Multi-Input Functional Encryption for Inner Products: Function-Hiding Realizations and Constructions without Pairings

Michel Abdalla Dario Catalano Dario Fiore
Romain Gay Bogdan Ursu

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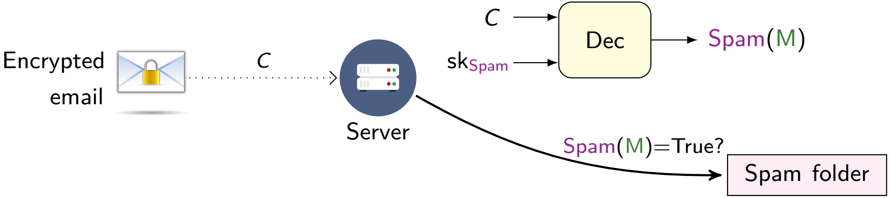
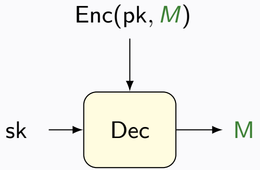
Motivation - Spam Server

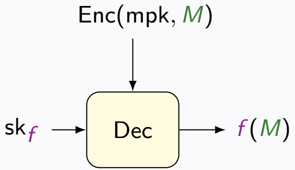


Beyond Public Key Encryption

Public key encryption [Diffie, Hellman 76]



Functional encryption [Boneh, Sahai, Waters 11]



Functional Encryption

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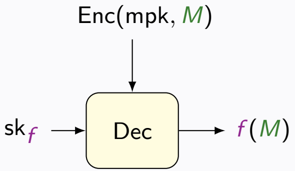
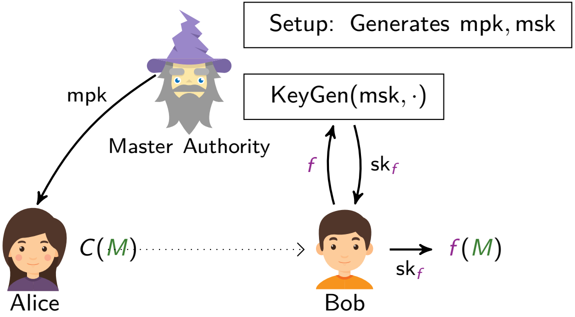
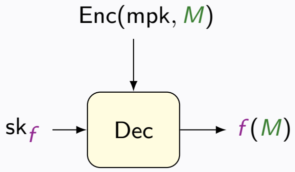




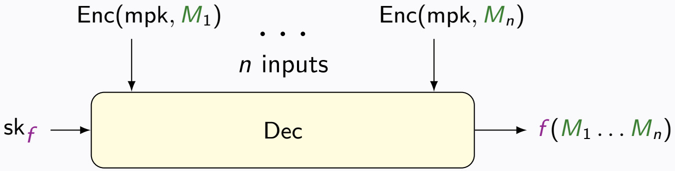
Multi-Input Functional Encryption

Functional encryption



Multi-input functional encryption

[Goldwasser, Gordon, Goyal, Jain, Katz, Liu, Sahai, Shi, Zhou 14]



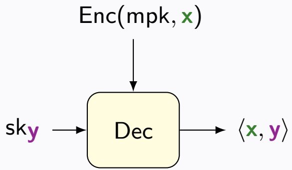
Independent ciphertexts

Inner-Product Functional Encryption

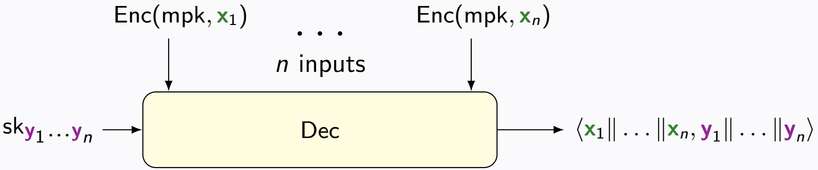
$f_{\mathbf{y}}(\cdot) = \langle \cdot, \mathbf{y} \rangle$

$$f_{\mathbf{y}_1\|\ldots\|\mathbf{y}_n}(\cdot, \ldots, \cdot) = \langle \mathbf{x}_1 \| \ldots \| \mathbf{x}_n, \mathbf{y}_1 \| \ldots \| \mathbf{y}_n \rangle$$

Inner-Product Functional encryption



Multi-input Inner-Product



Independent ciphertexts

Previous Work

Multi-input scheme	Classes of functions	Assumptions
[GGG+14, BLR+15, BGJS15] [AJ15, BKS16] FH	General functions	IO, Multilinear maps, ...

[AGRW17]	Inner products, poly inputs	SXDH in Pairing Groups
[DOT18] FH	Inner products unbounded poly inputs	SXDH in Pairing Groups

FH - function hiding

Previous Work + Our Contribution

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This work	Inner products, poly inputs	DDH, DCR or LWE
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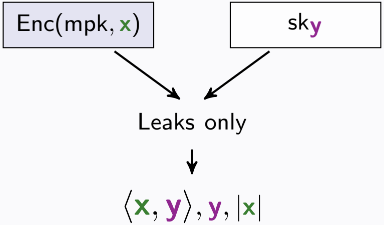
Security Goal

Security goal



Security of Multi-Input Functional Encryption

Security goal

$\mathsf{Enc}(\mathsf{mpk}, \mathbf{x}_1) \ldots \mathsf{Enc}(\mathsf{mpk}, \mathbf{x}_n)$

$\mathsf{sk}_{\mathbf{y}_1 \| \ldots \| \mathbf{y}_n}$

Leaks only

$$\langle \mathbf{x}_1 \| \ldots \| \mathbf{x}_n, \mathbf{y}_1 \| \ldots \| \mathbf{y}_n \rangle, \mathbf{y}_1 \| \ldots \| \mathbf{y}_n, \{|\mathbf{x}_i|\}$$

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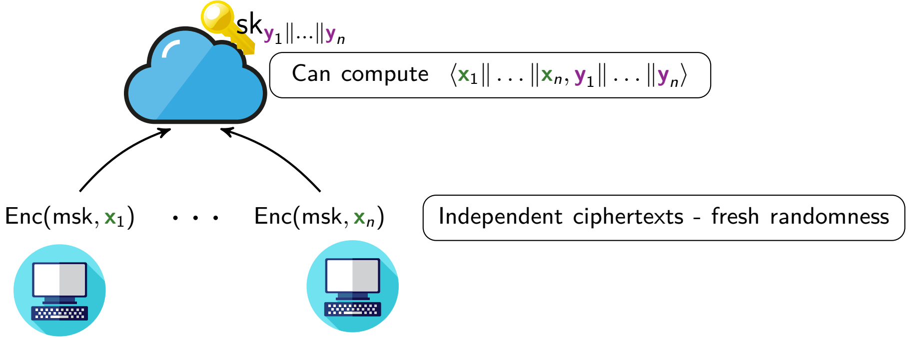
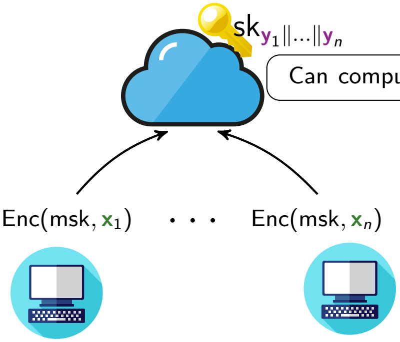
Leakage is more complex!

Multi-Input Inner-Product Encryption



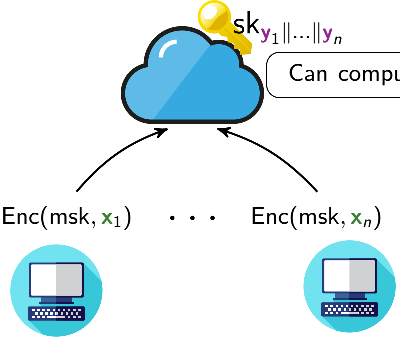
Multi-Input Inner-Product Encryption



Can compute $\langle \mathbf{x}_1\| \ldots \|\mathbf{x}_n, \mathbf{y}_1\| \ldots \|\mathbf{y}_n \rangle$

But nothing more about $\langle \mathbf{x}_i, \mathbf{y}_i \rangle$

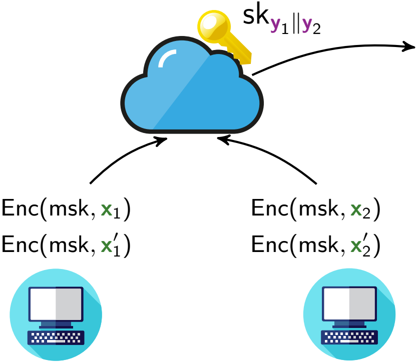
Public Key - Symmetric Key



Can compute $\langle \mathbf{x}_1 \| \ldots \| \mathbf{x}_n, \mathbf{y}_1 \| \ldots \| \mathbf{y}_n \rangle$

But nothing more about $\langle \mathbf{x}_i, \mathbf{y}_i \rangle$
Public key, encrypt $\mathbf{0}$

$$\langle \mathbf{0} \ldots \mathbf{0} \| \mathbf{x}_i \| \mathbf{0} \ldots \mathbf{0}, \mathbf{y}_1 \| \ldots \| \mathbf{y}_n \rangle = \langle \mathbf{x}_i, \mathbf{y}_i \rangle$$

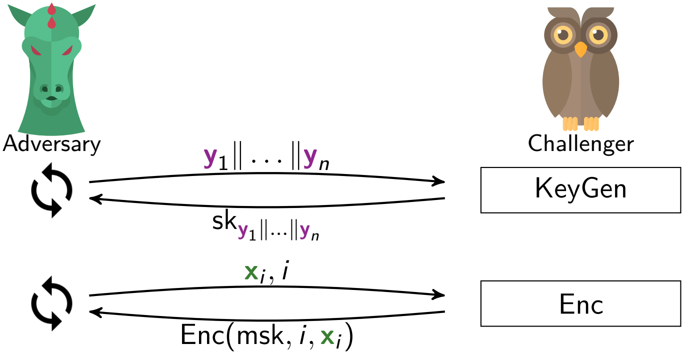


Can compute:
- $\langle \mathbf{x}_1 \| \mathbf{x}_2, \mathbf{y}_1 \| \mathbf{y}_2 \rangle$
- $\langle \mathbf{x}'_1 \| \mathbf{x}_2, \mathbf{y}_1 \| \mathbf{y}_2 \rangle$
- $\langle \mathbf{x}_1 \| \mathbf{x}'_2, \mathbf{y}_1 \| \mathbf{y}_2 \rangle$
- $\langle \mathbf{x}'_1 \| \mathbf{x}'_2, \mathbf{y}_1 \| \mathbf{y}_2 \rangle$

Example for $n = 2$

Difficulty: Allow ciphertext mixing but not key mixing!!!.

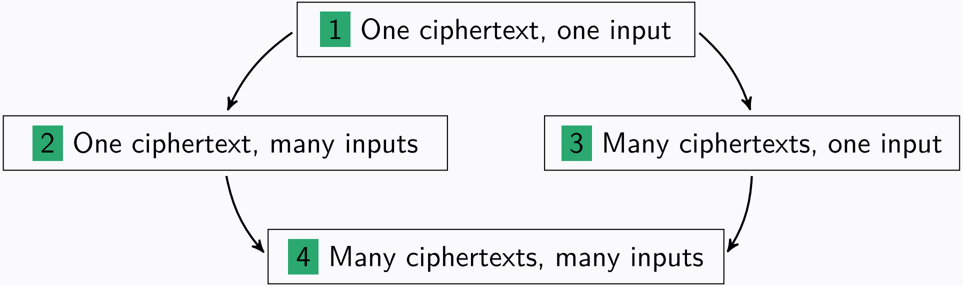
Multi-Input Inner-Product - Security



Adversary only learns $\langle \mathbf{x}_1 \| \dots \| \mathbf{x}_n, \mathbf{y}_1 \| \dots \| \mathbf{y}_n \rangle$ for all queried $(\mathbf{x}_i, i)$ and all queried $\mathbf{y}_1 \| \dots \| \mathbf{y}_n$.

Construction without Pairings

Roadmap



Symmetric setting one ciphertext $\not\Rightarrow$ many ciphertexts

1 One ciphertext, one input

$$\mathsf{msk} = \mathbf{u} \in \mathbb{Z}_q^m$$
$$\mathsf{Enc}_1(\mathsf{msk}, \mathbf{x}) = \mathbf{x} + \mathbf{u} \in \mathbb{Z}_q^m$$
$$\mathsf{KeyGen}_1(\mathsf{msk}, \mathbf{y}) = \langle \mathbf{u}, \mathbf{y} \rangle \in \mathbb{Z}_q, \mathbf{y}$$

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$$\mathsf{KeyGen}_1(\mathsf{msk}, \mathbf{y}) = \langle \mathbf{u}, \mathbf{y} \rangle \in \mathbb{Z}_q, \mathbf{y}$$

Decrypt with $\mathsf{sk}_{\mathbf{y}}$:

$$\langle \mathbf{x} + \mathbf{u}, \mathbf{y} \rangle - \langle \mathbf{u}, \mathbf{y} \rangle = \langle \mathbf{x}, \mathbf{y} \rangle + \cancel{\langle \mathbf{u}, \mathbf{y} \rangle} - \cancel{\langle \mathbf{u}, \mathbf{y} \rangle}$$

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Security:

$$(\mathbf{x} + \mathbf{u}, \langle \mathbf{u}, \mathbf{y} \rangle, \mathbf{y}) \equiv (\mathbf{w}, \langle \mathbf{w}, \mathbf{y} \rangle - \langle \mathbf{x}, \mathbf{y} \rangle, \mathbf{y})$$

Goal: only leakage on $\mathbf{x}$ is $\langle \mathbf{x}, \mathbf{y} \rangle$. ✓

2 One ciphertext, many inputs

1 One ciphertext, one input

$$\mathsf{msk} = \mathbf{u} \in \mathbb{Z}_q^m$$
$$\mathsf{Enc}_1(\mathsf{msk}, \mathbf{x}) = \mathbf{x} + \mathbf{u} \in \mathbb{Z}_q^m$$
$$\mathsf{KeyGen}_1(\mathsf{msk}, \mathbf{y}) = \langle \mathbf{u}, \mathbf{y} \rangle \in \mathbb{Z}_q, \mathbf{y}$$

2 One ciphertext, many inputs

$$\mathsf{msk} = \mathbf{u}_1 \ldots \mathbf{u}_n \in \mathbb{Z}_q^{n \times m}$$
$$\mathsf{Enc}_2(\mathsf{msk}, i, \mathbf{x}_i) = \mathbf{x}_i + \mathbf{u}_i \in \mathbb{Z}_q^m$$
$$\mathsf{KeyGen}_2(\mathsf{msk}, \mathbf{y}_1 \ldots \mathbf{y}_n) = \sum_{i=1}^n \langle \mathbf{u}_i, \mathbf{y}_i \rangle \in \mathbb{Z}_q, \mathbf{y}_1 \ldots \mathbf{y}_n$$

1 One ciphertext, one input	2 One ciphertext, many inputs
$\mathsf{msk} = \mathbf{u} \in \mathbb{Z}_q^m$	$\mathsf{msk} = \mathbf{u}_1 \dots \mathbf{u}_n \in \mathbb{Z}_q^{n \times m}$
$\mathsf{Enc}_1(\mathsf{msk}, \mathbf{x}) = \mathbf{x} + \mathbf{u} \in \mathbb{Z}_q^m$	$\mathsf{Enc}_2(\mathsf{msk}, i, \mathbf{x}_i) = \mathbf{x}_i + \mathbf{u}_i \in \mathbb{Z}_q^m$
$\mathsf{KeyGen}_1(\mathsf{msk}, \mathbf{y}) = \langle \mathbf{u}, \mathbf{y} \rangle \in \mathbb{Z}_q, \mathbf{y}$	$\mathsf{KeyGen}_2(\mathsf{msk}, \mathbf{y}_1 \dots \mathbf{y}_n) = \sum_{i=1}^n \langle \mathbf{u}_i, \mathbf{y}_i \rangle \in \mathbb{Z}_q, \mathbf{y}_1 \dots \mathbf{y}_n$

Dec: $\sum_{i=1}^n \langle \mathbf{x}_i + \mathbf{u}_i, \mathbf{y}_i \rangle - \sum_{i=1}^n \langle \mathbf{u}_i, \mathbf{y}_i \rangle = \langle \mathbf{x}_1 \dots \mathbf{x}_n, \mathbf{y}_1, \dots \mathbf{y}_n \rangle$

3 Many ciphertexts, one input

1 One ciphertext, one input	3 Many ciphertexts, one input [ABDP15]
$\mathsf{msk} = \mathbf{u} \in \mathbb{Z}_q^m$	$\mathsf{msk} = \mathbf{v} \in \mathbb{Z}_q^m$
$\mathsf{Enc}_1(\mathsf{msk}, \mathbf{x}) = \mathbf{x} + \mathbf{u} \in \mathbb{Z}_q^m$	$\mathsf{Enc}_3(\mathsf{msk}, \mathbf{x}) = g^r, g^{\mathbf{x}+r\mathbf{v}} \in \mathbb{G}^{m+1}$
$\mathsf{KeyGen}_1(\mathsf{msk}, \mathbf{y}) = \langle \mathbf{u}, \mathbf{y} \rangle \in \mathbb{Z}_q, \mathbf{y}$	$\mathsf{KeyGen}_3(\mathsf{msk}, \mathbf{y}) = \langle \mathbf{v}, \mathbf{y} \rangle \in \mathbb{Z}_q, \mathbf{y}$

$\mathbb{G}$ prime group of order q

Using [ALS16], this step can also be based on LWE or DCR.

Construction without Pairings

1 One ciphertext, one input

$$\begin{aligned} \mathsf{msk} &= \mathbf{u} \\ \mathsf{Enc}_1(\mathsf{msk}, \mathbf{x}) &= \mathbf{x} + \mathbf{u} \\ \mathsf{KeyGen}_1(\mathsf{msk}, \mathbf{y}) &= \langle \mathbf{u}, \mathbf{y} \rangle, \mathbf{y} \end{aligned}$$

2 One ciphertext, many inputs

$$\begin{aligned} \mathsf{msk} &= \mathbf{u}_1 \dots \mathbf{u}_n \\ \mathsf{Enc}_2(\mathsf{msk}, i, \mathbf{x}_i) &= \mathbf{x}_i + \mathbf{u}_i \\ \mathsf{KeyGen}_2(\mathsf{msk}, \mathbf{y}_1 \dots \mathbf{y}_n) &= \\ \textstyle\sum_{i=1}^{n} \langle \mathbf{u}_i, \mathbf{y}_i \rangle, \mathbf{y}_1 \dots \mathbf{y}_n \end{aligned}$$

3 Many ciphertexts, one input

$$\begin{aligned} \mathsf{msk} &= \mathbf{v} \\ \mathsf{Enc}_3(\mathsf{msk}, \mathbf{x}) &= g^r, g^{\mathbf{x}+r\mathbf{v}} \in \mathbb{G}^{m+1} \\ \mathsf{KeyGen}_3(\mathsf{msk}, \mathbf{y}) &= \langle \mathbf{v}, \mathbf{y} \rangle, \mathbf{y} \end{aligned}$$

4 Many ciphertexts, many inputs

$$\begin{aligned} \mathsf{msk} &= \mathbf{u}_i, \mathbf{v}_i \\ \mathsf{Enc}_4(\mathsf{msk}, i, \mathbf{x}) &= \mathsf{Enc}_3(\mathsf{Enc}_2(\mathsf{msk}, i, \mathbf{x}_i)) \\ \mathsf{KeyGen}_4(\mathsf{msk}, \mathbf{y}_1 \dots \mathbf{y}_n) &= \textstyle\sum_{i=1}^{n} \langle \mathbf{u}_i, \mathbf{y}_i \rangle, \mathsf{KeyGen}_3(\mathbf{y}_i) \end{aligned}$$

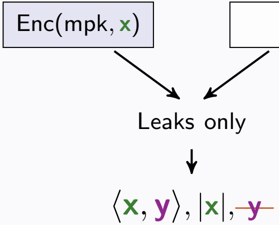
Our Construction Without Pairings

Pairing-free construction

- removed bilinear groups
- adaptive security
- support larger messages
- efficient schemes (linearly-sized ciphertexts and decryption keys)
- instantiations from DDH, LWE or DCR.
- polynomial number of slots

Function-Hiding Scheme

Security goal



New multi-input function-hiding scheme for the inner product ✓

- Adaptively secure
- poly-many inputs

Function-Hiding Scheme

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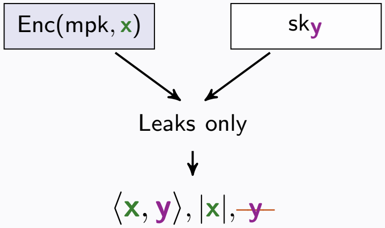

New multi-input function-hiding scheme for the inner product ✓

- Adaptively secure
- poly-many inputs

Multi-input inner-product	Number of inputs	Assumptions
[AGRW17]	poly inputs	Pairing Groups
[DOT18] FH	unbounded poly inputs	Pairing Groups
This work FH	poly inputs	Pairing Groups

FH - function hiding

Future Work

Adapt our techniques for other classes of functions?

Thank you!

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