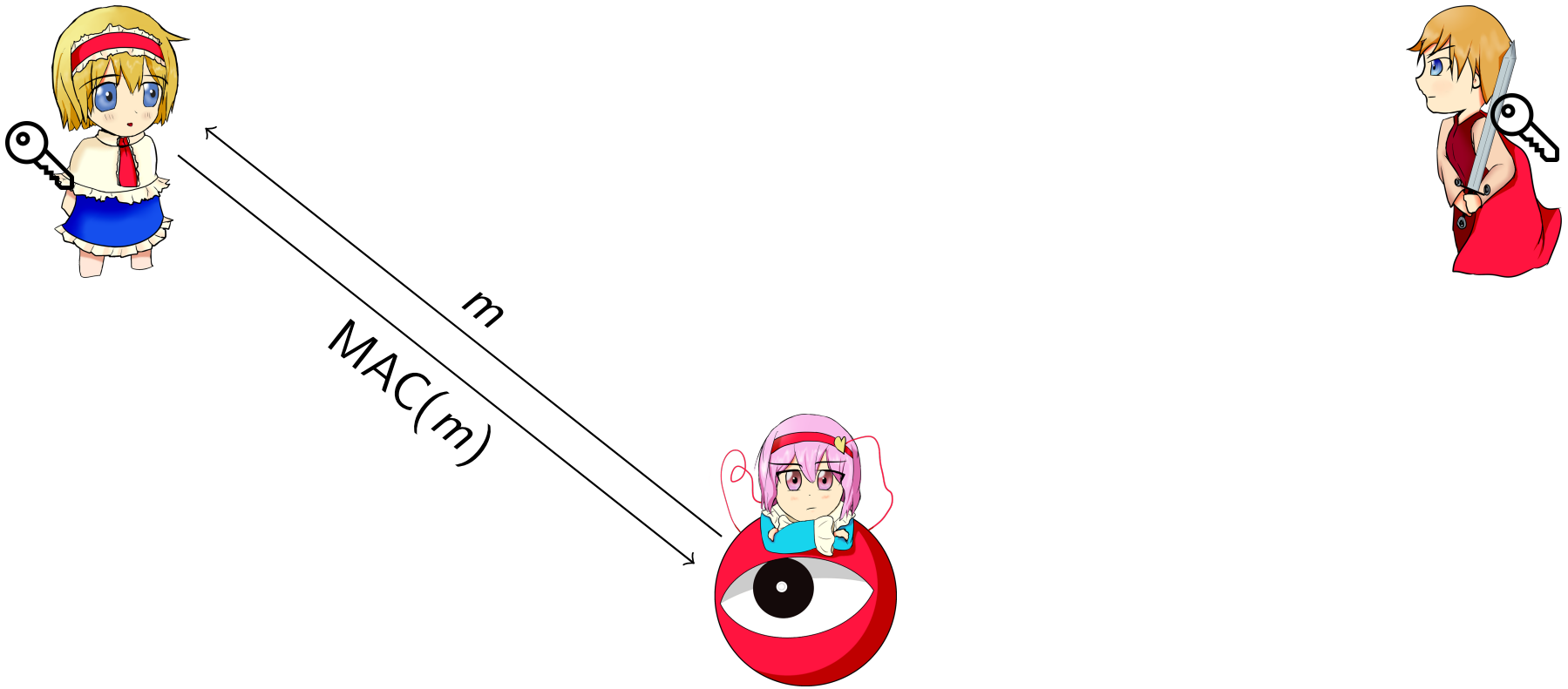

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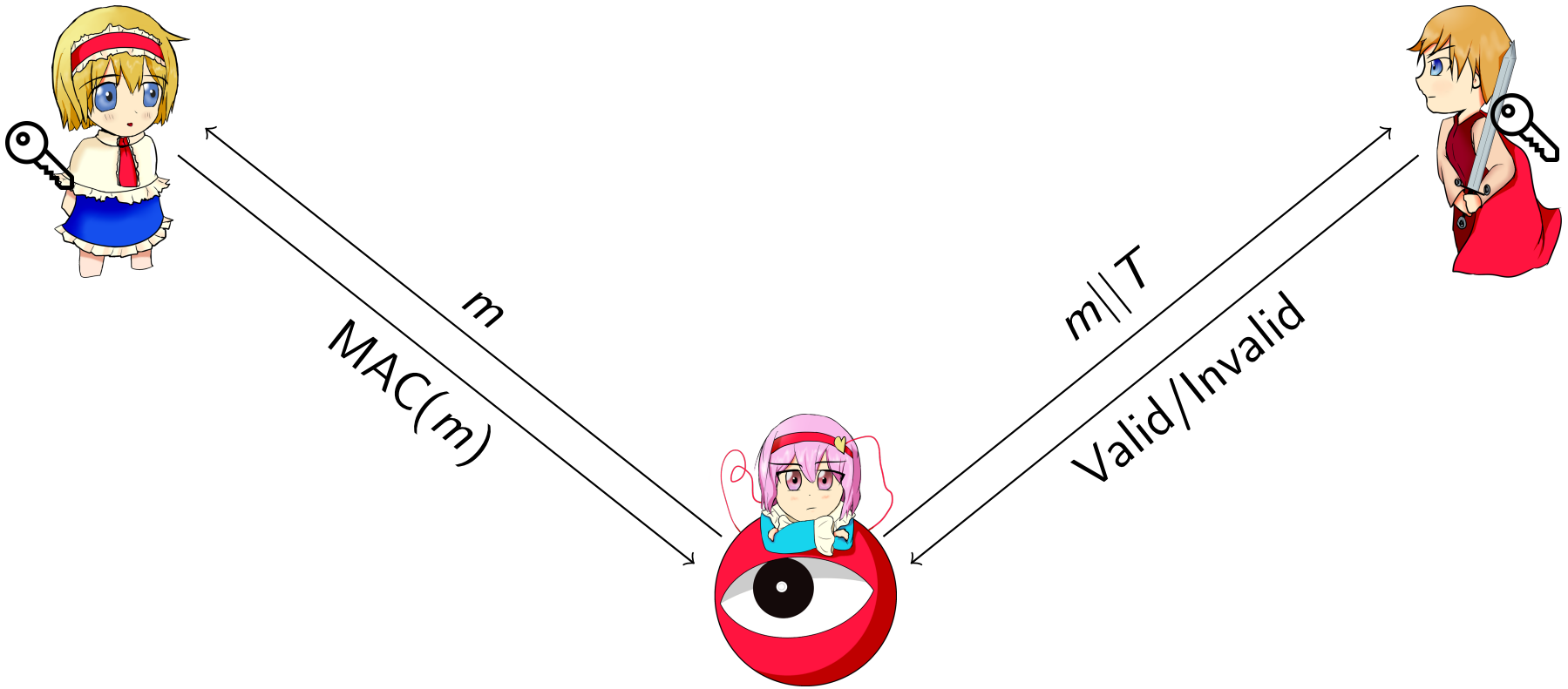

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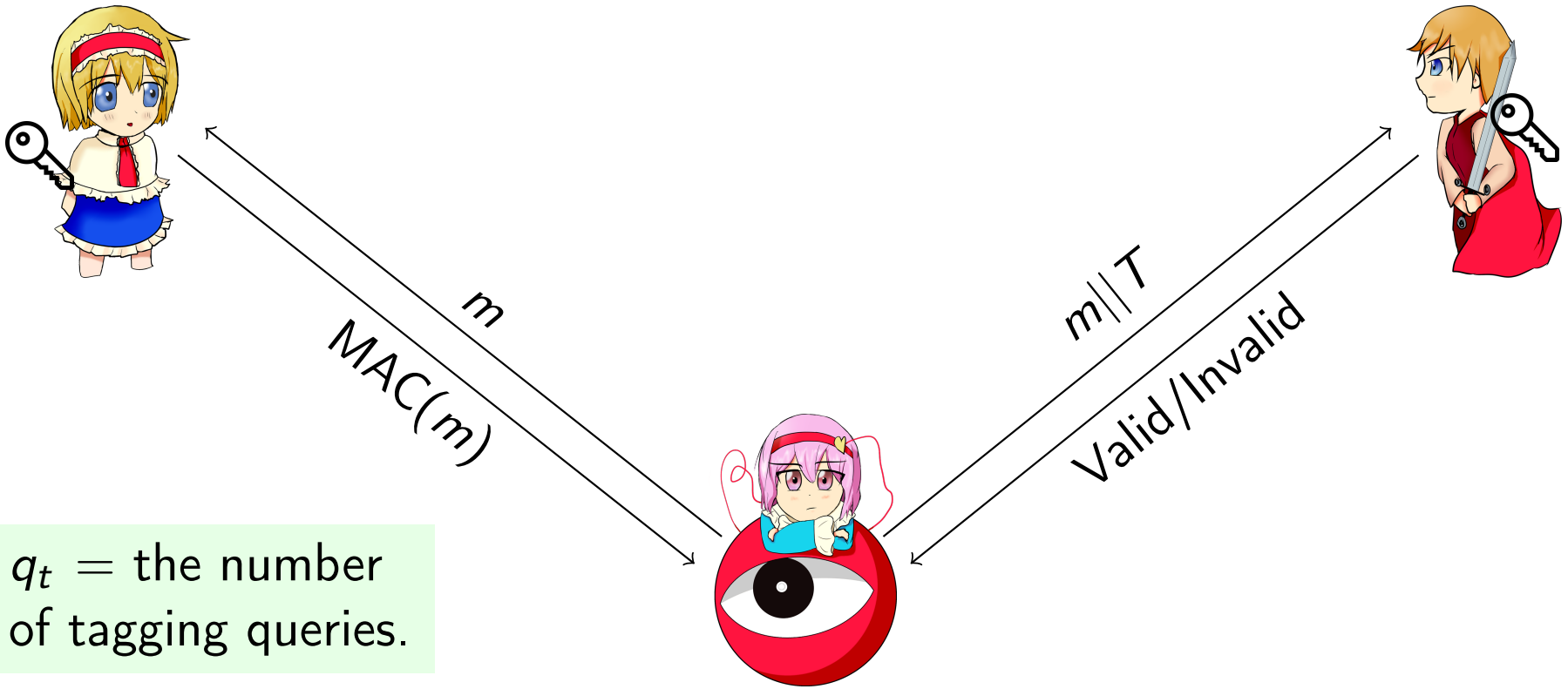

q_t = the number of tagging queries.

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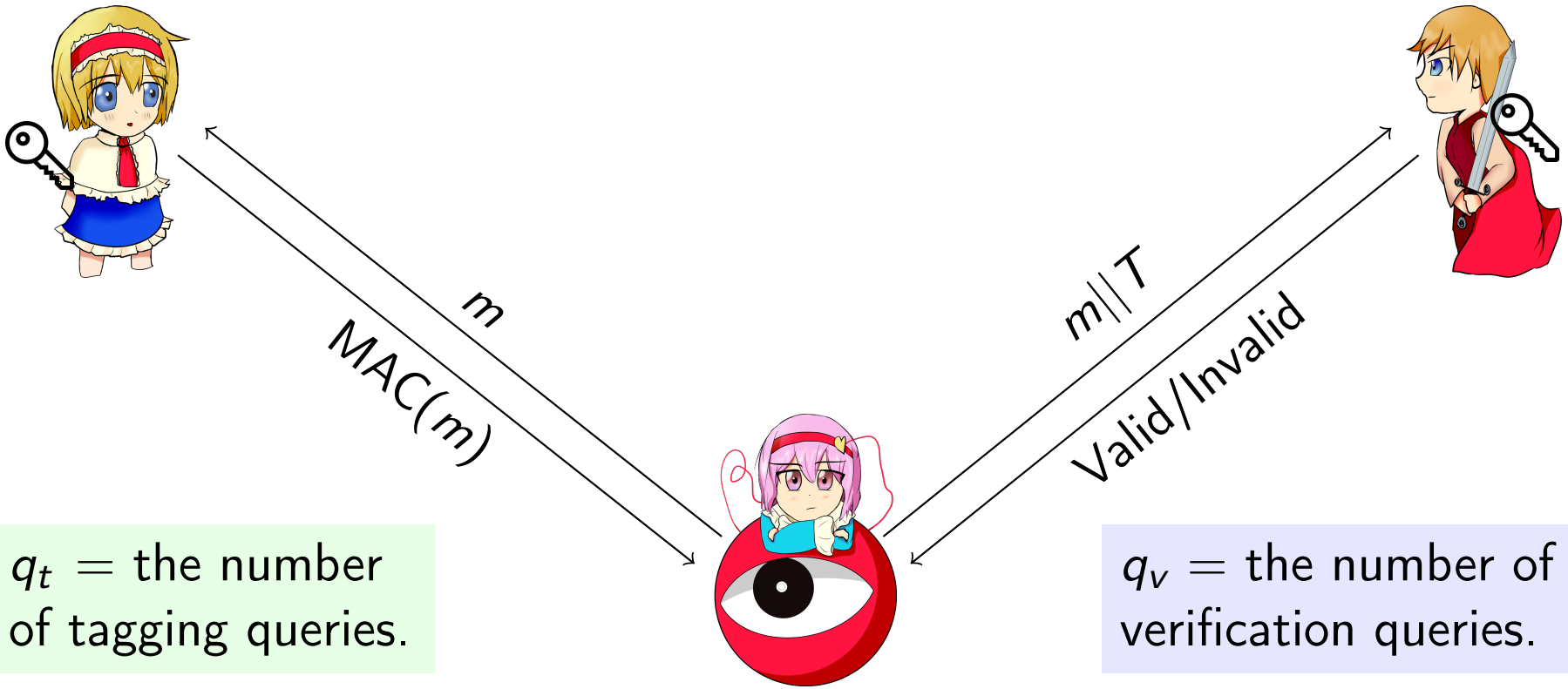

q_t = the number of tagging queries.

q_v = the number of verification queries.

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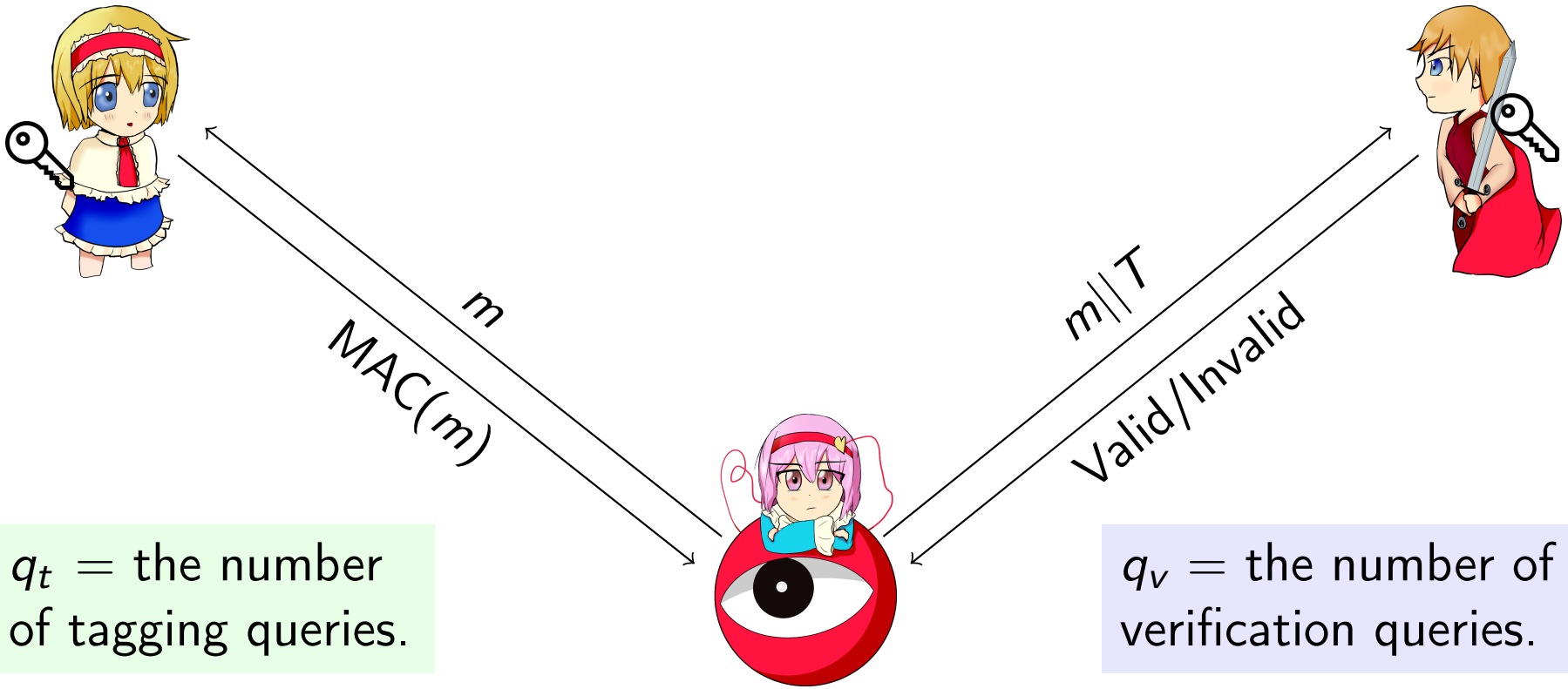


q_t = the number of tagging queries.

q_v = the number of verification queries.

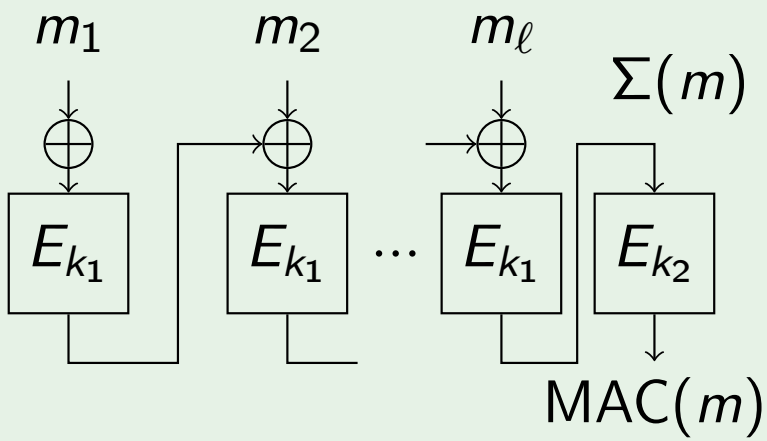
Can Eve forge a valid tag for a message that Alice never saw?

Case of ECBC

Properties of ECBC for all messages m, m', c:

$$
\begin{aligned}
& \mathrm{MAC}(m) = \mathrm{MAC}(m') \\
\Longrightarrow \quad & E_{k_2}\big(\Sigma(m)\big) = E_{k_2}\big(\Sigma(m')\big) \\
\Longrightarrow \quad & \Sigma(m) = \Sigma(m') \\
\Longrightarrow \quad & \Sigma(m||c) = \Sigma(m'||c) \\
\Longrightarrow \quad & \mathrm{MAC}(m||c) = \mathrm{MAC}(m'||c)
\end{aligned}
$$

ECBC mode



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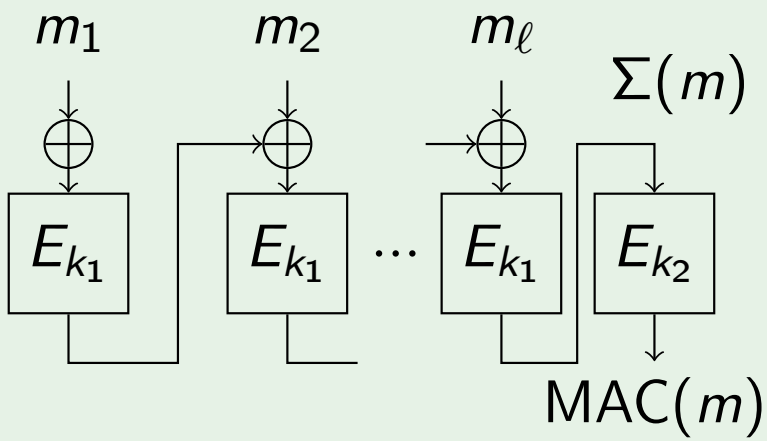
ECBC mode

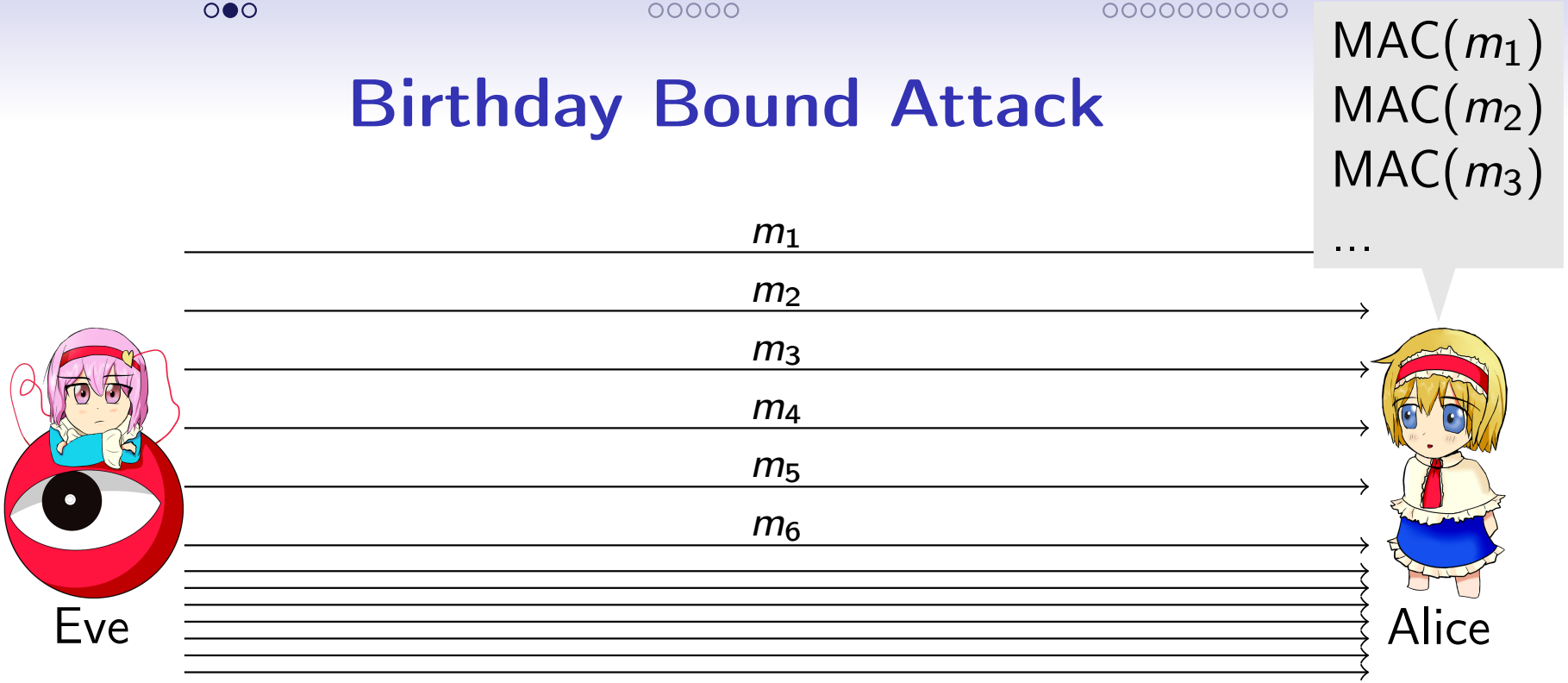


Simple collision approach

Look for a pair of messages X,Y that satisfies:

$$\Sigma(X) = \Sigma(Y) \iff \mathrm{MAC}(X) \oplus \mathrm{MAC}(Y) = 0$$

Birthday Bound Attack



Looking for collisions

Eve looks for $\mathrm{MAC}(m_i) = \mathrm{MAC}(m_j)$ for some $i \neq j$.
She has $\simeq q_t^2$ pairs for an n-bit relationship so chances grow as:

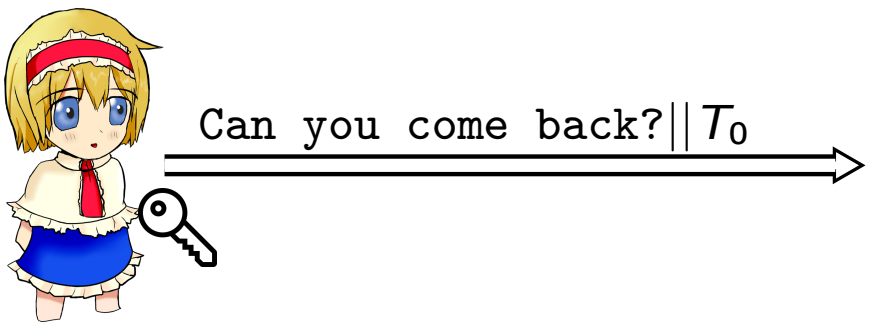
$$\mathbf{Adv}(\mathcal{A}) \simeq \frac{q_t^2}{2^n}$$

Forgery from collisions

Expansion property

$$\mathrm{MAC}(m) = \mathrm{MAC}(m') \implies \mathrm{MAC}(m||c) = \mathrm{MAC}(m'||c)\ \forall c$$

Collision found:
$\mathrm{MAC}(\texttt{You must}) =$
$\mathrm{MAC}(\texttt{No, don't})$



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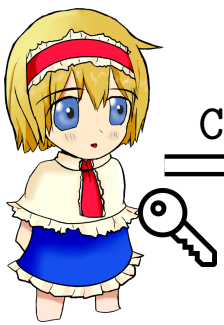
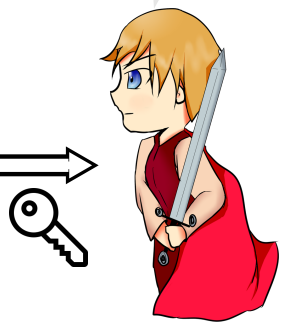
Correct tag.
Will read.



$\texttt{Can you come back?}||T_0$



Forgery from collisions

Expansion property

$$\mathrm{MAC}(m) = \mathrm{MAC}(m') \implies \mathrm{MAC}(m||c) = \mathrm{MAC}(m'||c)\ \forall c$$

Tell Bob **he must** come back!

Collision found:
$\mathrm{MAC}(\texttt{You must}) =$
$\mathrm{MAC}(\texttt{No, don't})$

Oh you are right!

Forgery from collisions

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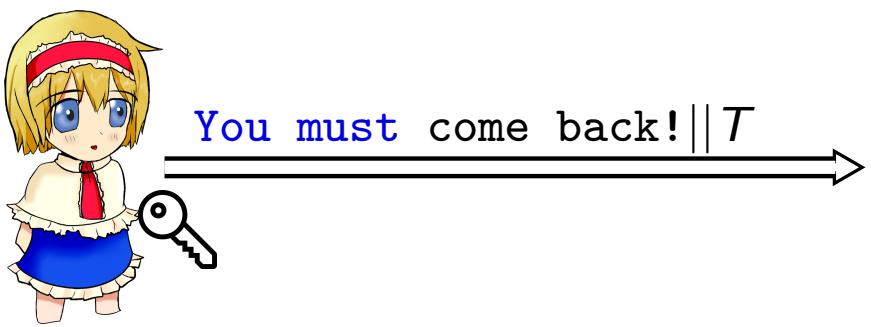

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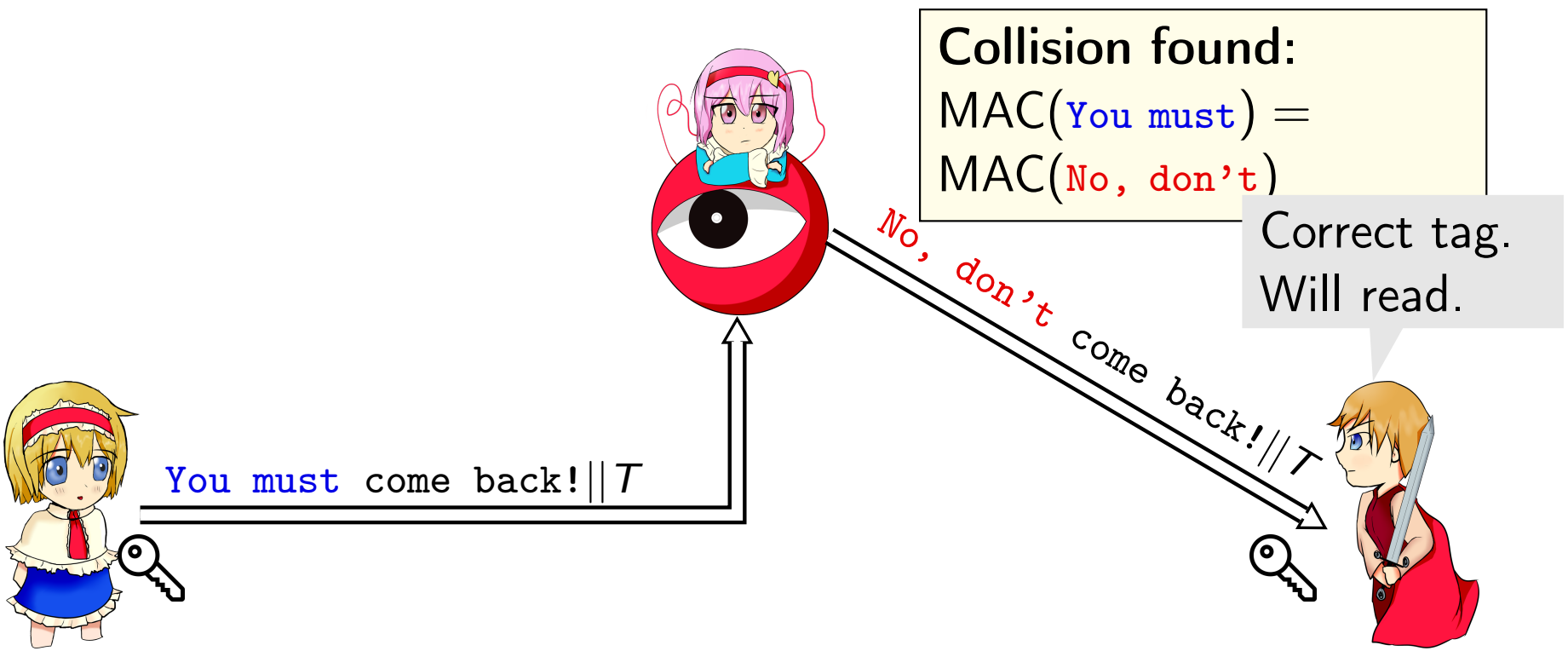

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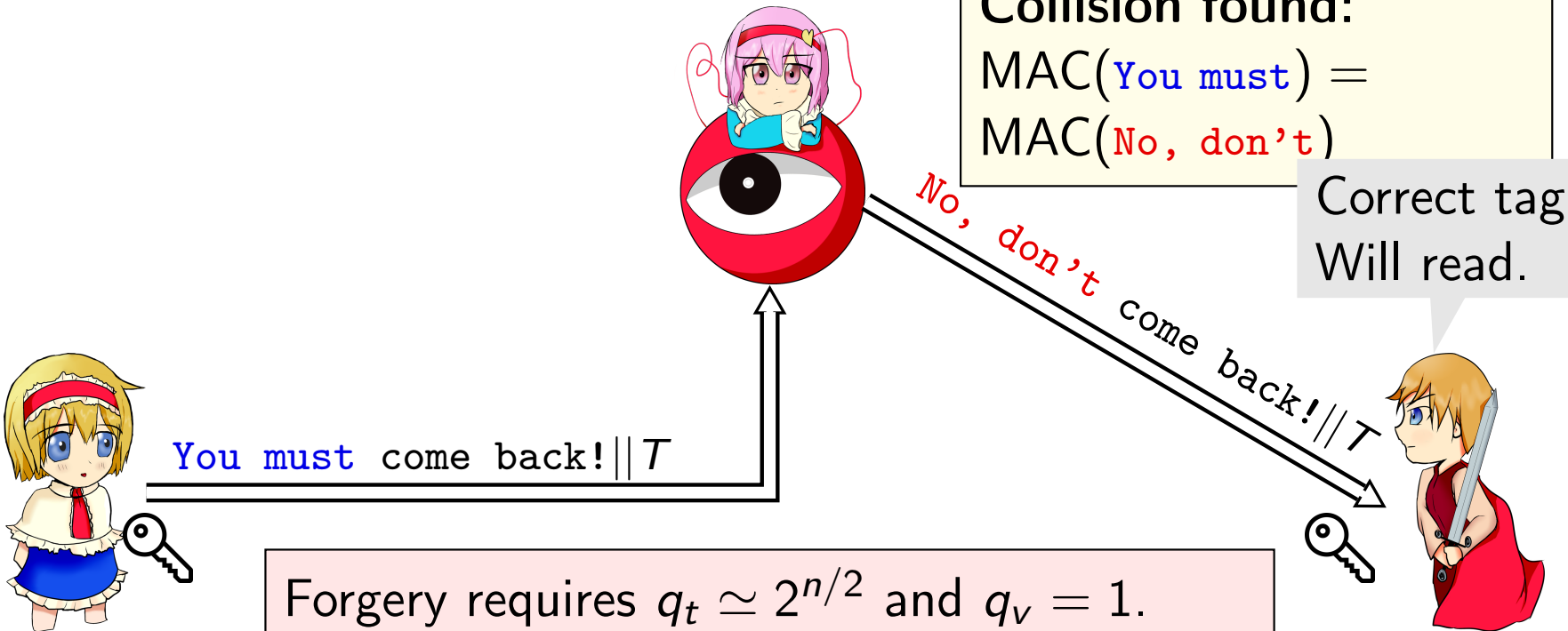

Forgery requires $q_t \simeq 2^{n/2}$ and $q_v = 1$.
Not secure beyond birthday bound $(2^{n/2})$

Going beyond

Problem

How to build a deterministic MAC scheme secure when $q_t > 2^{n/2}$?

Not so easy: This birthday bound attack is generic to all deterministic iterated MAC constructions with an n-bit internal state [Preneel, van Oorschot, CRYPTO'95].

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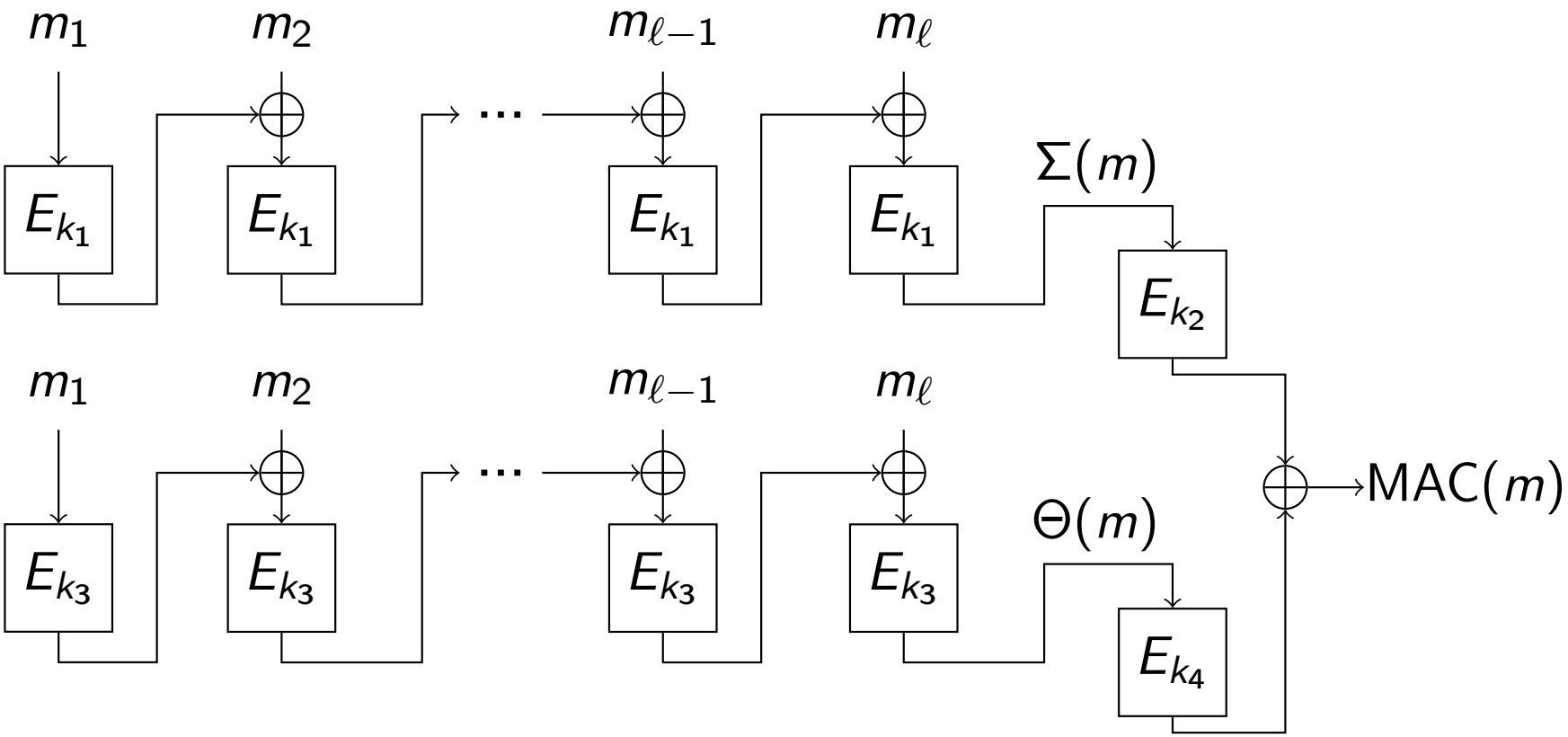
Idea: Double the size of the internal state to $2n$ bits.

Double-Block-Hash-Then-Sum Approach

XOR the two half-states at the end to recover an n-bit MAC.
Important research effort exploring this idea including:
SUM-ECBC, PMAC+, 3kf9, LightMAC+, GCM-SIV2, 1kPMAC+

Example: SUM-ECBC [Yasuda, CT-RSA'10]



$$\mathrm{MAC}(m) = E_{k_2}\big(\Sigma(m)\big) \oplus E_{k_4}\big(\Theta(m)\big)$$

This paper

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Many of those schemes are proven secure when $q_t < 2^{2n/3}$.
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Results

A generic approach leading to an attack on all cited schemes using $q_v = 1$ and $q_t \simeq 2^{3n/4}$.

4-way collision for double-hash-then-sum schemes

Look for a quadruple of messages X, Y, Z, T that satisfies:

$$\mathcal{R}(X,Y,Z,T) := \begin{cases} \Sigma(X) = \Sigma(Y) \\ \Theta(Y) = \Theta(Z) \\ \Sigma(Z) = \Sigma(T) \\ \Theta(T) = \Theta(X) \end{cases}$$

$$\mathcal{R}(X,Y,Z,T) \implies \mathrm{MAC}(X) \oplus \mathrm{MAC}(Y) \oplus \mathrm{MAC}(Z) \oplus \mathrm{MAC}(T) = 0$$

$$\begin{array}{ccccc} & & = & & \\ & \overset{\Theta(X)}{\ulcorner} & & \overset{\Theta(T)}{\urcorner} & \\ \mathrm{MAC}(X) = E(\Sigma(X)) \oplus E'(\Theta(X)) & & & E'(\Theta(T)) \oplus E(\Sigma(T)) = \mathrm{MAC}(T) \\ \| & & & & \| \\ \mathrm{MAC}(Y) = E(\Sigma(Y)) \oplus E'(\Theta(Y)) & & & E'(\Theta(Z)) \oplus E(\Sigma(Z)) = \mathrm{MAC}(Z) \\ & \underset{\Theta(Y)}{\llcorner} & & \underset{\Theta(Z)}{\lrcorner} & \\ & & = & & \end{array}$$

4-way collision for double-hash-then-sum schemes

With carefully crafted sets of messages for X, Y, Z, T:

$$\begin{cases} \Sigma(X) = \Sigma(Y) \\ \Theta(Y) = \Theta(Z) \\ \Sigma(Z) = \Sigma(T) \end{cases} \implies \Theta(T) = \Theta(X).$$

Thus $\mathcal{R}(X, Y, Z, T) \iff \begin{cases} \Sigma(X) = \Sigma(Y) \\ \Theta(Y) = \Theta(Z) \\ \Sigma(Z) = \Sigma(T) \end{cases}$ a $3n$-bit condition.

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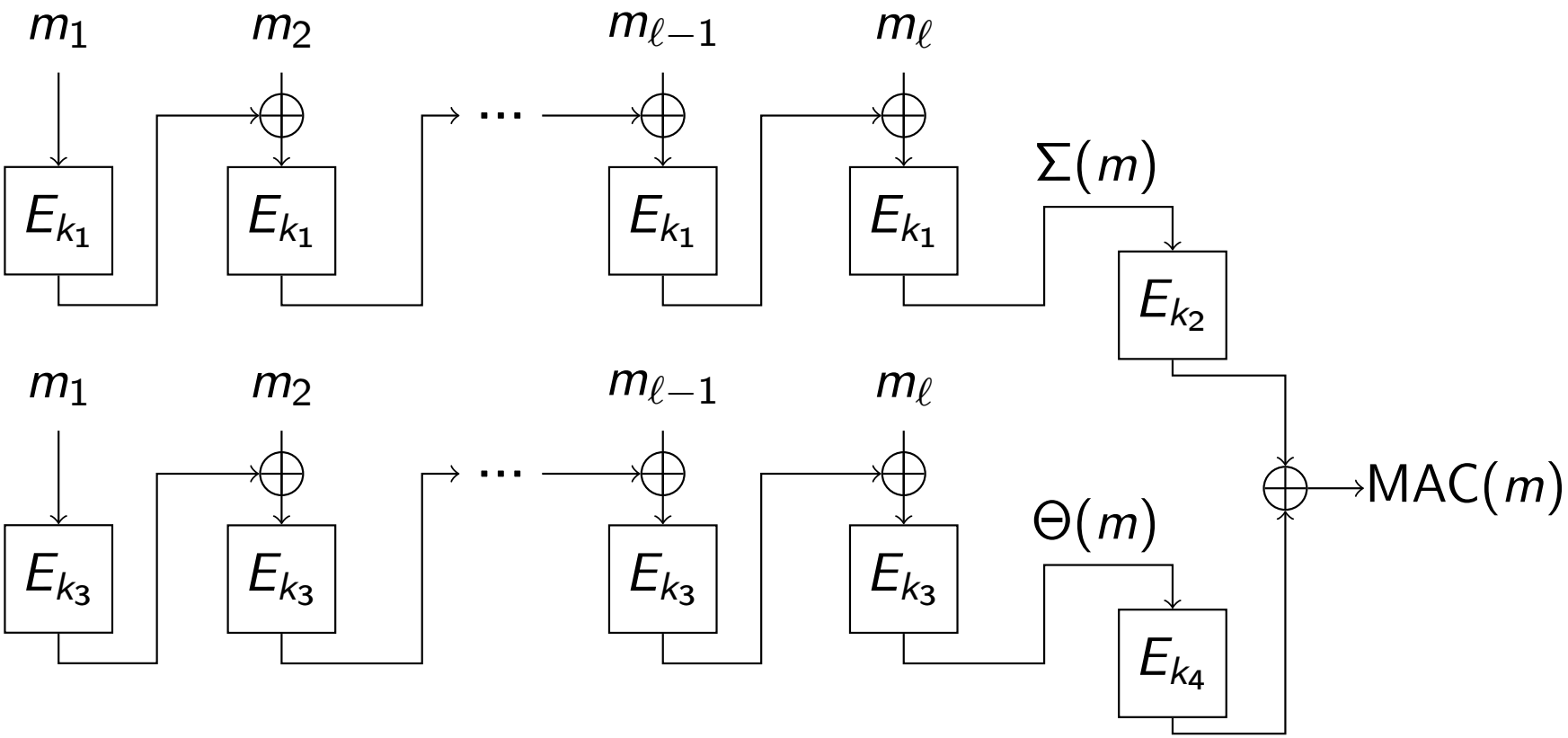
Query complexity

There are $\simeq q_t^4$ quadruples for a $3n$-bit condition.
A good one with high probability after $q_t \simeq 2^{3n/4}$ queries.

Attack on SUM-ECBC



$$\mathrm{MAC}(m) = E_{k_2}\big(\Sigma(m)\big) \oplus E_{k_4}\big(\Theta(m)\big)$$

Crafting the messages

$$X = 0||x; \qquad Y = 1||y; \qquad Z = 0||z; \qquad T = 1||t;$$

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$$\iff \begin{cases} x \oplus E_{k_1}(0) = y \oplus E_{k_1}(1) \\ y \oplus E_{k_3}(1) = z \oplus E_{k_3}(0) \\ z \oplus E_{k_1}(0) = t \oplus E_{k_1}(1) \\ t \oplus E_{k_3}(1) = x \oplus E_{k_3}(0) \end{cases} \iff \begin{cases} x \oplus y \oplus z \oplus t = 0 \\ x \oplus y = E_{k_1}(0) \oplus E_{k_1}(1) \\ x \oplus t = E_{k_3}(0) \oplus E_{k_3}(1) \end{cases}$$

$\mathcal{R}(X, Y, Z, T)$ is indeed a $3n$-bit condition on the quadruple.

Filtering quadruples

$$\mathcal{R} \iff \begin{cases} x \oplus y \oplus z \oplus t = 0 \\ x \oplus y = E_{k_1}(0) \oplus E_{k_1}(1) \\ x \oplus t = E_{k_3}(0) \oplus E_{k_3}(1) \end{cases}$$

Observable Filters

The first equation of $\mathcal{R}$ in addition to the sum of MACs:

$$\begin{cases} x \oplus y \oplus z \oplus t = 0 \\ \mathrm{MAC}(0||x) \oplus \mathrm{MAC}(1||y) \oplus \mathrm{MAC}(0||z) \oplus \mathrm{MAC}(1||t) = 0 \end{cases}$$

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Not enough

It is a $2n$-bit filter for $q_t^4 \simeq 2^{3n}$ quadruples.
2^n quadruples to randomly pass the filter for only 1 respecting $\mathcal{R}$.

Amplifying the filter

$$\mathcal{R}\big((0||x),(1||y),(0||z),(1||t)\big) \iff \begin{cases} x \oplus y \oplus z \oplus t = 0 \\ x \oplus y = E_{k_1}(0) \oplus E_{k_1}(1) \\ x \oplus t = E_{k_3}(0) \oplus E_{k_3}(1) \end{cases}$$

$$\mathcal{R} \iff \begin{cases} (x \oplus 1) \oplus (y \oplus 1) \oplus (z \oplus 1) \oplus (t \oplus 1) = 0 \\ (x \oplus 1) \oplus (y \oplus 1) = E_{k_1}(0) \oplus E_{k_1}(1) \\ (x \oplus 1) \oplus (t \oplus 1) = E_{k_3}(0) \oplus E_{k_3}(1) \end{cases}$$

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Related solutions

$\mathcal{R}\big((0||x),(1||y),(0||z),(1||t)\big) \iff$
$\mathcal{R}\big((0||x \oplus 1),(1||y \oplus 1),(0||z \oplus 1),(1||t \oplus 1)\big)$

In particular if we have a good solution x, y, z, t then it verifies:

$$\mathrm{MAC}(0||x \oplus 1) \oplus \mathrm{MAC}(1||y \oplus 1) \oplus \mathrm{MAC}(0||z \oplus 1) \oplus \mathrm{MAC}(1||t \oplus 1) = 0$$

Finding a good quadruple

Find a quadruple (x, y, z, t) such that:

$$
\begin{array}{llllll}
x & \oplus y & \oplus z & \oplus t & = 0 \\
\mathrm{MAC}(0||x) & \oplus \mathrm{MAC}(1||y) & \oplus \mathrm{MAC}(0||z) & \oplus \mathrm{MAC}(1||t) & = 0 \\
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\end{array}
$$

1. Query and build the following 4 lists of size $2^{3n/4}$:

$$
\begin{aligned}
L_1 &= \{x||\,\mathrm{MAC}(0||x)||\,\mathrm{MAC}(0||x \oplus 1)\} \\
L_2 &= \{y||\,\mathrm{MAC}(1||y)||\,\mathrm{MAC}(1||y \oplus 1)\} \\
L_3 &= \{z||\,\mathrm{MAC}(0||z)||\,\mathrm{MAC}(0||z \oplus 1)\} \\
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2. Find $\ell_1, \ell_2, \ell_3, \ell_4$ in L_1, L_2, L_3, L_4 respectively such that $\ell_1 \oplus \ell_2 \oplus \ell_3 \oplus \ell_4 = 0$.

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Algorithm cost

Step 1 costs $q_t = \mathcal{O}(2^{3n/4})$ queries and as much memory.

Step 2 is about solving an instance of the 4-XOR problem.
Solve it in $\mathcal{O}(2^{3n/4})$ memory and $\mathcal{O}(2^{3n/2})$ time.

Optimizing time complexity

SUM-ECBC and GCM-SIV2: optimize the time complexity at the cost of queries.

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SUM-ECBC and GCM-SIV2: optimize the time complexity at the cost of queries.

Related solutions

$\mathcal{R}\big((0||x),(1||y),(0||z),(1||t)\big) \iff$
$\mathcal{R}\big((0||x \oplus c),(1||y \oplus c),(0||z \oplus c),(1||t \oplus c)\big)\ \forall c$

So $\mathcal{R} \implies \forall c$:
$\mathrm{MAC}(0||x \oplus c) \oplus \mathrm{MAC}(1||y \oplus c) \oplus \mathrm{MAC}(0||z \oplus c) \oplus \mathrm{MAC}(1||t \oplus c) = 0$

Optimizing time complexity

Let $\mathcal{C} = \{c : c < 2^{3n/7}\}$ we sum the relations:

$$\bigoplus \begin{cases} \mathrm{MAC}(0||x \oplus 0) \oplus \mathrm{MAC}(1||y \oplus 0) \oplus \mathrm{MAC}(0||z \oplus 0) \oplus \mathrm{MAC}(1||t \oplus 0) = 0 \\ \mathrm{MAC}(0||x \oplus 1) \oplus \mathrm{MAC}(1||y \oplus 1) \oplus \mathrm{MAC}(0||z \oplus 1) \oplus \mathrm{MAC}(1||t \oplus 1) = 0 \\ \mathrm{MAC}(0||x \oplus 2) \oplus \mathrm{MAC}(1||y \oplus 2) \oplus \mathrm{MAC}(0||z \oplus 2) \oplus \mathrm{MAC}(1||t \oplus 2) = 0 \\ \mathrm{MAC}(0||x \oplus 3) \oplus \mathrm{MAC}(1||y \oplus 3) \oplus \mathrm{MAC}(0||z \oplus 3) \oplus \mathrm{MAC}(1||t \oplus 3) = 0 \\ \mathrm{MAC}(0||x \oplus 4) \oplus \mathrm{MAC}(1||y \oplus 4) \oplus \mathrm{MAC}(0||z \oplus 4) \oplus \mathrm{MAC}(1||t \oplus 4) = 0 \\ \ldots \end{cases}$$

Optimizing time complexity

Let $\mathcal{C} = \{c : c < 2^{3n/7}\}$ we sum the relations:

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Only the most significant $4n/7$ bits of x, y, z, t are meaningful and must respect a $3 \cdot 4n/7 = 12n/7$-bit relationship.

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For $|L| = 2^{3n/7}$ the 4-XOR problem takes $\mathcal{O}(2^{6n/7})$ time.
One element requires $2^{3n/7}$ queries, a total of $\mathcal{O}(2^{6n/7})$ queries.

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Previously we used $\mathcal{O}(2^{3n/2})$ time and $\mathcal{O}(2^{3n/4})$ queries.
Thus this optimization uses less time but more queries.

Forgery from quadruples

$\Sigma(m)$ and $\Theta(m)$ are built the same way as simple ECBC's $\Sigma(m)$.
In particular for all suffixes c:

$$\Sigma(m) = \Sigma(m') \implies \Sigma(m||c) = \Sigma(m'||c)$$

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Expansion property SUM-ECBC

$\mathcal{R}(X, Y, Z, T) \implies \mathcal{R}(X||c, Y||c, Z||c, T||c)\ \forall c$

Therefore Eve can forge in a very similar manner.

Forgery from quadruples

Expansion property SUM-ECBC (reminder)

$$\mathcal{R}(X, Y, Z, T) \implies \mathcal{R}(X||c, Y||c, Z||c, T||c)\ \forall c$$

Quadruple found:
MAC(`You should`)
MAC(`Plz help`)
MAC(`You must`)
MAC(`Plz never`)

Tell Bob he **should** come back!

T_1

`You should come back!`$||T_1$

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Correct tag.
Will read.

`You should come back!` $||\ T_1$

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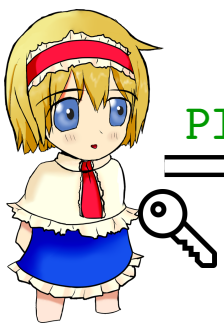
Plz help tell Bob to come back!

T_1, T_2



`Plz help come back!`$||T_2$

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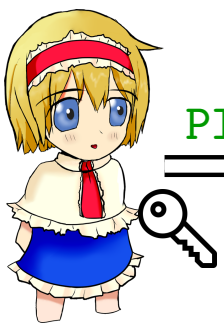

`Plz help come back!`$||T_2$

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$$\mathcal{R}(X, Y, Z, T) \implies \mathcal{R}(X||c, Y||c, Z||c, T||c)\ \forall c$$

Quadruple found:
MAC(`You should`)
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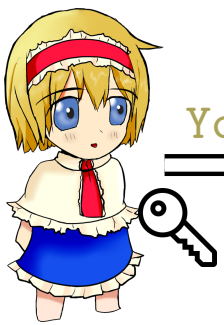
MAC(`You must`)
MAC(`Plz never`)

Tell Bob he **must** come back!

T_1, T_2, T_3
$T_4 = T_1 \oplus T_2 \oplus T_3$



`You must come back!`$||T_3$

Forgery from quadruples

Expansion property SUM-ECBC (reminder)

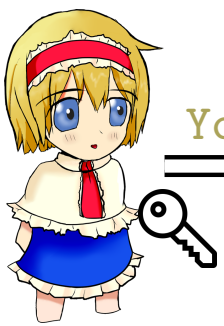
$\mathcal{R}(X, Y, Z, T) \implies \mathcal{R}(X||c, Y||c, Z||c, T||c)\ \forall c$

Quadruple found:
MAC(`You should`)
MAC(`Plz help`)
MAC(`You must`)
MAC(`Plz never`)

T_1, T_2, T_3
$T_4 = T_1 \oplus T_2 \oplus T_3$

`Plz never come back!`$||T_4$

Correct tag.
Will read.



`You must come back!`$||T_3$

Conclusion

Main results:

- Most of our attacks use $2^{3n/4}$ queries and $2^{3n/2}$ time.
- Variant for SUM-ECBC & GCM-SIV2: $2^{6n/7}$ queries and time.

Conclusion

Main results:

- Most of our attacks use $2^{3n/4}$ queries and $2^{3n/2}$ time.
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Additionally:

- Withdrawn 1kf9 shown to allow Birthday Bound Attacks and therefore is not a BBB scheme.
- Recent results on security of LightMAC+ [Naito, CT-RSA'18] proved wrong by our attack.

Conclusion

Mode	Attacks (this work) Queries	Time	Type
SUM-ECBC	$\mathcal{O}(2^{3n/4})$	$\tilde{\mathcal{O}}(2^{3n/2})$	Universal
	$\mathcal{O}(2^{6n/7})$	$\tilde{\mathcal{O}}(2^{6n/7})$	Universal
GCM-SIV2	$\mathcal{O}(2^{3n/4})$	$\tilde{\mathcal{O}}(2^{3n/2})$	Universal
	$\mathcal{O}(2^{6n/7})$	$\tilde{\mathcal{O}}(2^{6n/7})$	Universal
PMAC+	$\mathcal{O}(2^{3n/4})$	$\tilde{\mathcal{O}}(2^{3n/2})$	Existential
LightMAC+	$\mathcal{O}(2^{3n/4})$	$\tilde{\mathcal{O}}(2^{3n/2})$	Existential
1kPMAC+	$\mathcal{O}(2^{3n/4})$	$\tilde{\mathcal{O}}(2^{3n/2})$	Existential
3kf9	$\mathcal{O}(\sqrt[4]{n} \cdot 2^{3n/4})$	$\tilde{\mathcal{O}}(2^{5n/4})$	Universal
1kf9	$\mathcal{O}(2^{n/2})$	$\tilde{\mathcal{O}}(2^{n/2})$	Universal

Except 1kf9, all above schemes have a proof that they are secure while $q_t < 2^{2n/3}$. We showed they are not secure when $q_t \geq 2^{3n/4}$.
Open question: What happens when $2^{2n/3} \leq q_t < 2^{3n/4}$?