

# Non-Malleable Codes for Partial Functions with Manipulation Detection

Aggelos Kiayias Feng-Hao Liu

Yiannis Tselekounis

Edin. & FAU

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# Outline



- Introduction to non-malleable codes
- Adversarial model, motivation
- Results, constructions
- Intuition

# Encoding schemes



#### An encoding scheme is a pair of algorithms (Enc, Dec), satisfying correctness:

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#### Error-correction codes: guarantee correctness in the presence of faults



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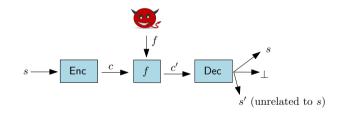


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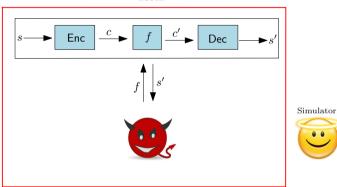




# cEnc c'Dec s- $\rightarrow s'$ |s'|

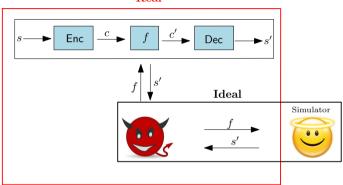
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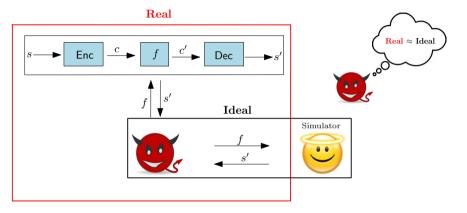




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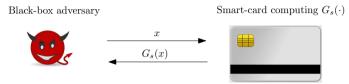
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# Application of NMC

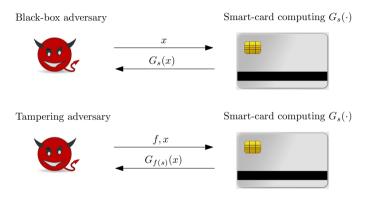




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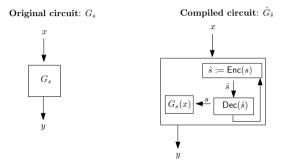
# $\label{eq:Application} \mbox{Application of NMC}$





# Application of NMC

Assuming (Enc, Dec) is a non-malleable code w.r.t.  $\mathcal{F}$ .



**Non-malleability**: for any  $f \in \mathcal{F}$ ,  $f(\hat{s})$  is simulatable and independent of s





#### Non-malleability is impossible against arbitrary tampering function classes

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For instance, consider a class containing the function f(c) := Enc(Dec(c) + 1)

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**Proposed function classes**: Split-state functions [ADL14, DKO13, ADKO15, LL12, AAG<sup>+</sup>16, DPW10, KLT16], bit-wise tampering and permutations [DPW10, AGM<sup>+</sup>15a, AGM<sup>+</sup>15b], bounded-size function classes [FMVW14], bounded depth/fan-in circuits [BDKM16], space-bounded tampering [FHMV17,BDKM18], block-wise tampering [CKM11,CGM<sup>+</sup>15], AC0 circuits, bounded-depth decision trees and streaming adversaries [BDKM18], small-depth circuits [BDGMT18], and others.

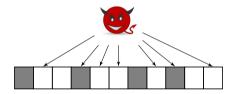


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This work: Partial functions

# NMC for Partial Functions



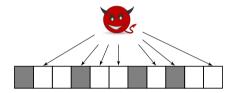


We allow read/write access to arbitrary subsets of codeword locations, with bounded cardinality.

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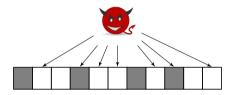
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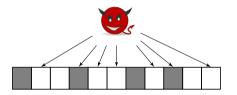




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# Basic definitions





- Information rate: the ratio of message to codeword, length, as the message length goes to infinity.
- Access rate: the fraction of the number of bits (symbols) the attacker is allowed to access over, the total codeword length.

# Main Goal



Is it possible to construct efficient (high information rate) non-malleable codes for partial functions, while allowing the attacker to access almost the entire codeword (high access rate)?



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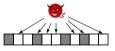


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• Constant functions are excluded from the model, thus it potentially allows stronger primitives





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- Our results imply efficient All-Or-Nothing-Transforms under standard assumptions

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• Non-malleability for partial functions with concrete access rate 1 is impossible



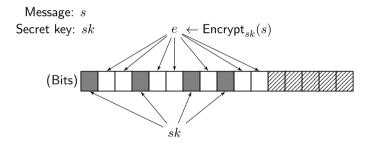
- Non-malleability for partial functions with concrete access rate 1 is impossible
- Impossibility on the information-theoretic setting [CG14]: assuming constant access/information rate, security is achievable only with constant probability



Towards an encryption-based solution:

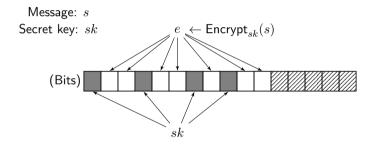


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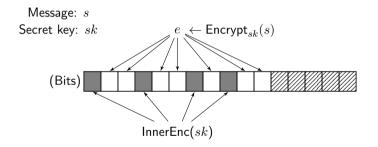
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Security breaks by accessing O(|sk|/|s|) codewords bits



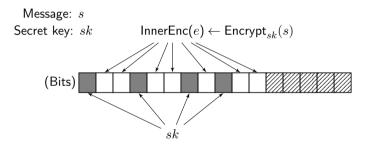
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#### **Question**: Is it possible to achieve access rate greater than O(|sk|/|c|)?



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# **More generally**: Can we achieve access rate greater than what our weakest primitive sustains?

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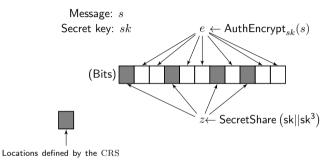
#### Main observation: the structure of the codeword is fixed and known to the attacker



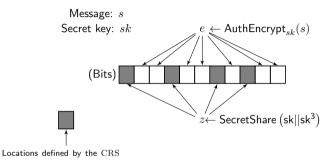
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Idea: hide the structure via randomization

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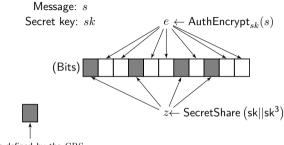






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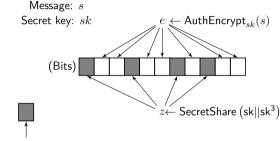




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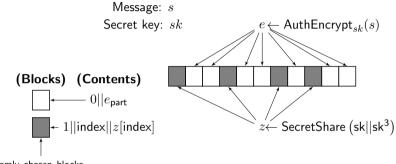


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- Due to the shuffling, the attacker learns nothing about  $sk, sk^3$ . Let  $(sk, sk^3) \xrightarrow{f} (sk', sk'')$
- If  $(sk, sk^3) \neq (sk', sk'')$ , then  $\Pr[sk'^3 = sk''] \leq \text{negl}$ , otherwise we can recover sk
- Thus, if  $sk \neq sk'$  or  $sk^3 \neq sk''$ , the simulator outputs  $\perp$ , otherwise, security follows by the authenticity property of the encryption scheme

#### Removing the CRS







Block size: log(k)

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#### Conclusions



• Stronger notion: Non-malleable codes with manipulation detection (MD-NMC)

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- $\bullet$  Constructions: efficient  $\mathrm{MD}\text{-}\mathrm{NMC}$  for partial functions
- **Applications**: tamper-resilient cryptography (boolen/aritmetic circuits), secure communication over adversarial channels (Wire-Tap channels), AONTs



## Thank you!

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