Non-Uniform Bounds in the Random-Permutation, Ideal-Cipher, and Generic-Group Models

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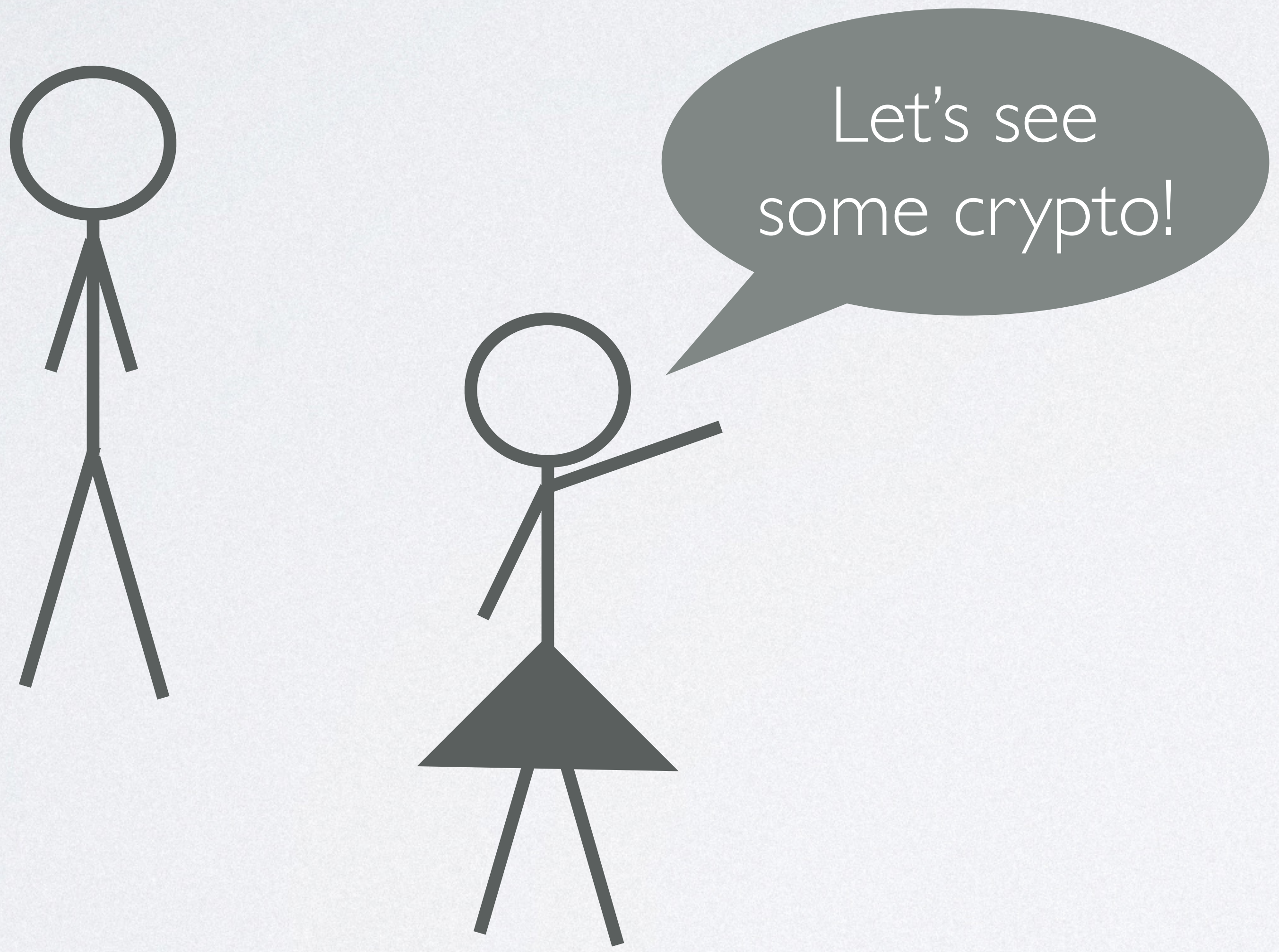
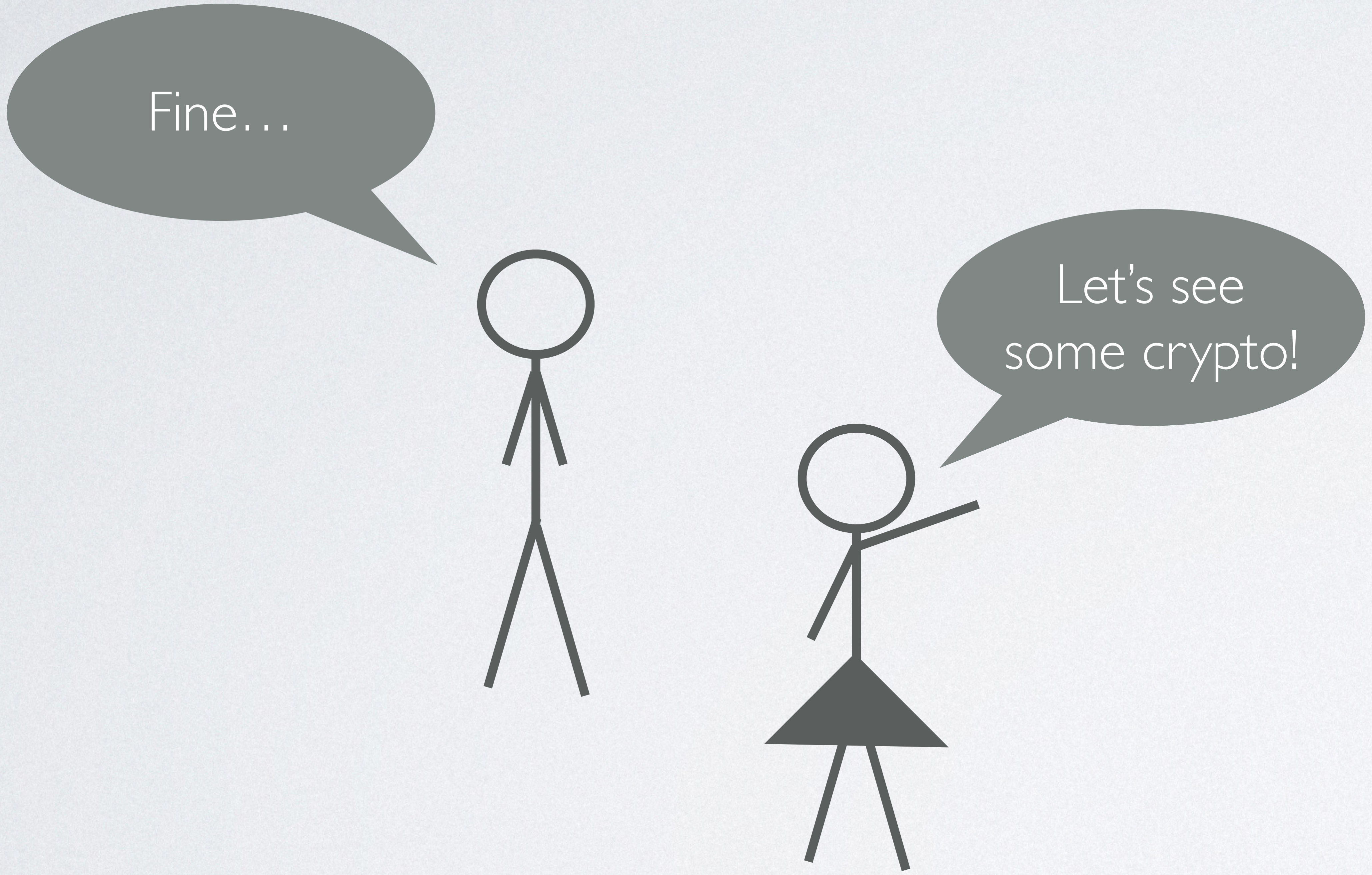
New York University

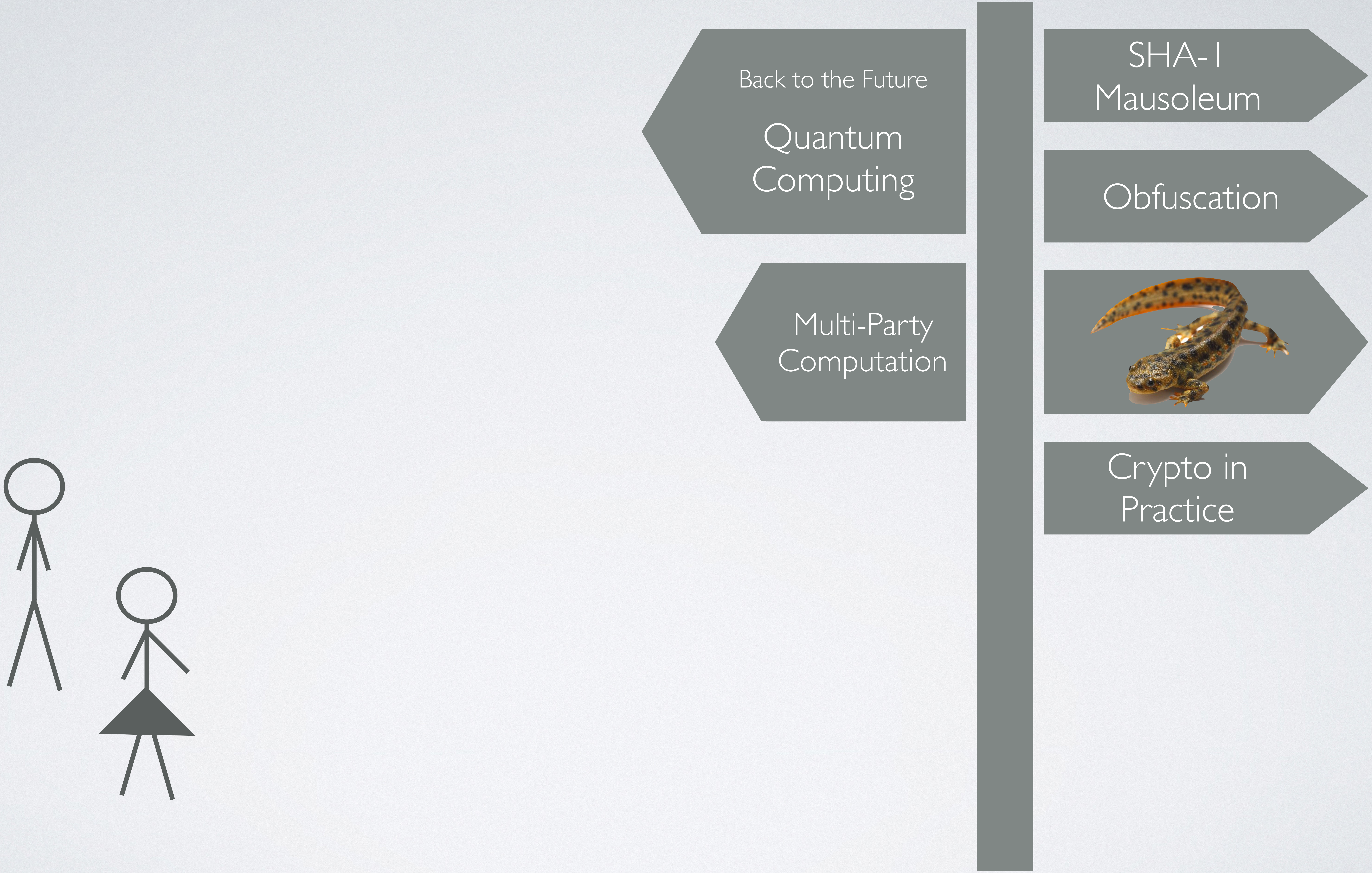
Joint work with:

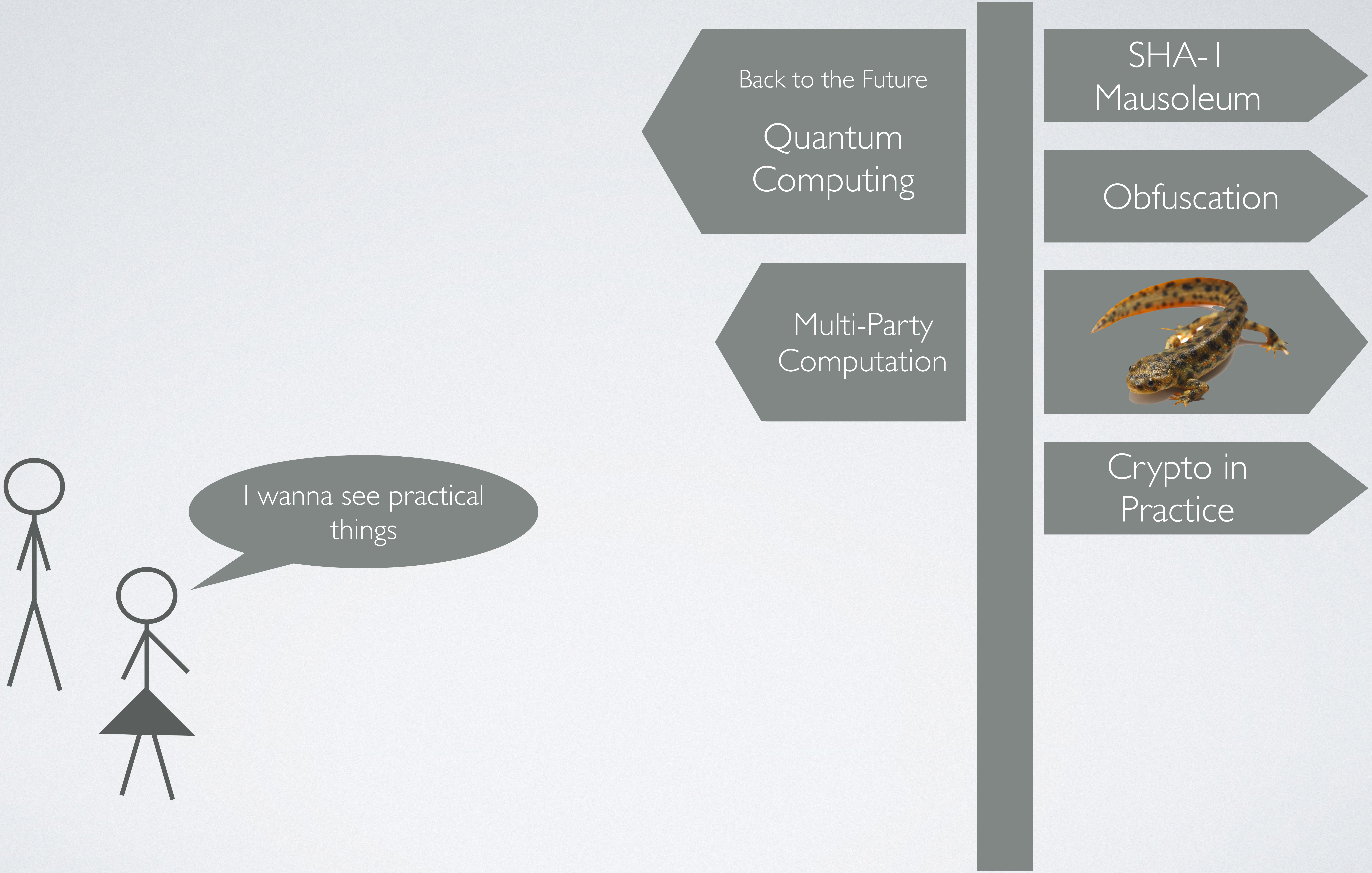
Yevgeniy Dodis
New York University

Siyao Guo
Northeastern University


Let's see
some crypto!


Fine…
Let's see some crypto!


Back to the Future
Quantum Computing
Multi-Party Computation
SHA-1 Mausoleum
Obfuscation
Crypto in Practice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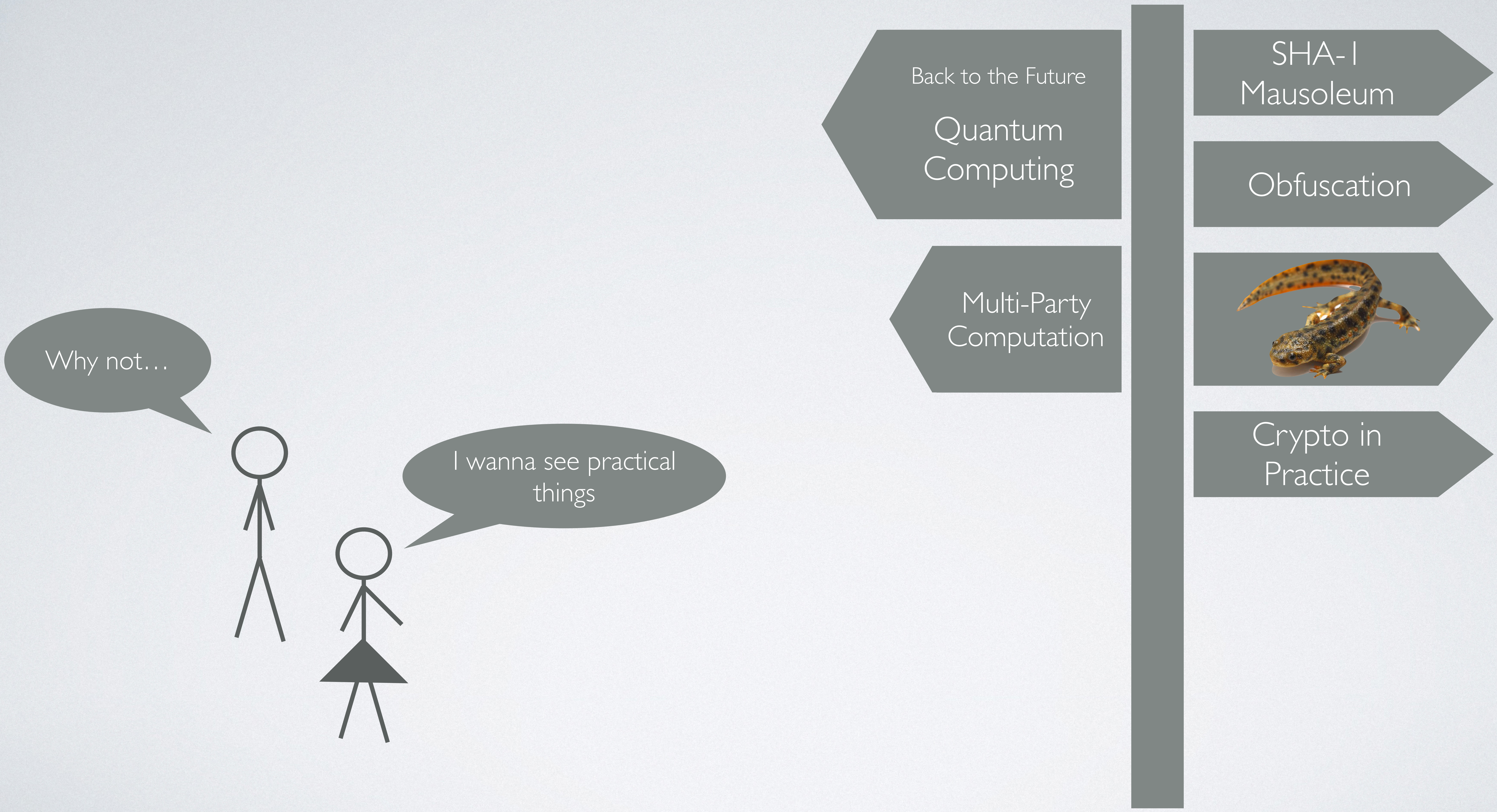
I wanna see practical things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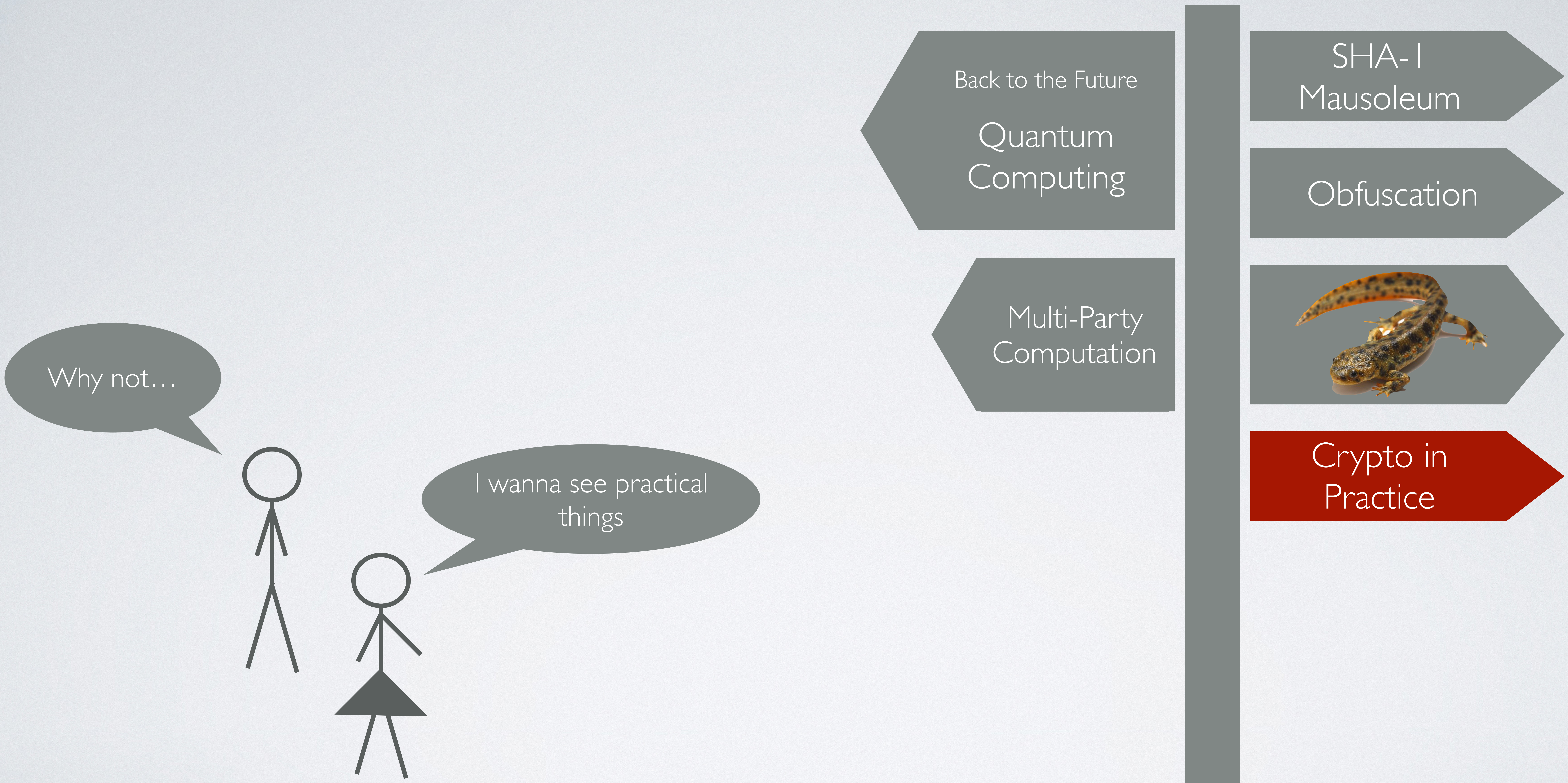
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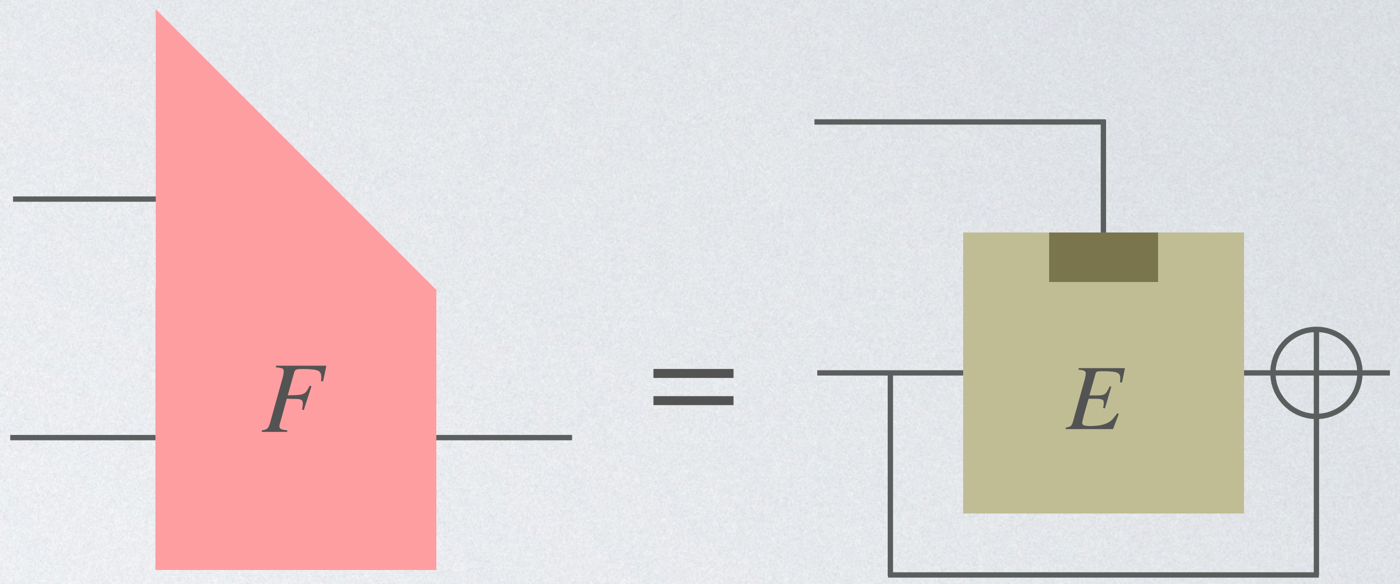
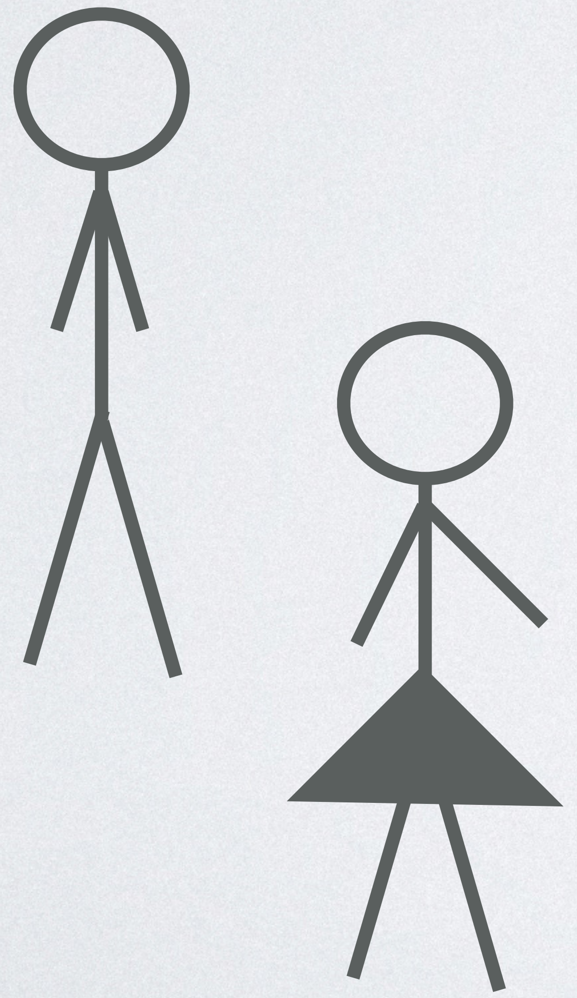
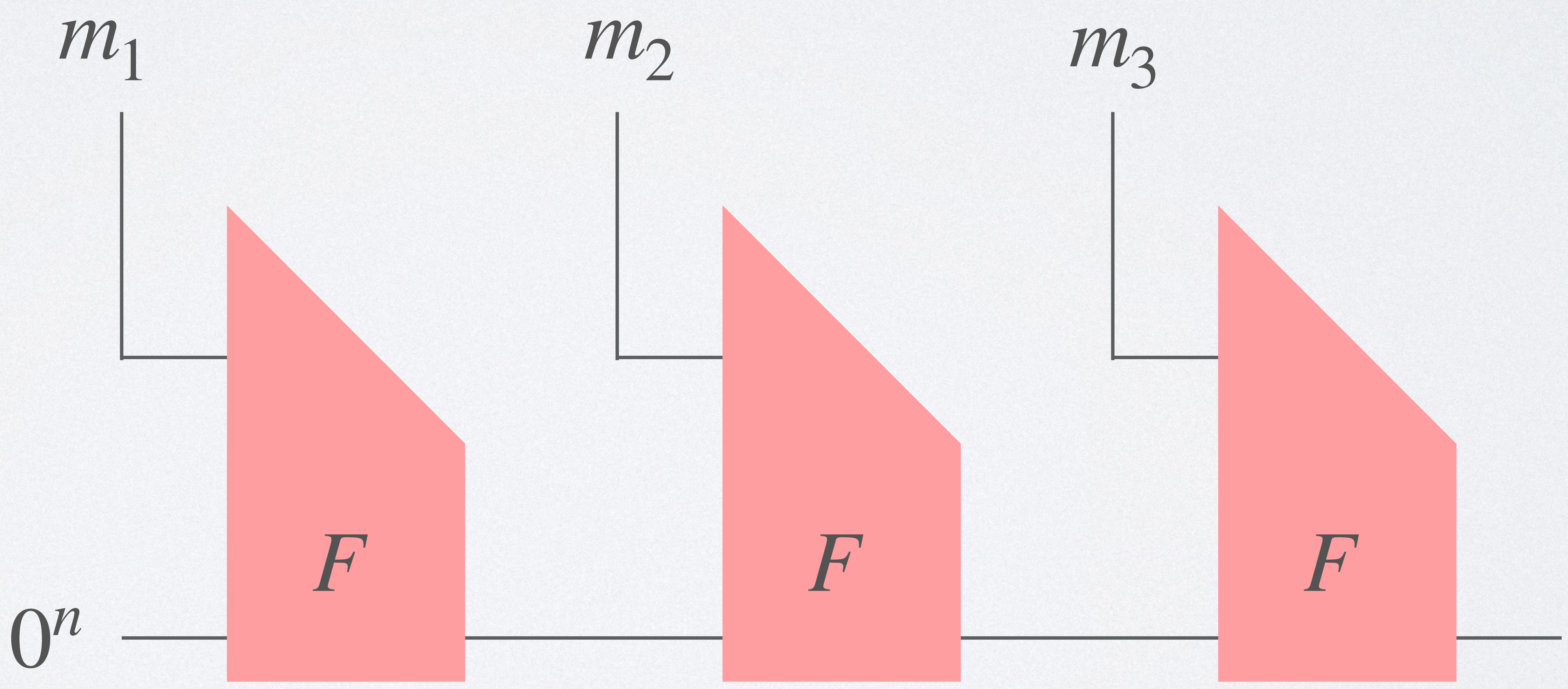
Exhibit A: Merkle-Damgard with Davies-Meyer (SHA-2)







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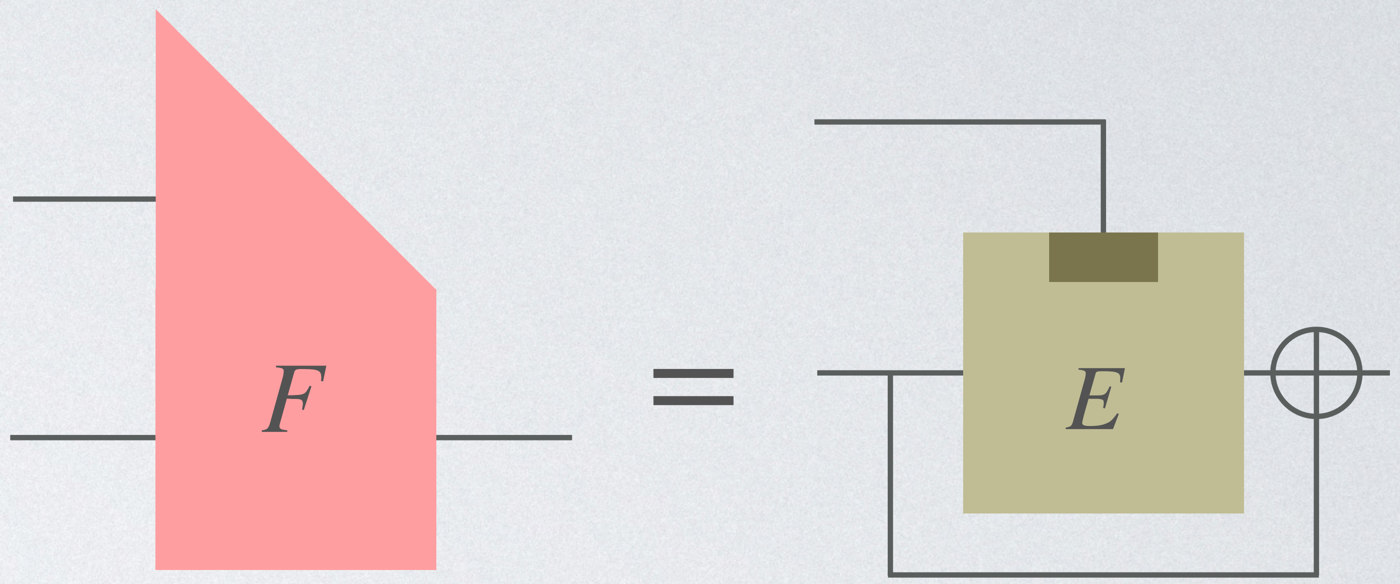
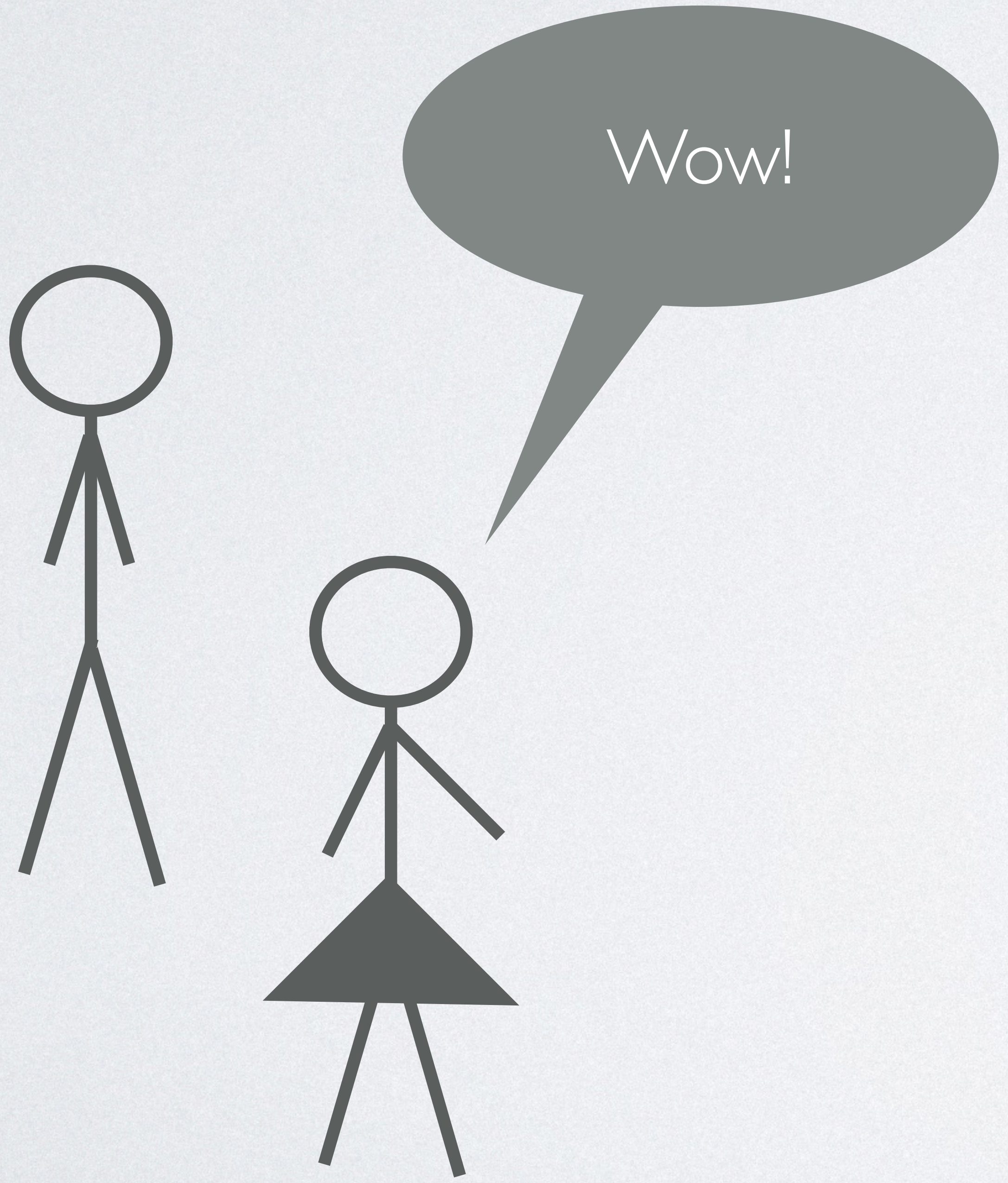




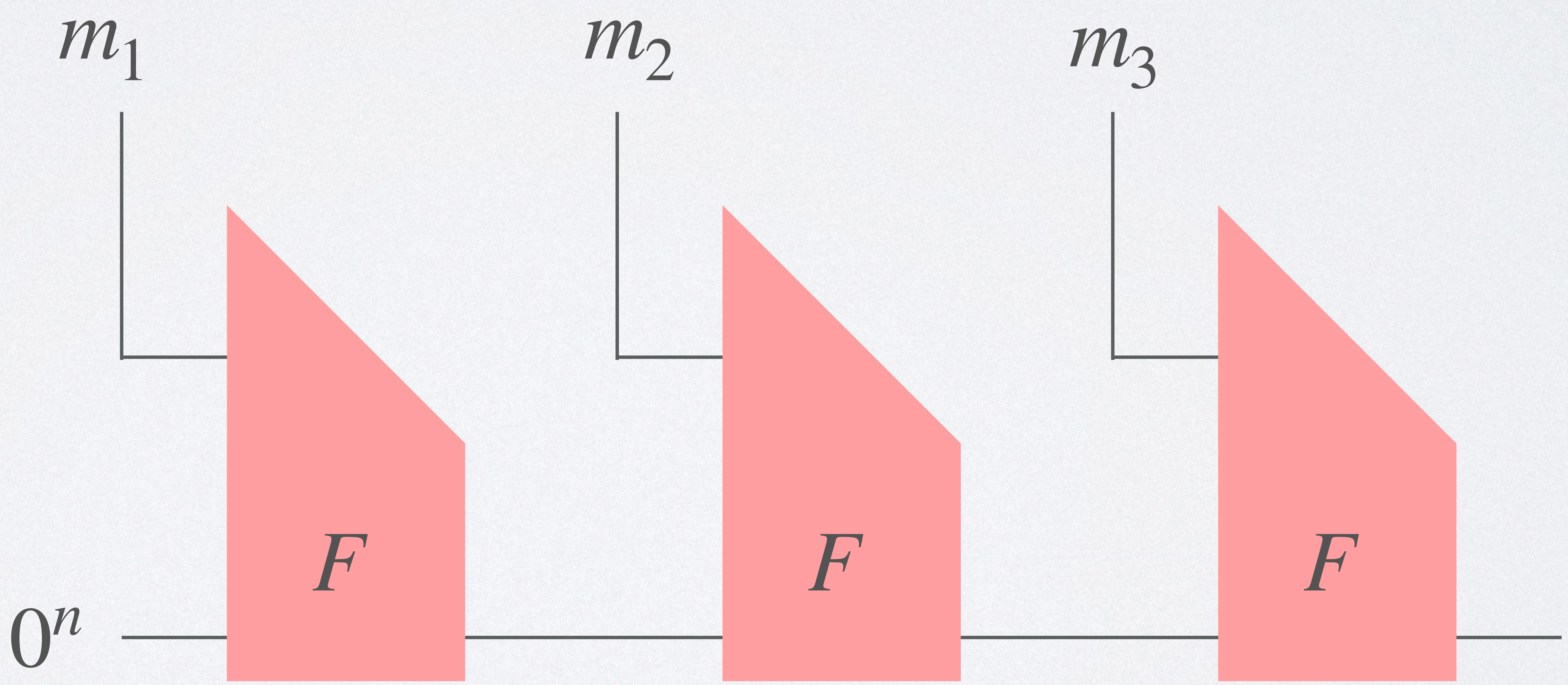
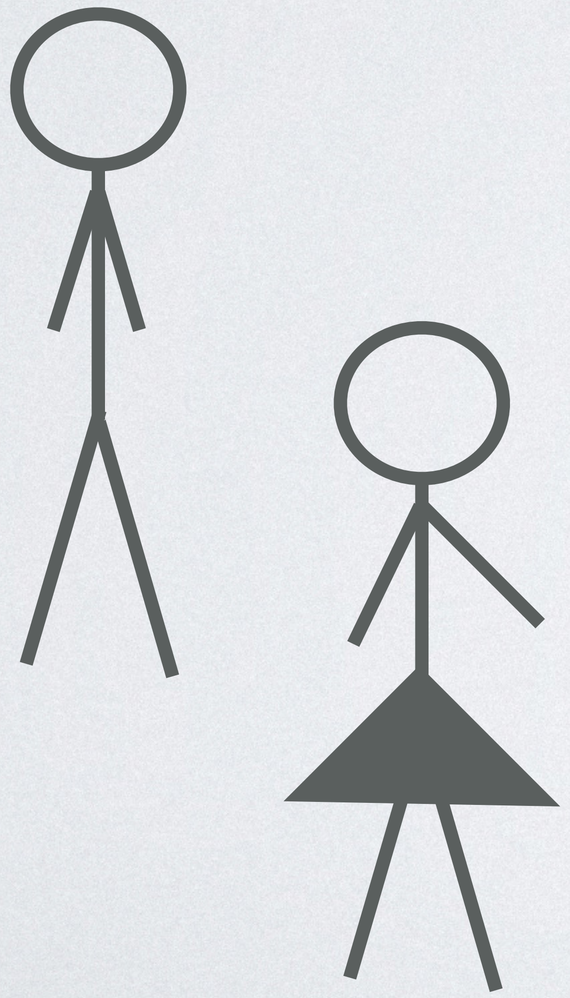
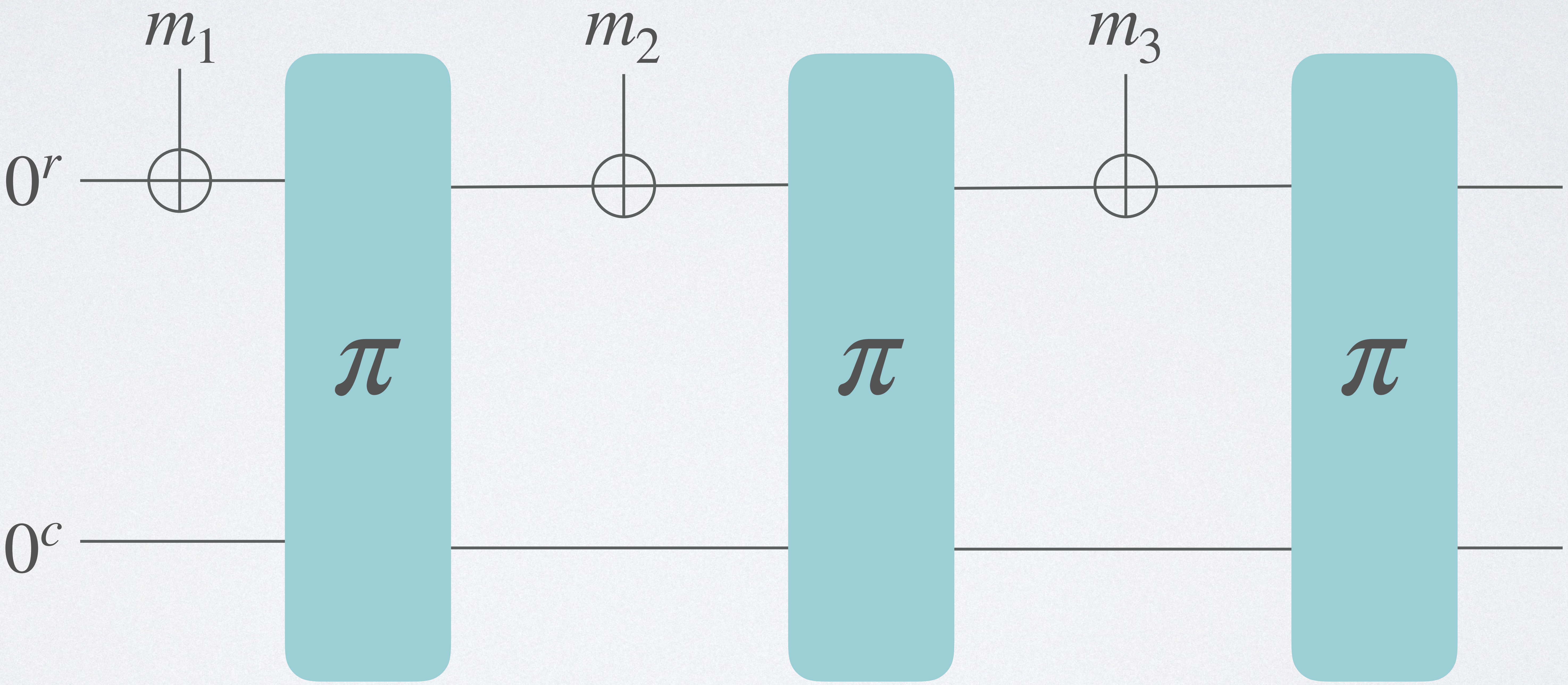


Exhibit 75: Sponge Construction (SHA-3)





Used as:

- CRHF
- MAC
- PRF
- PRNG
- etc.

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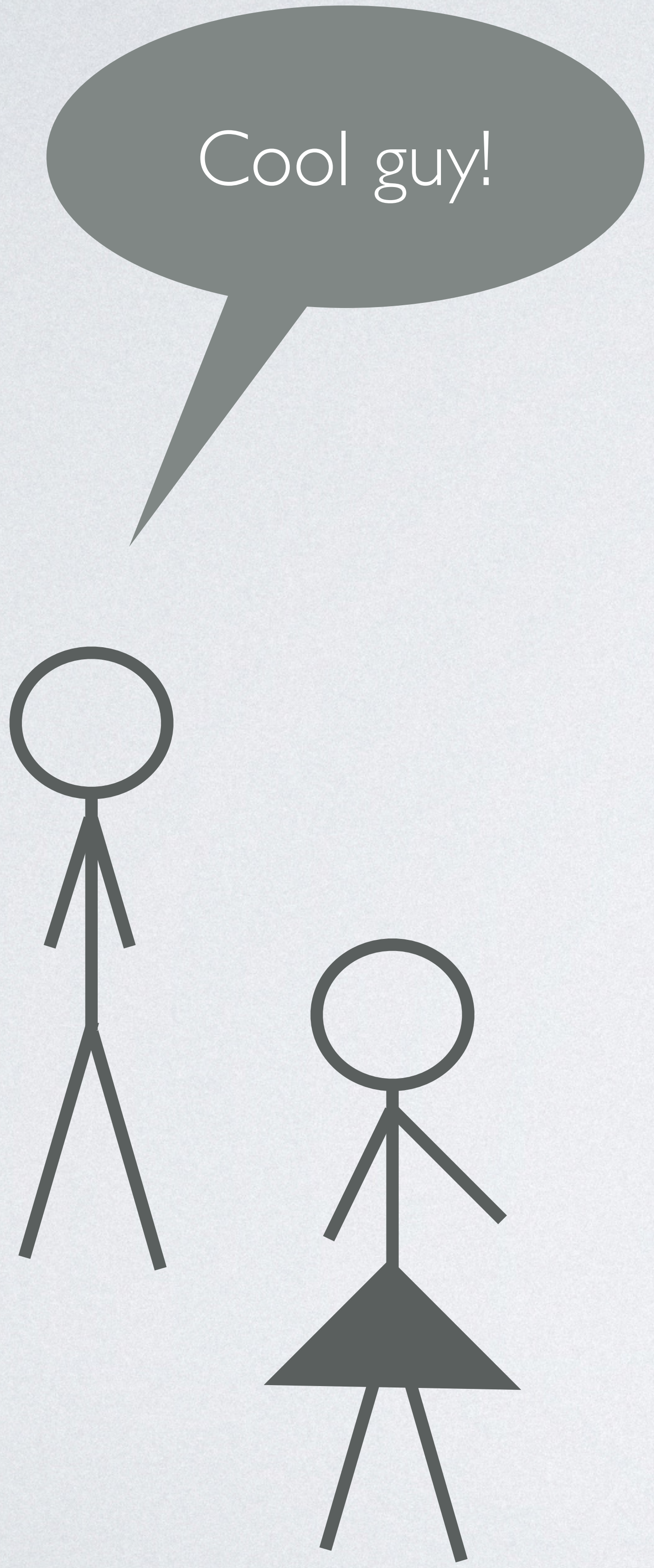

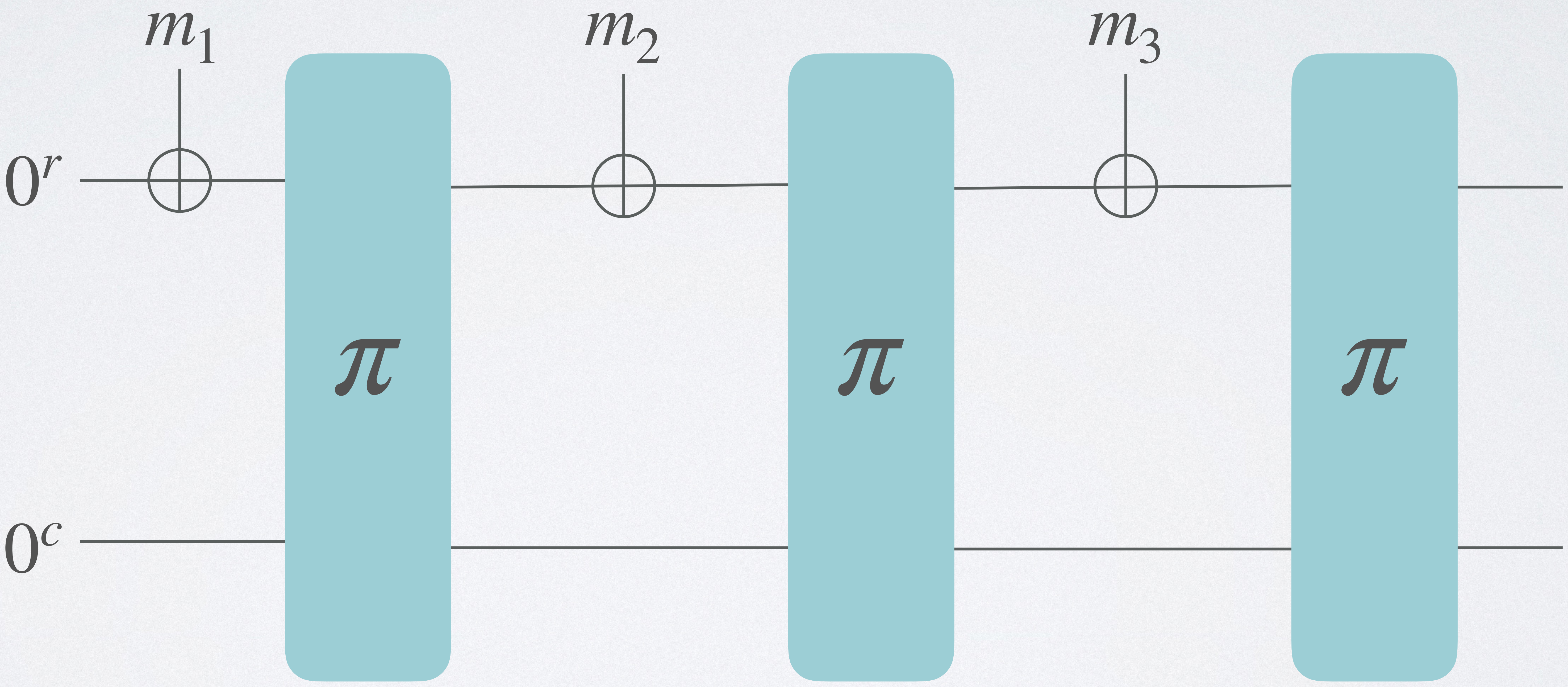

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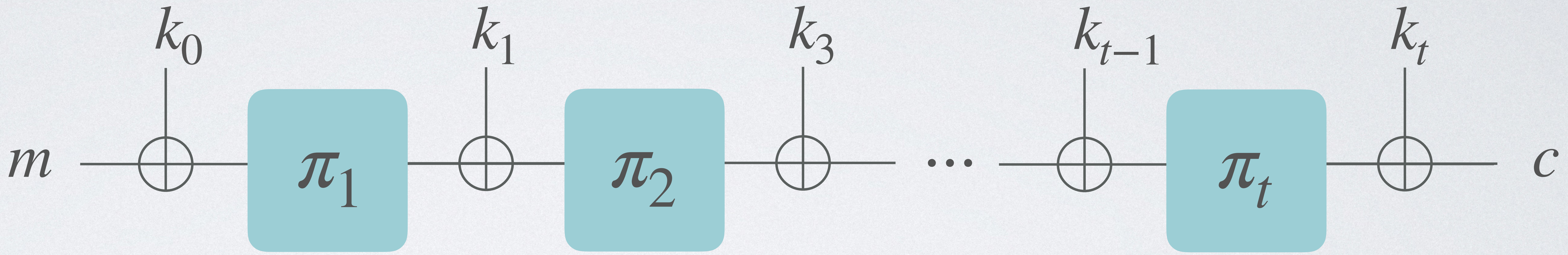
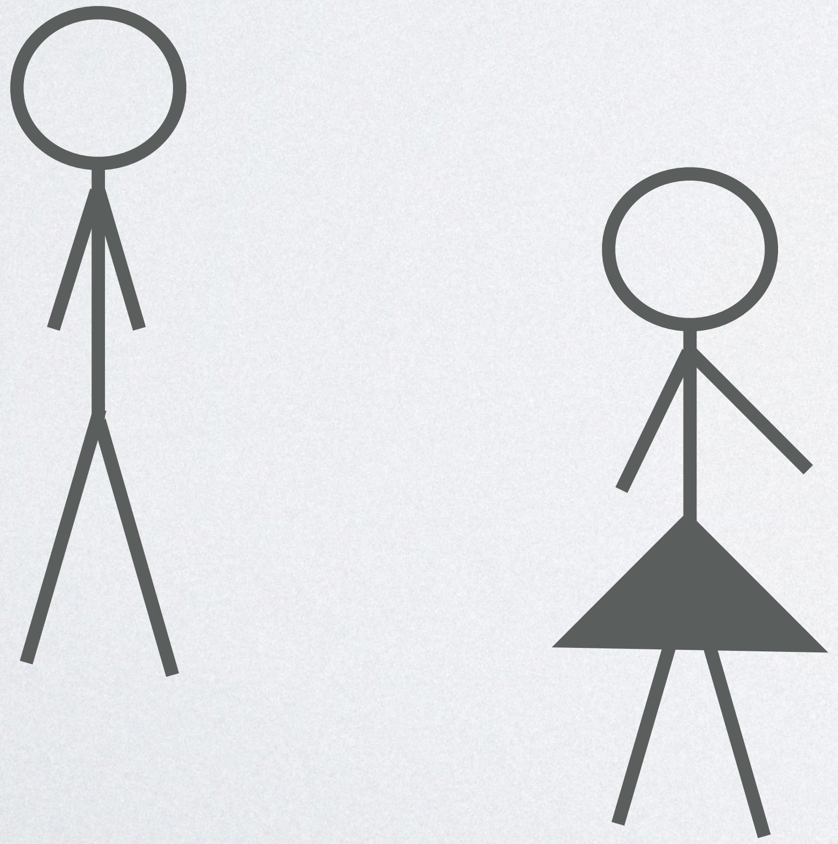
Item E12: Key-Alternating Ciphers (AES)





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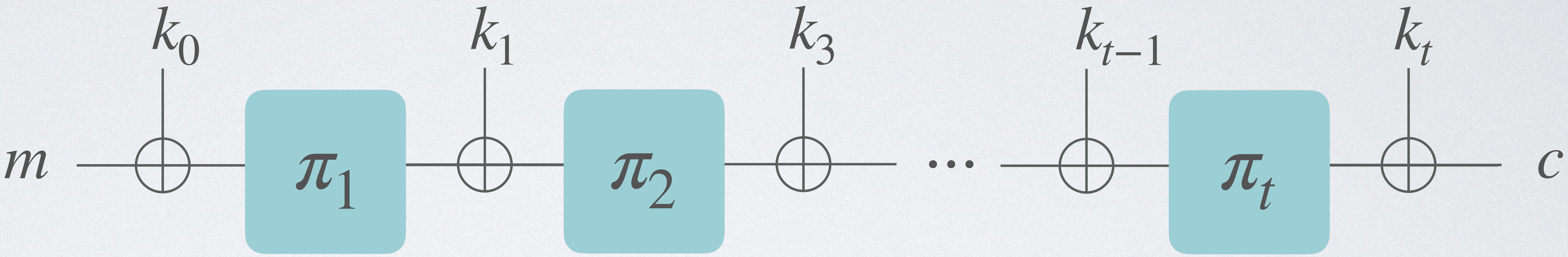

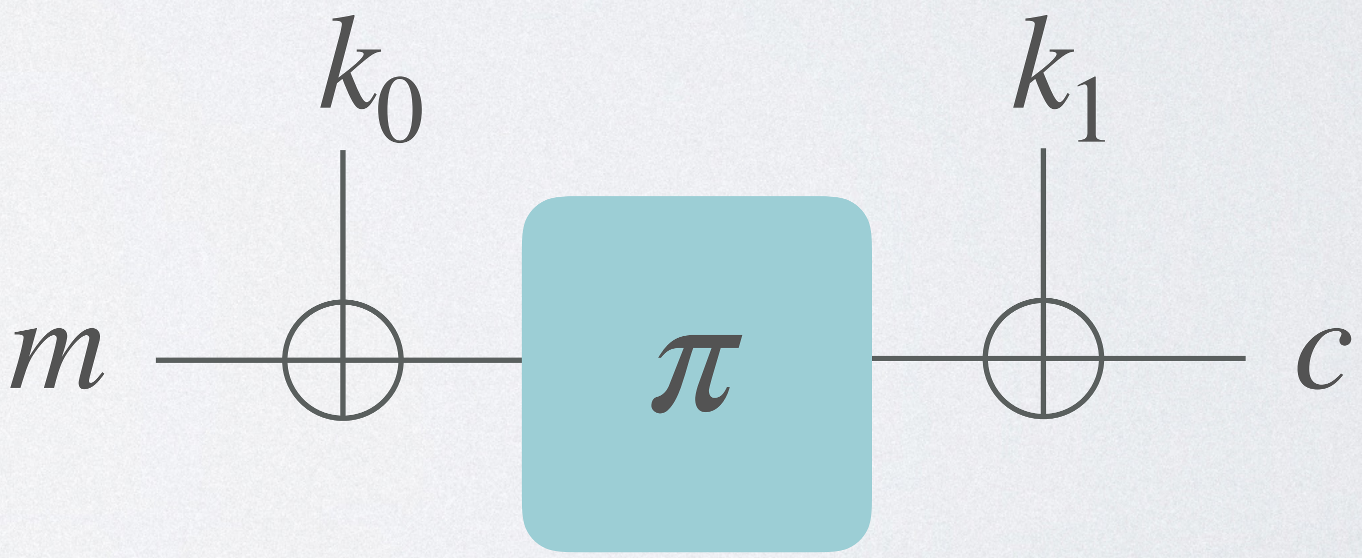

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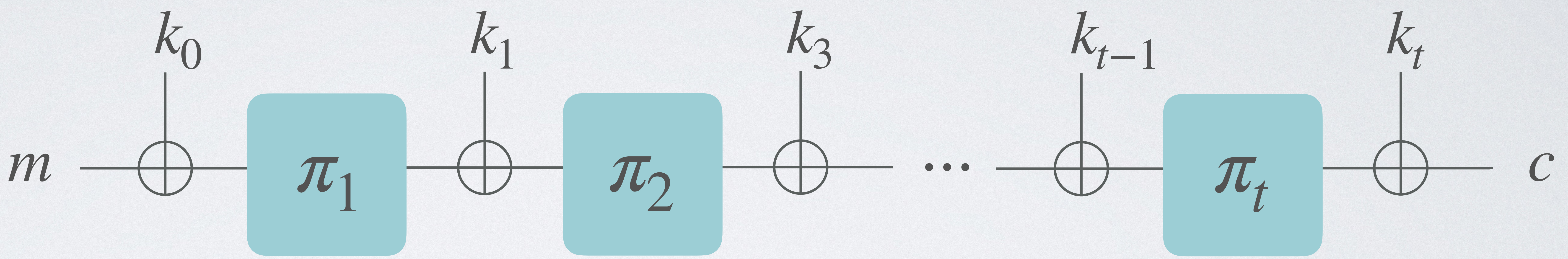
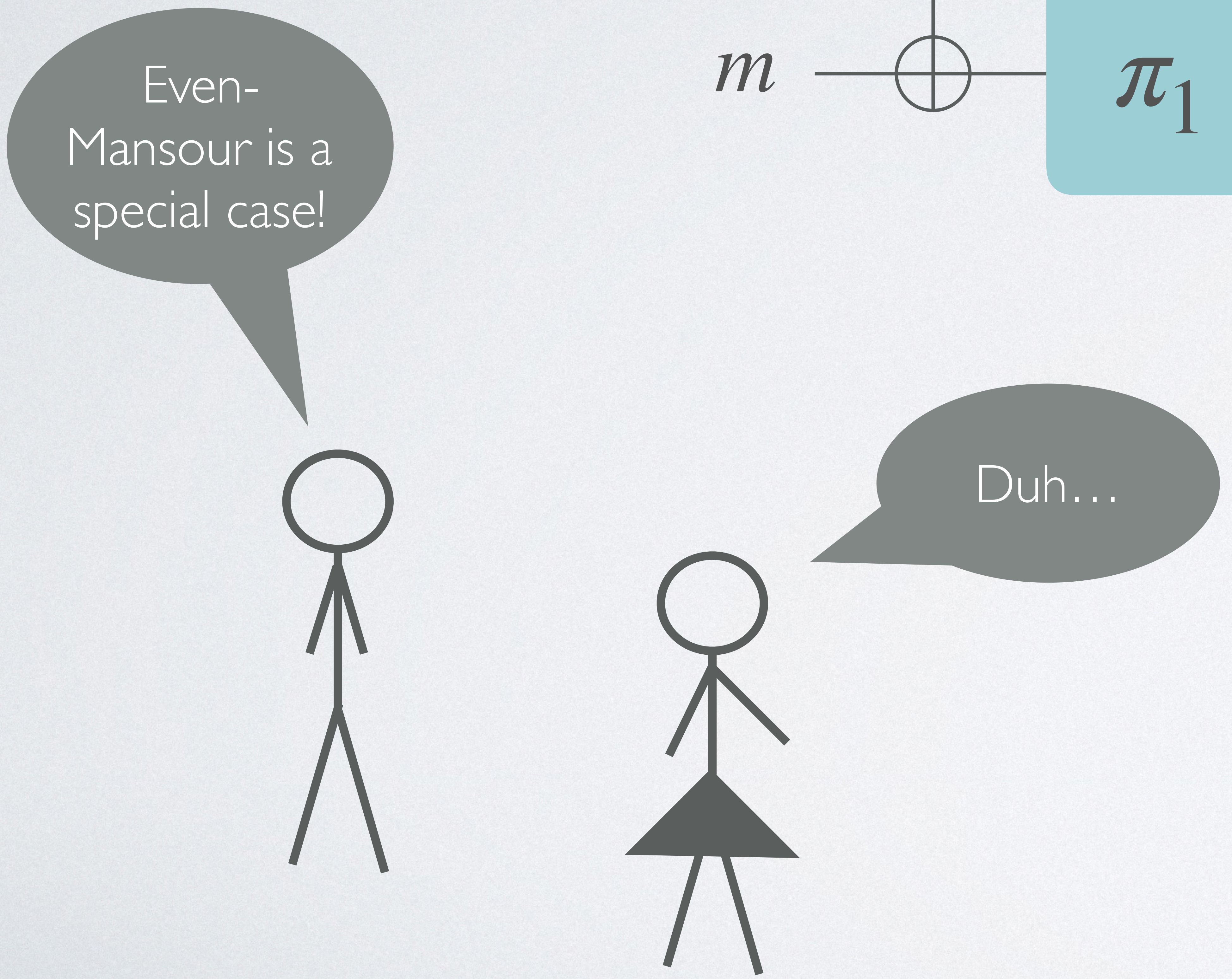




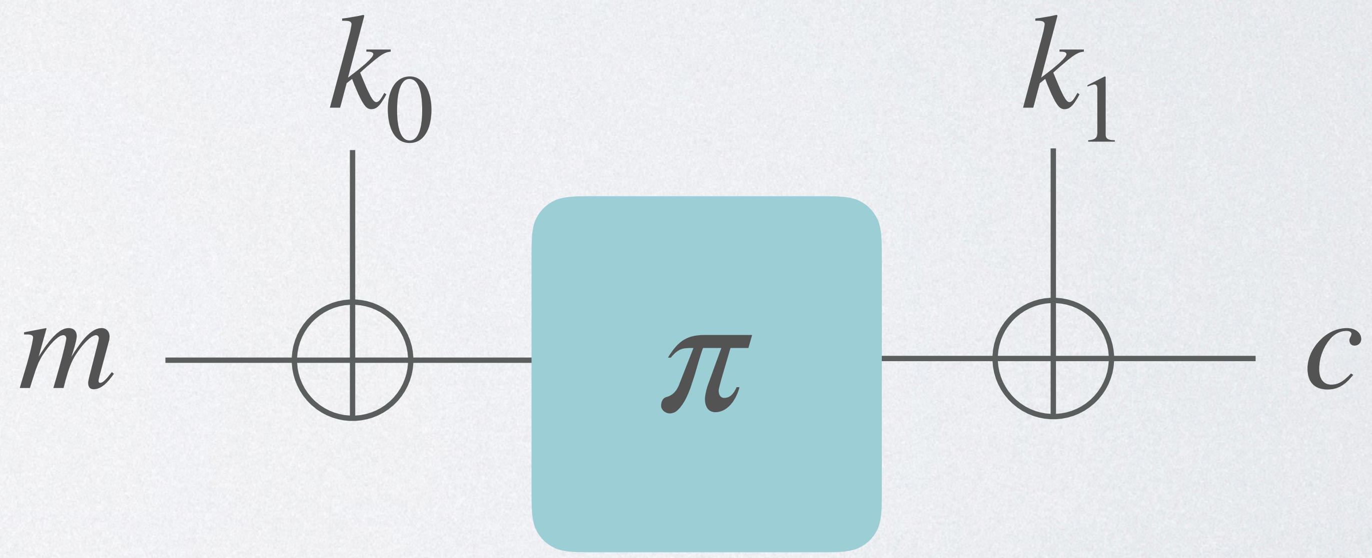
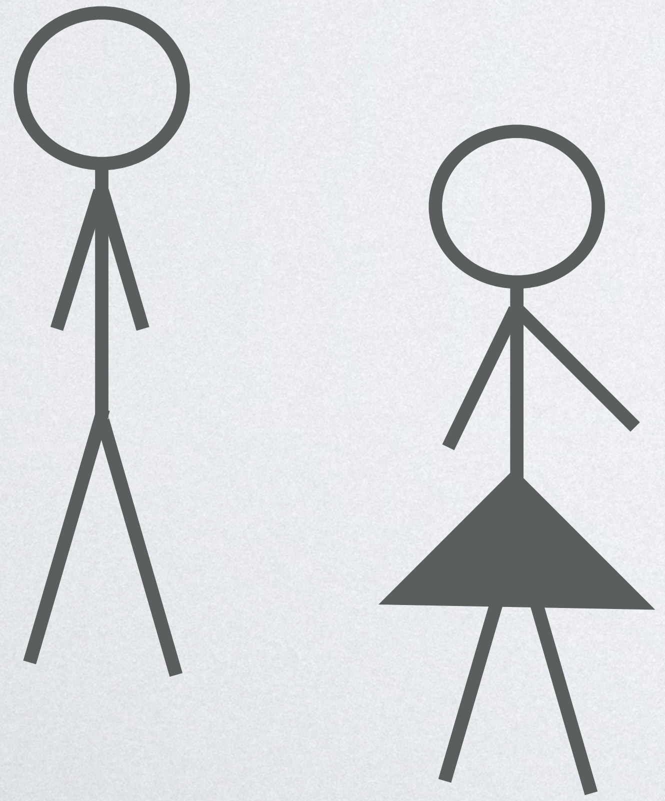


Exhibit S: Discrete Logarithms

Cyclic group $G = \langle g \rangle$ of order N



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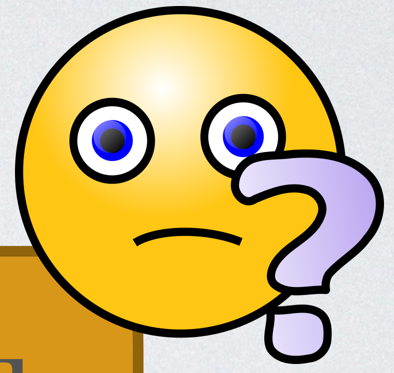
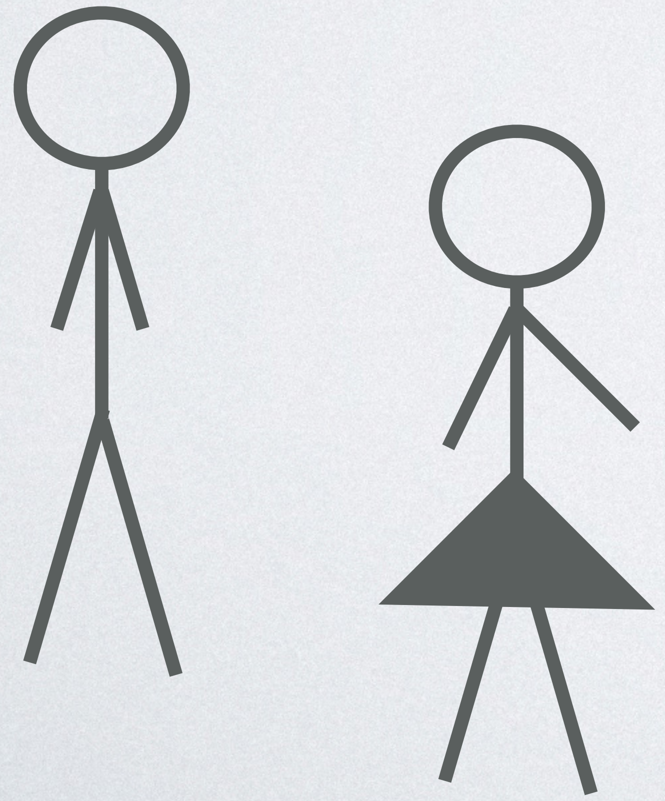


$x \leftarrow [N]$

$y = g^x$



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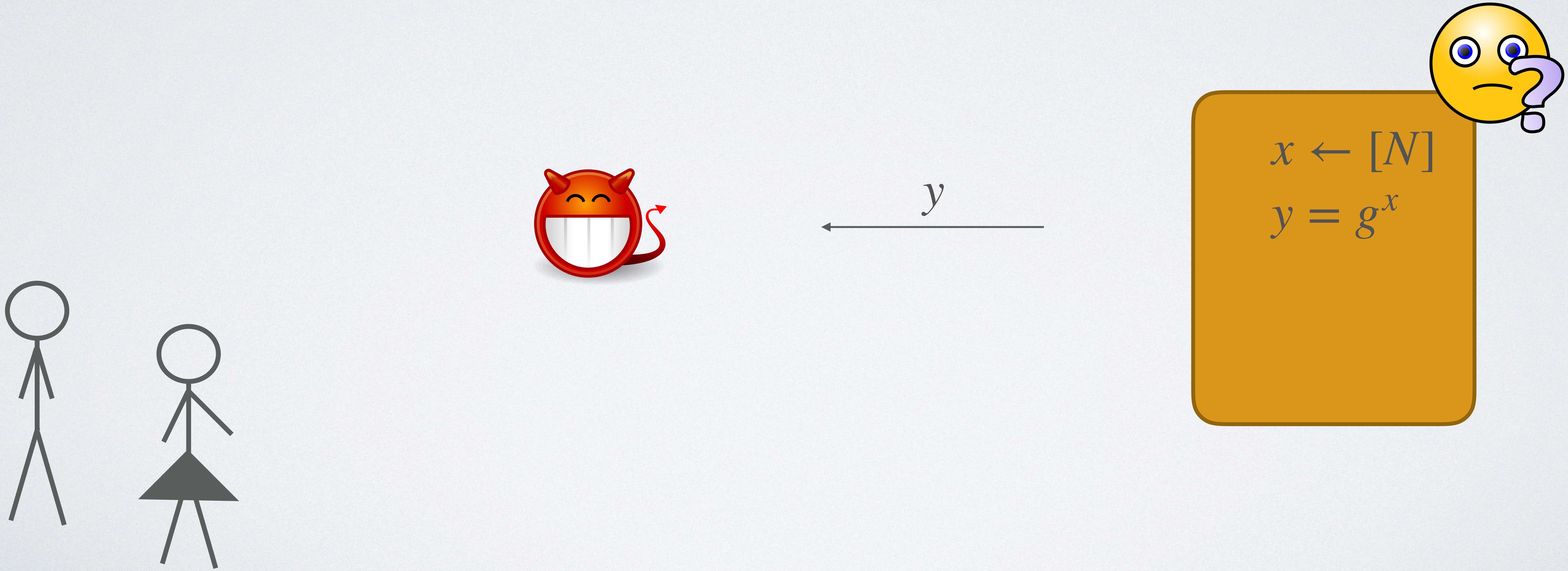

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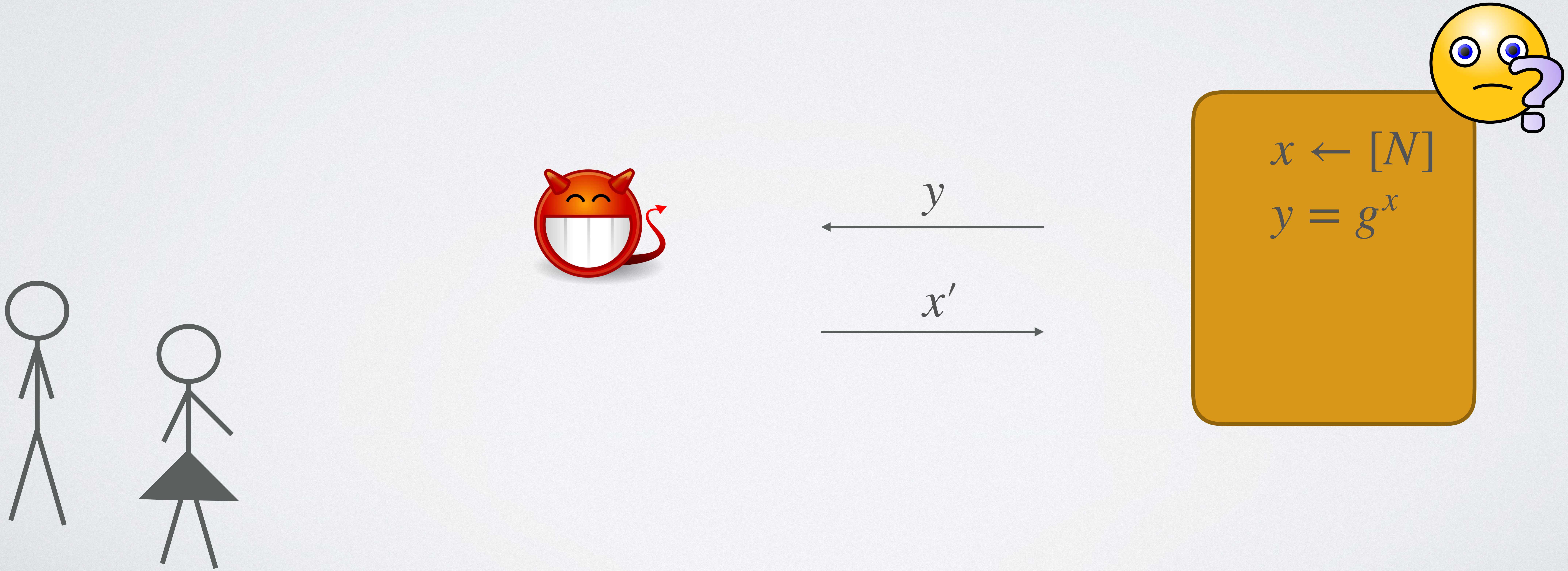

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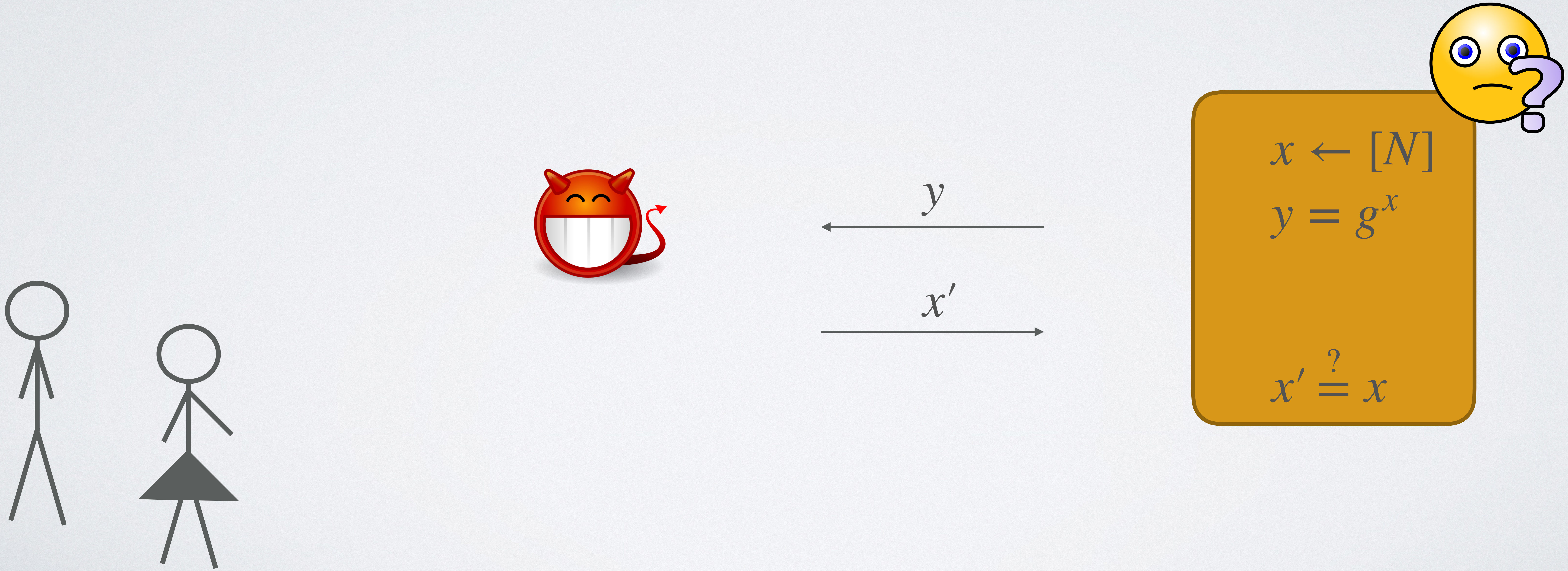
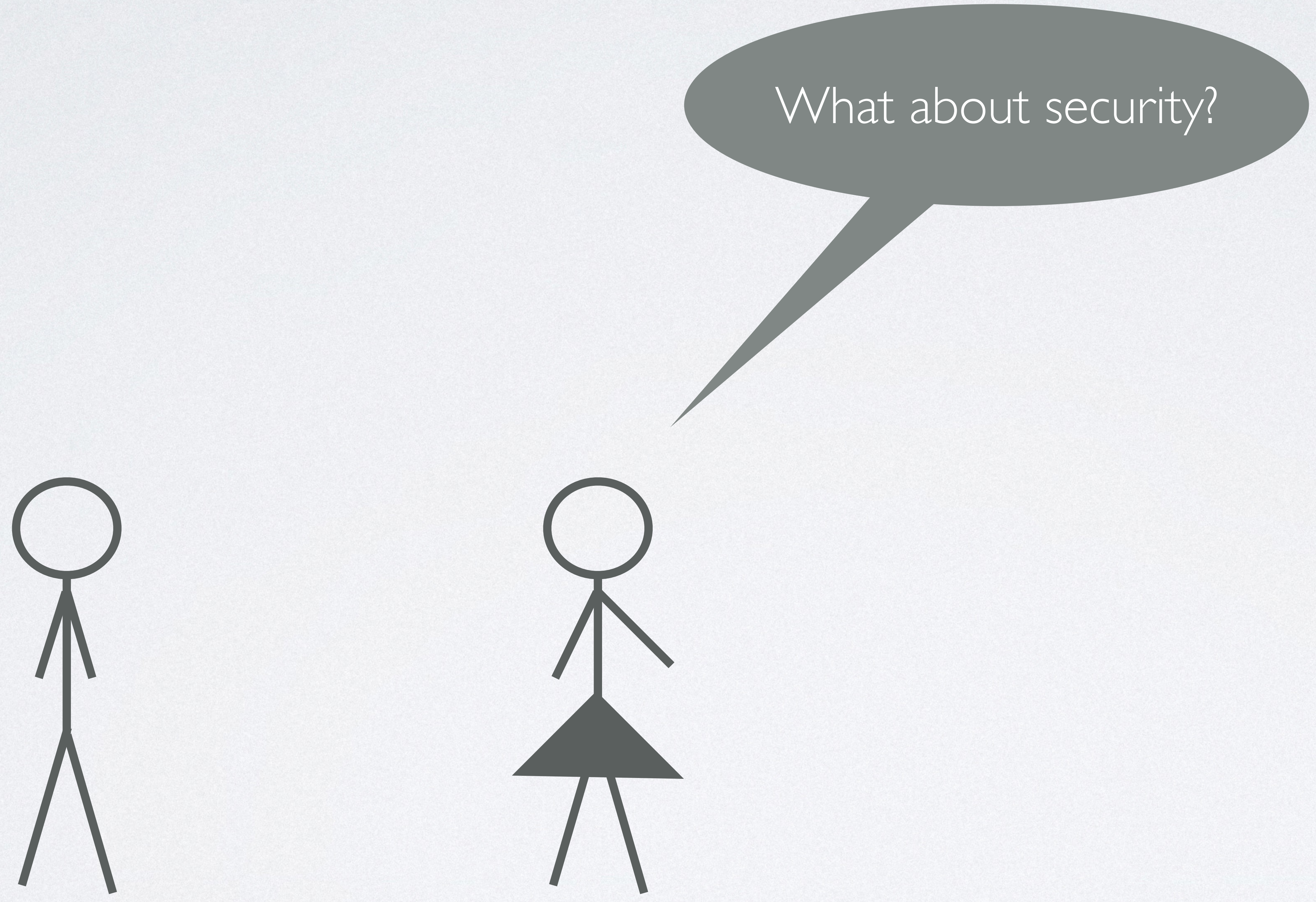
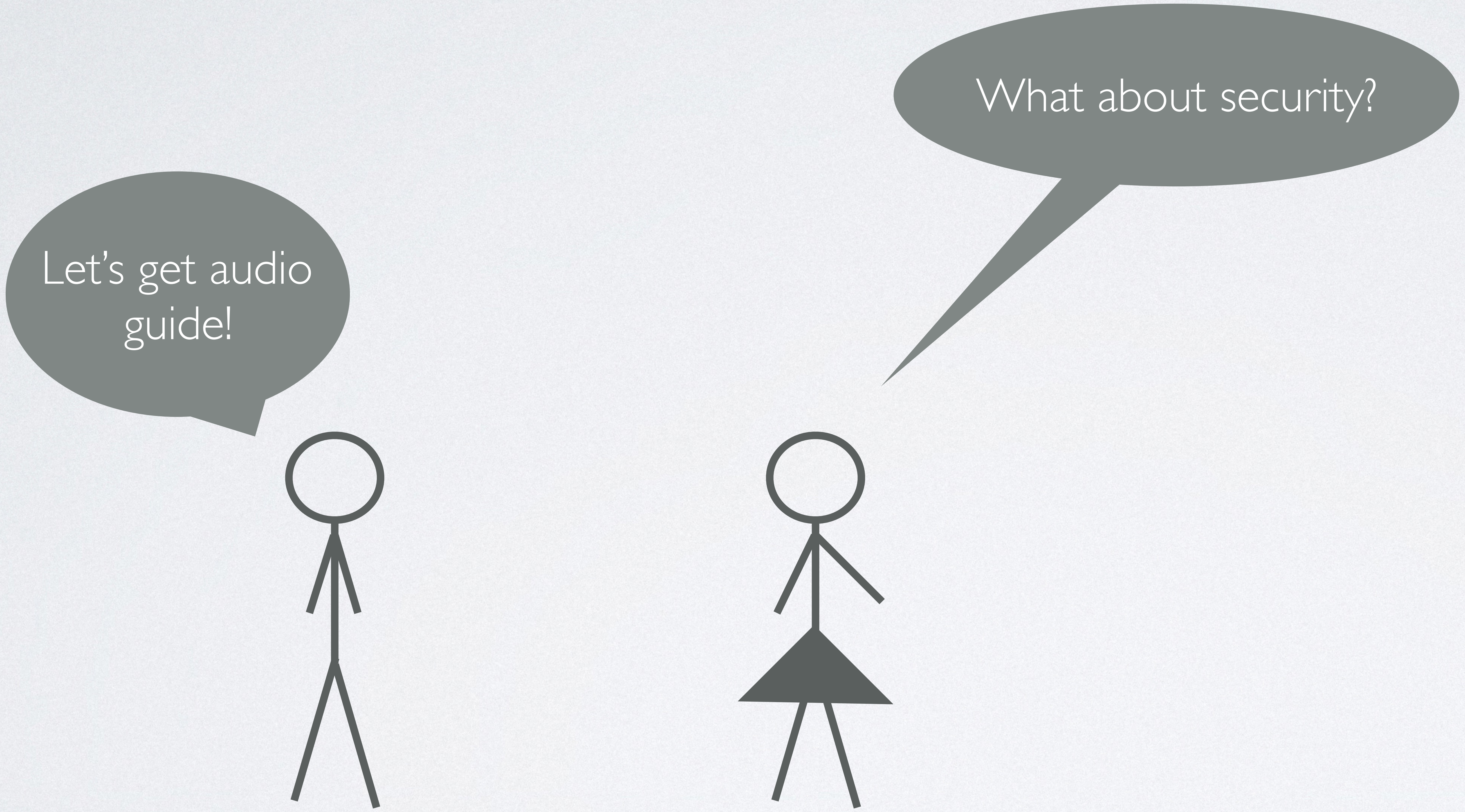
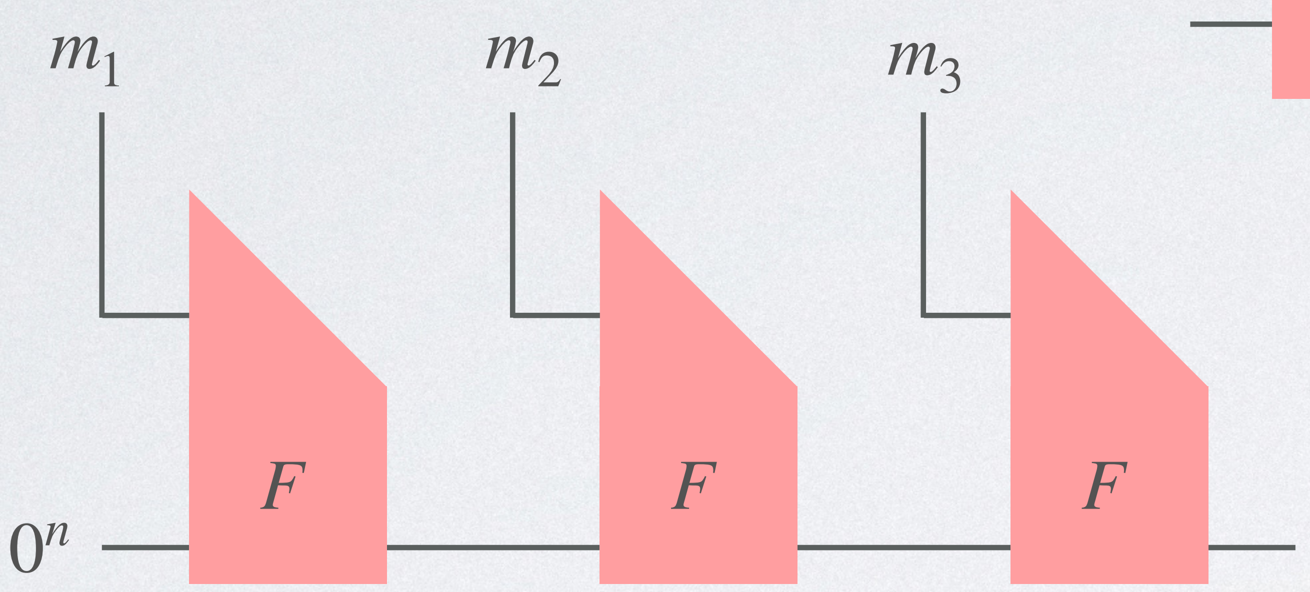
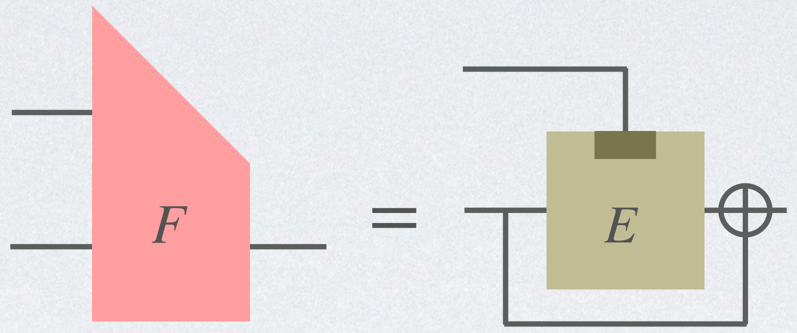


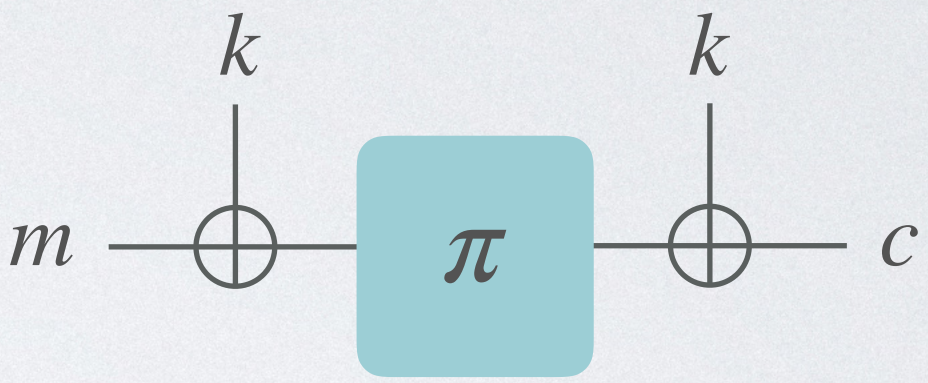

What about security?

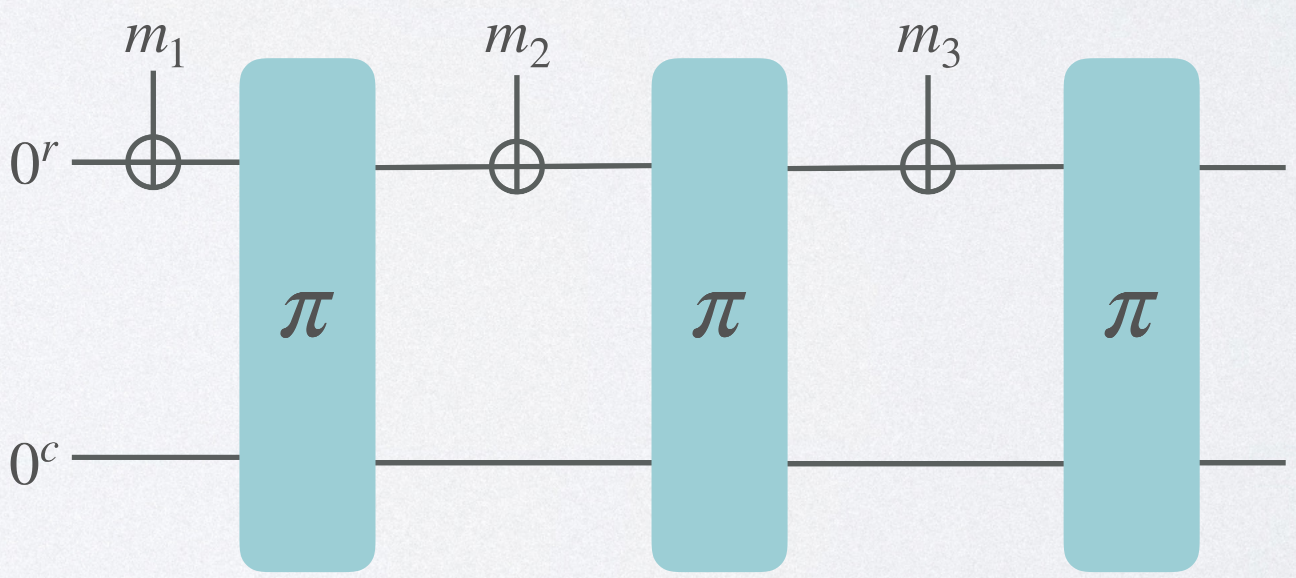

Let's get audio guide!
What about security?

Symmetric Crytpography









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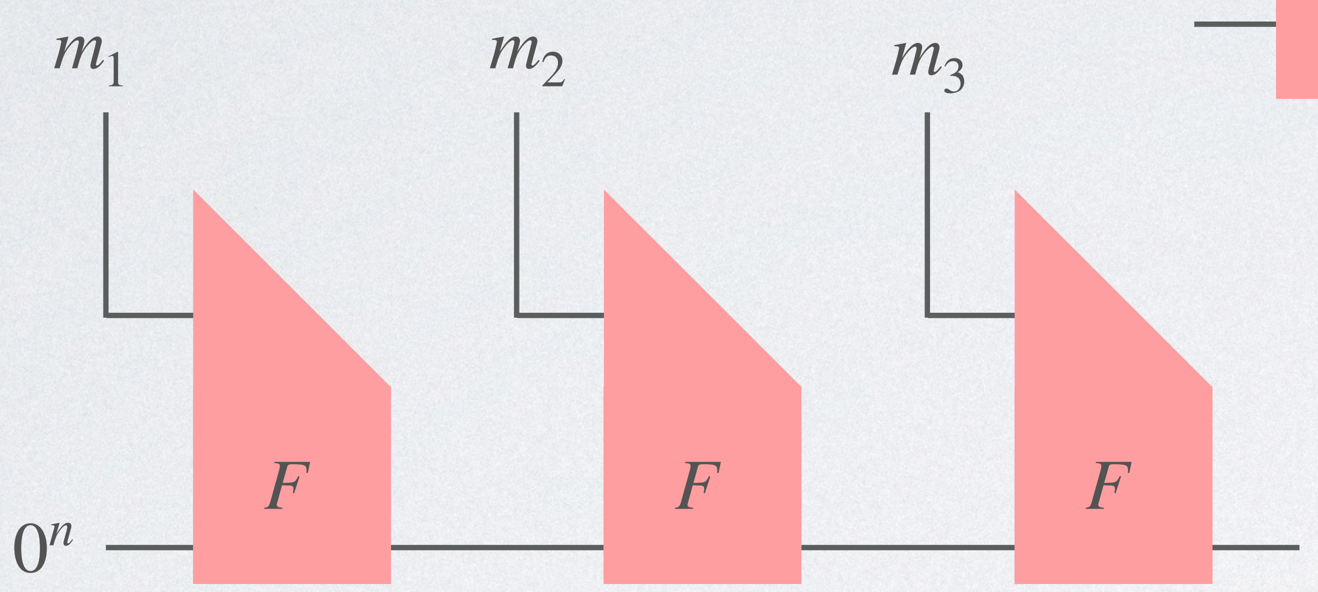
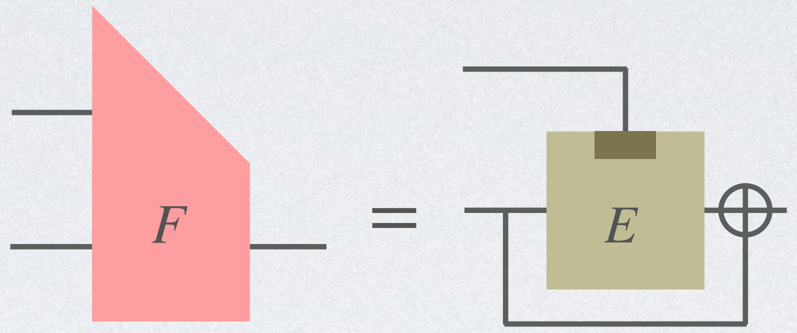
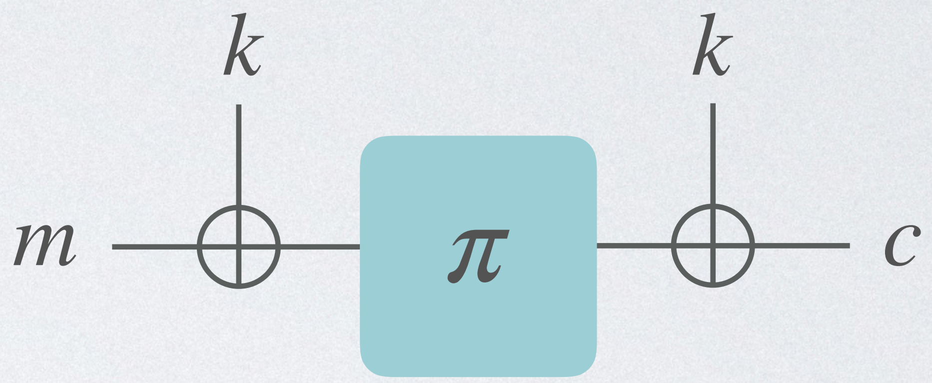
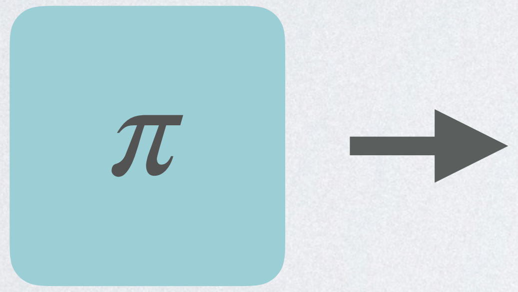
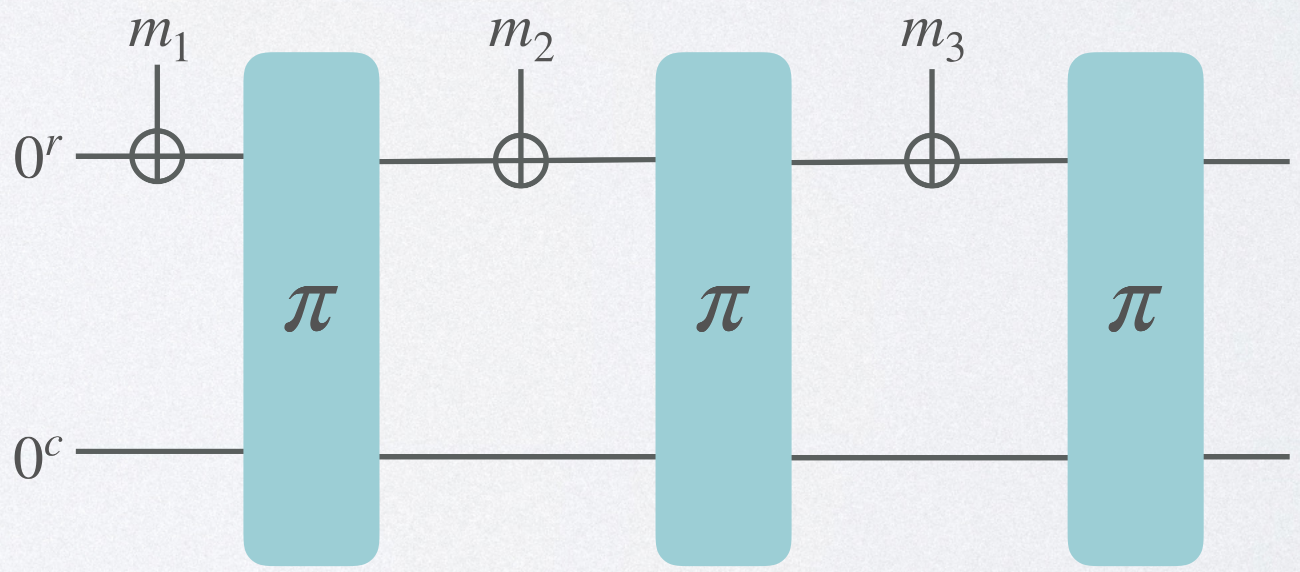








Random Permutation



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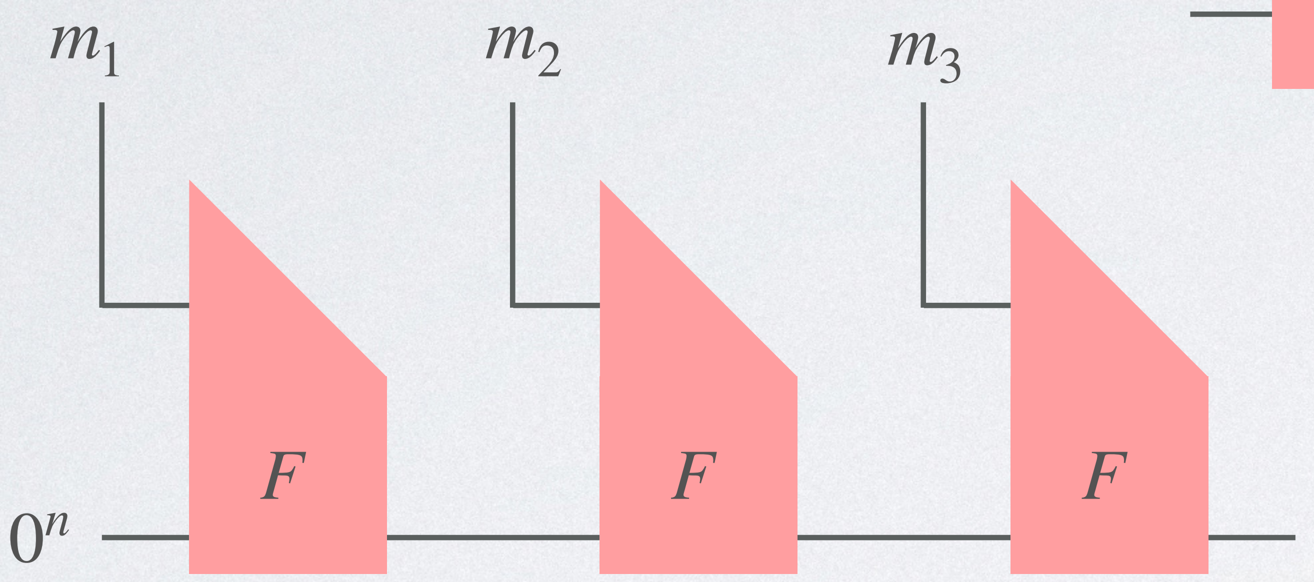
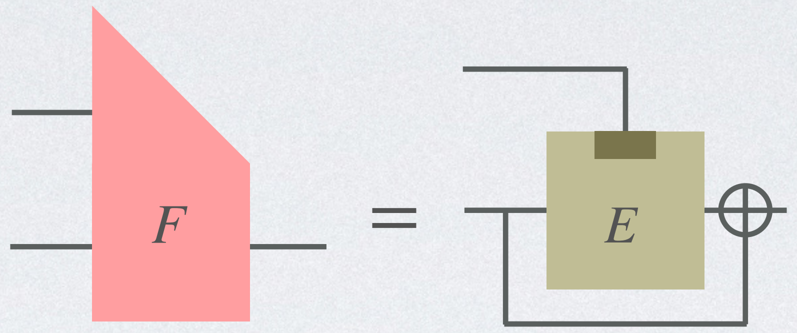
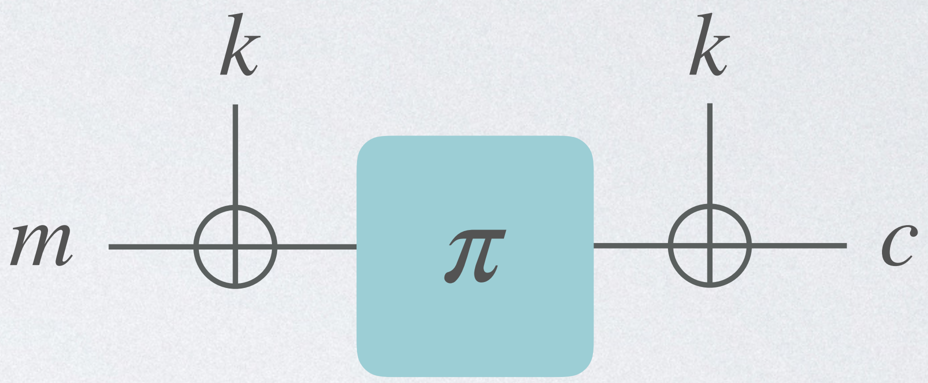
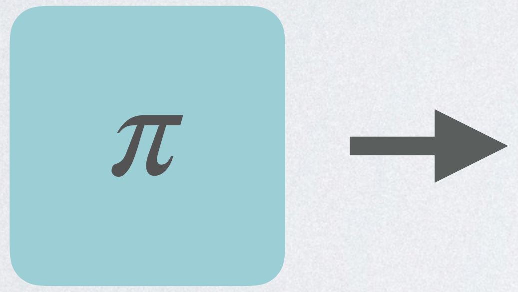


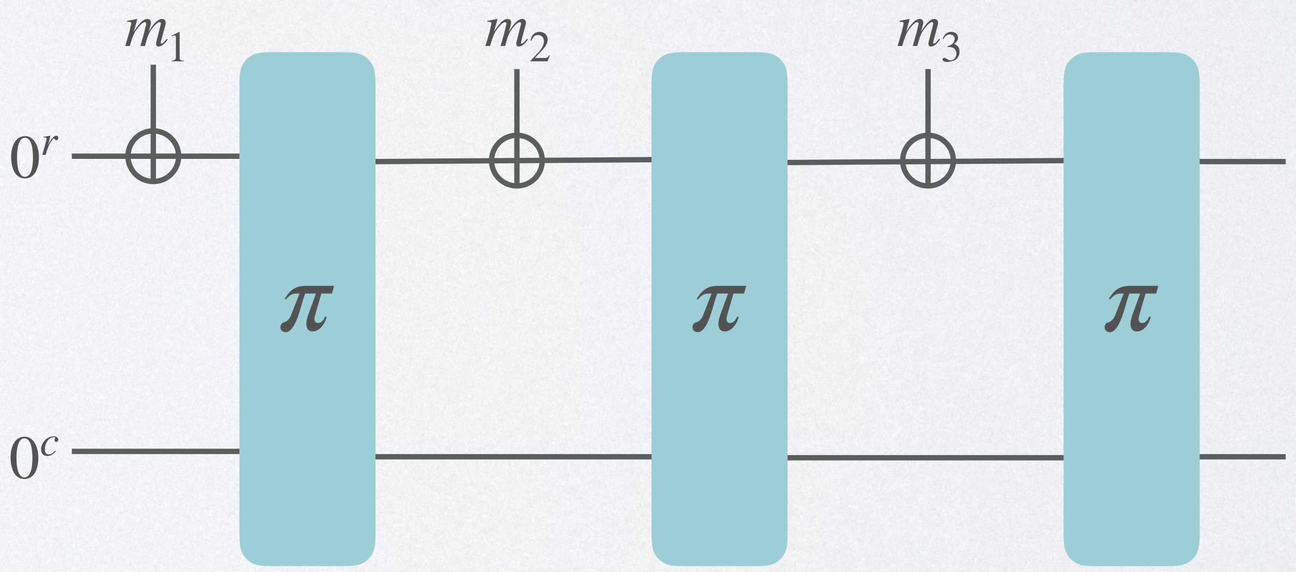






Random Permutation

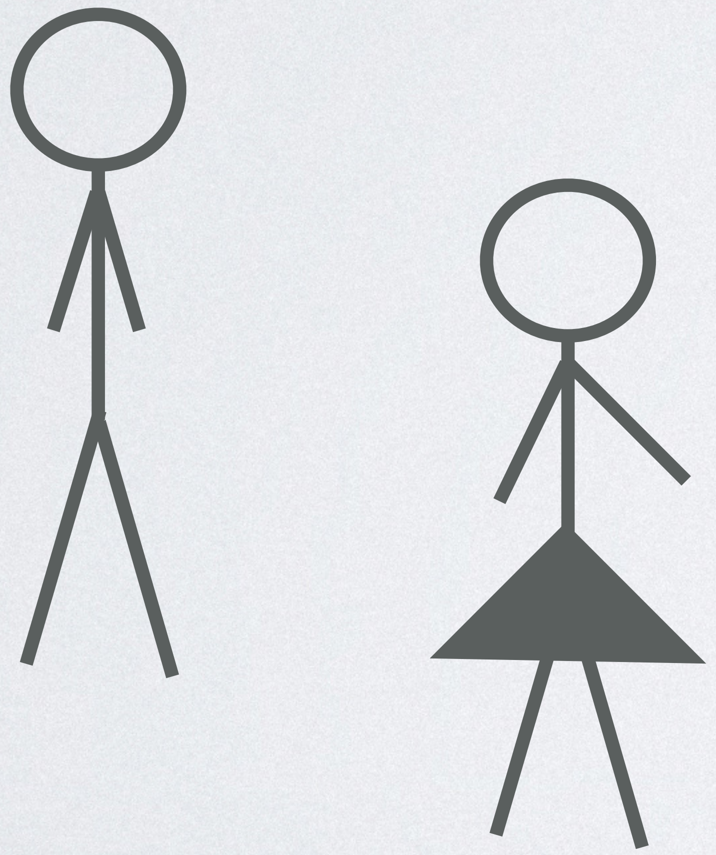
Ideal Cipher



Idealized-Model Methodology

For "natural" applications:

Security in idealized model
=
Security in standard model
using best possible instantiation



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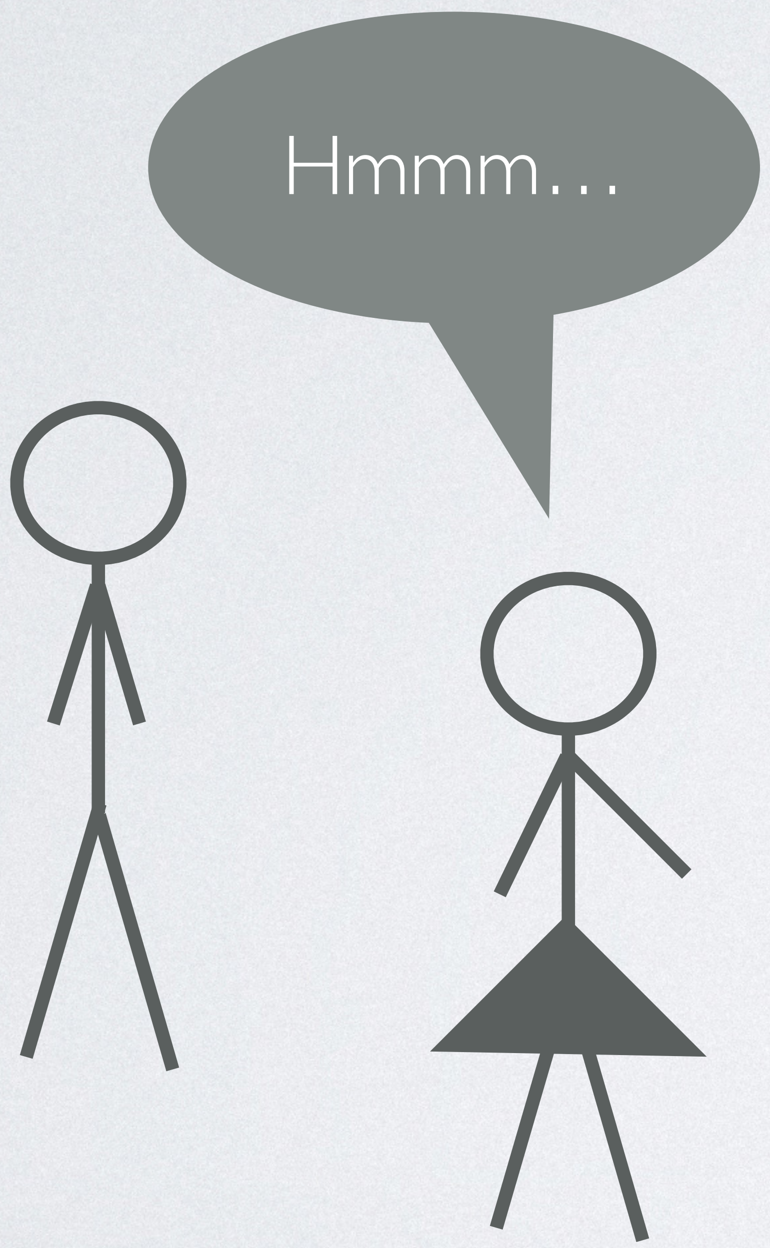

For "natural" applications:

Security in idealized model
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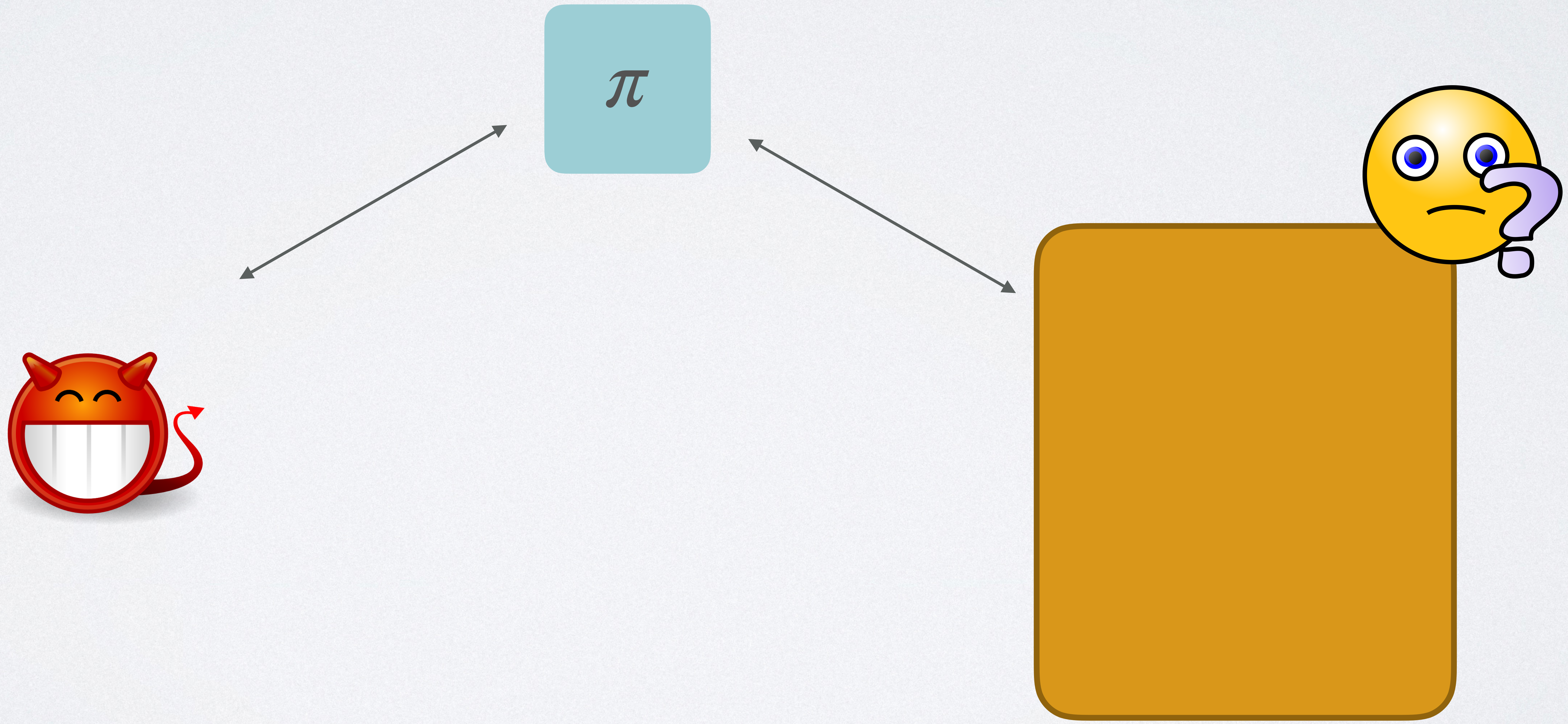
using best possible instantiation

Toy Example: One-Way Permutations

Random permutation $\pi : [N] \to [N]$



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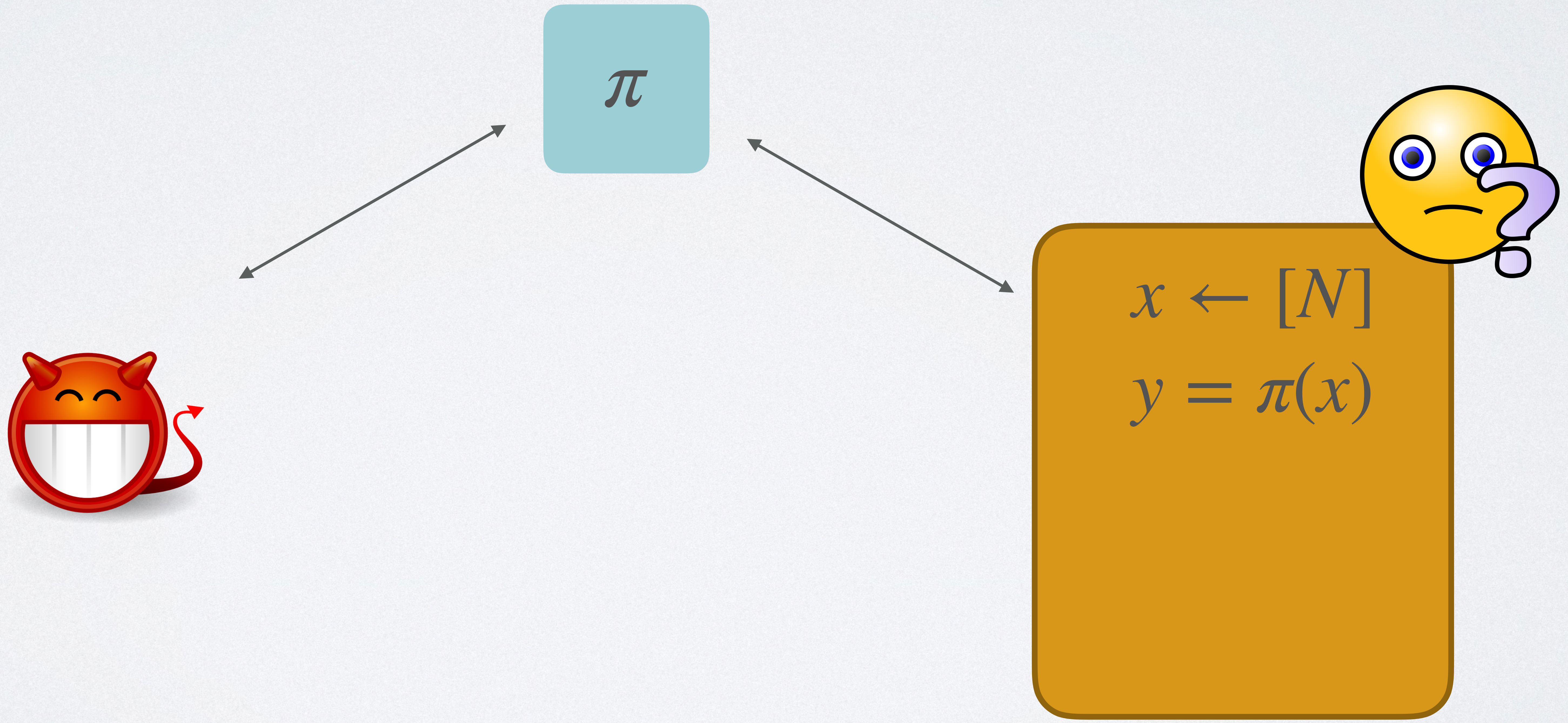

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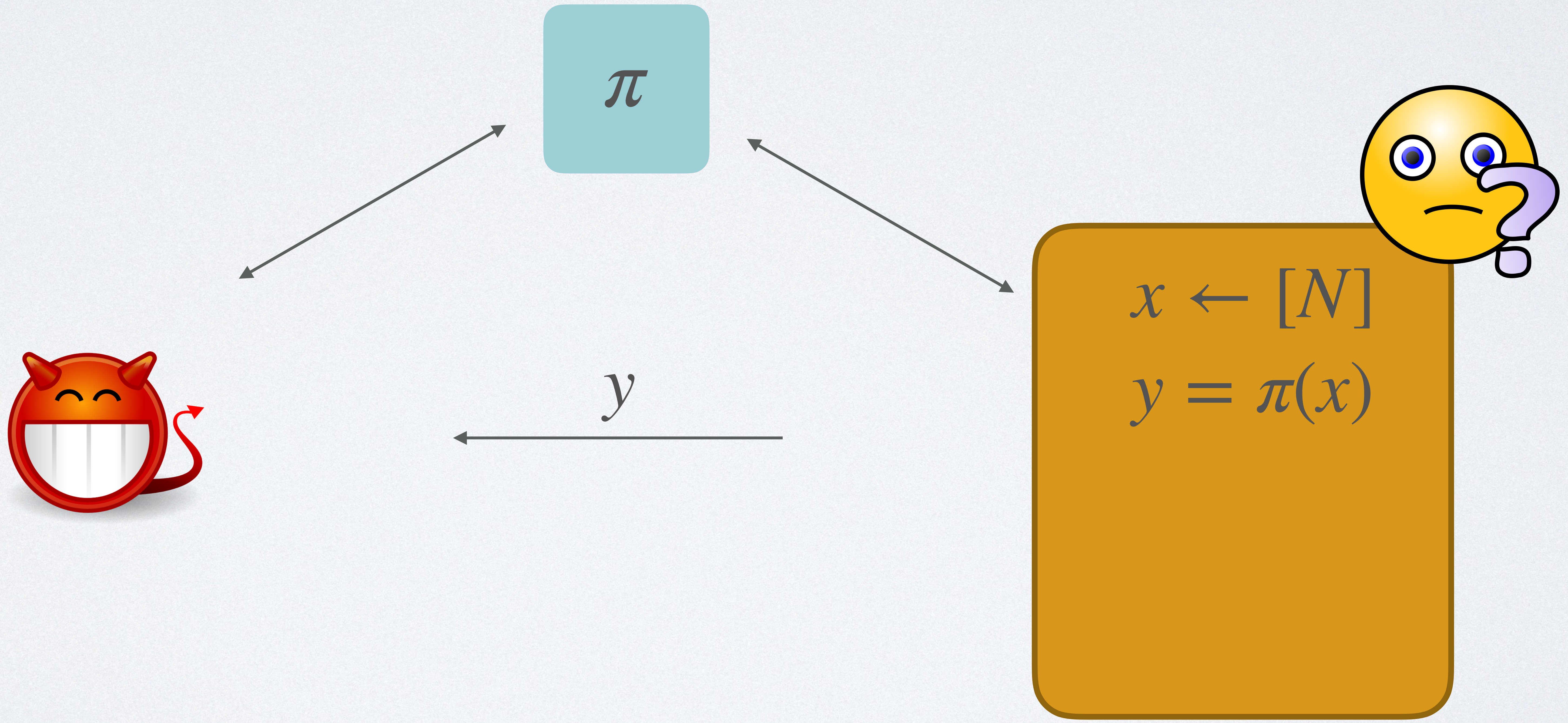

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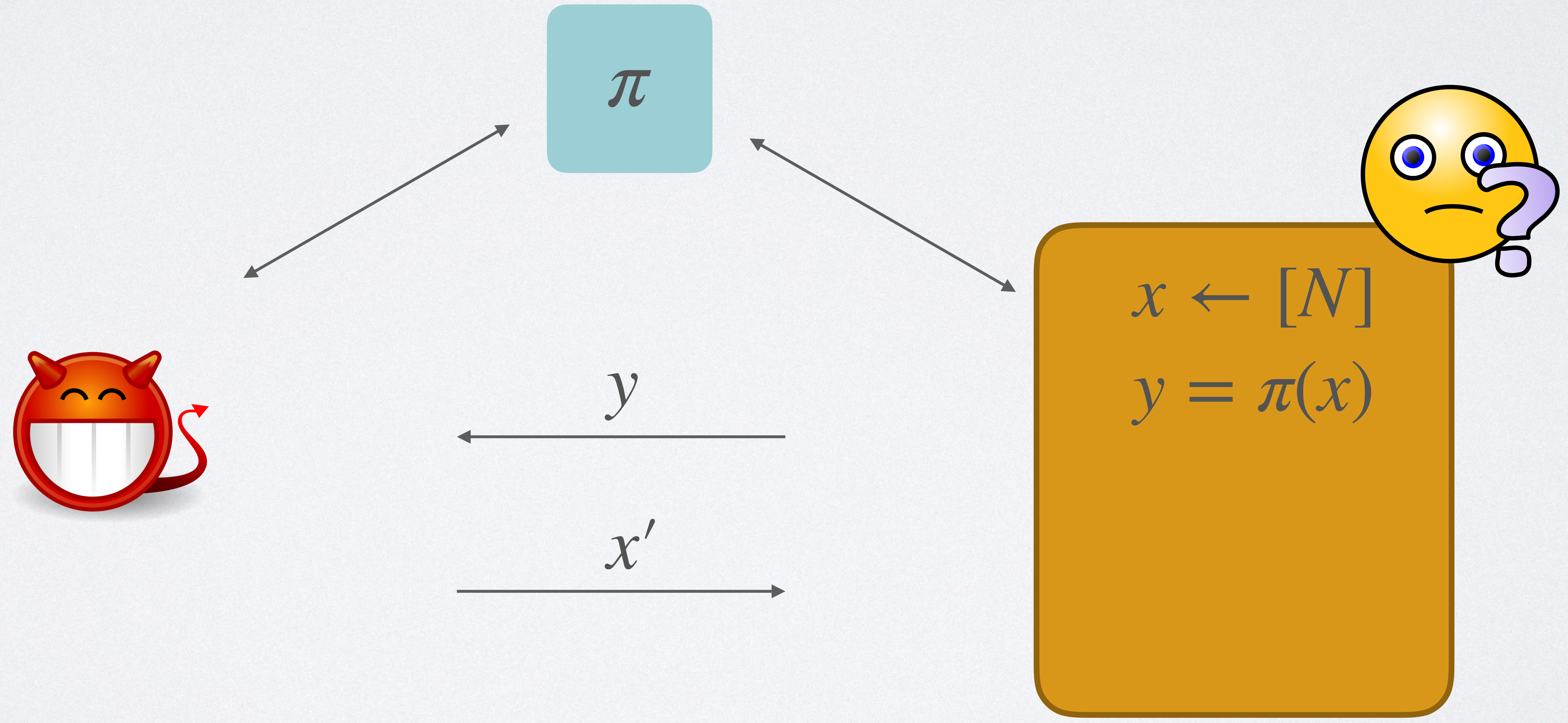

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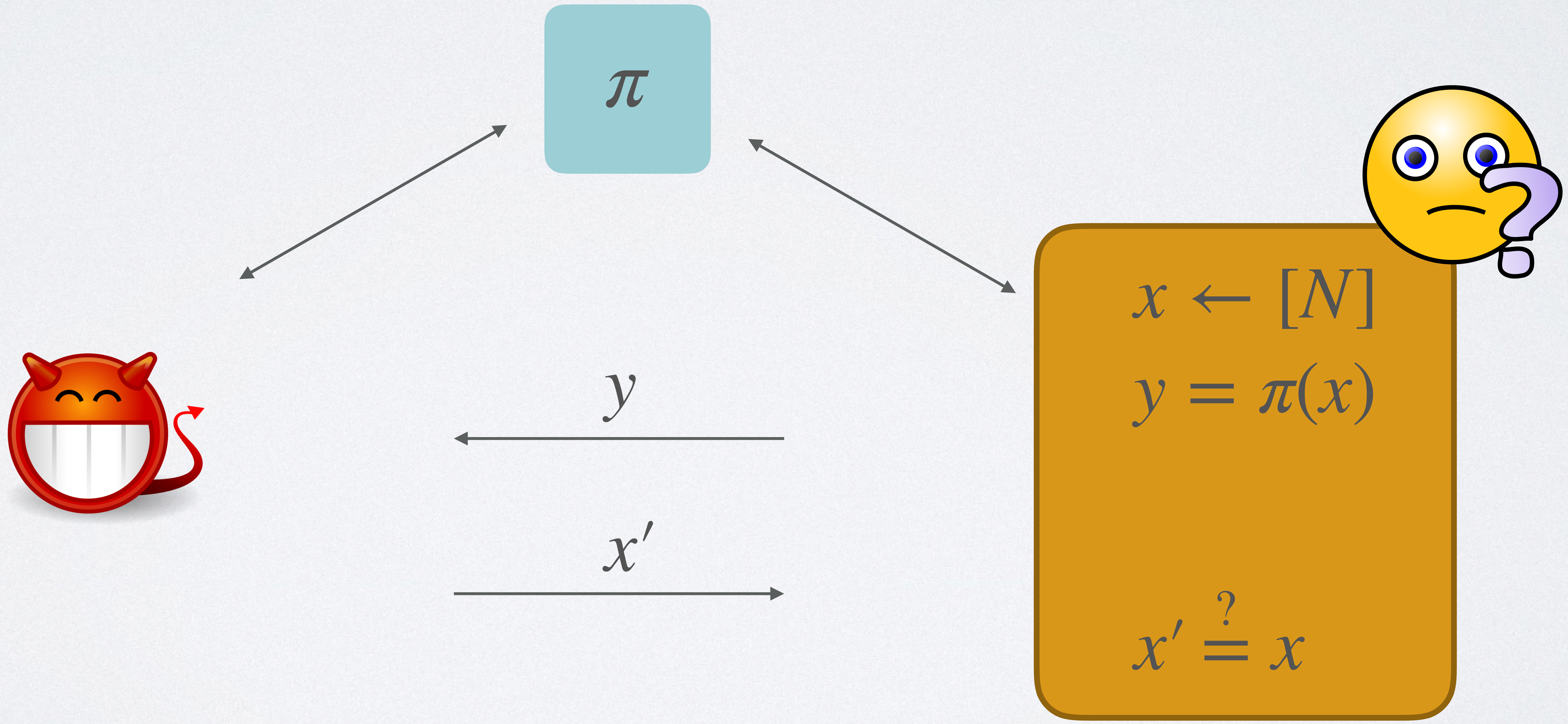

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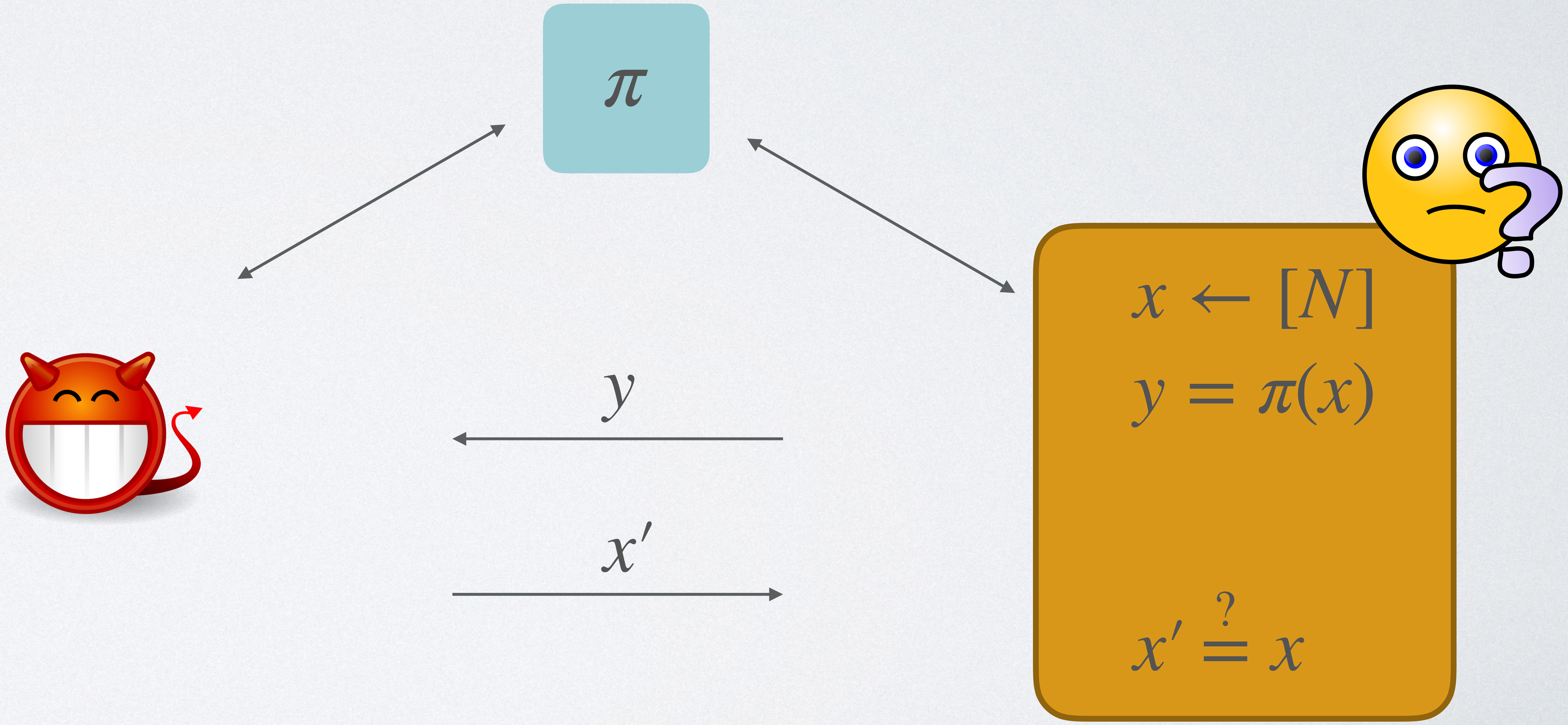

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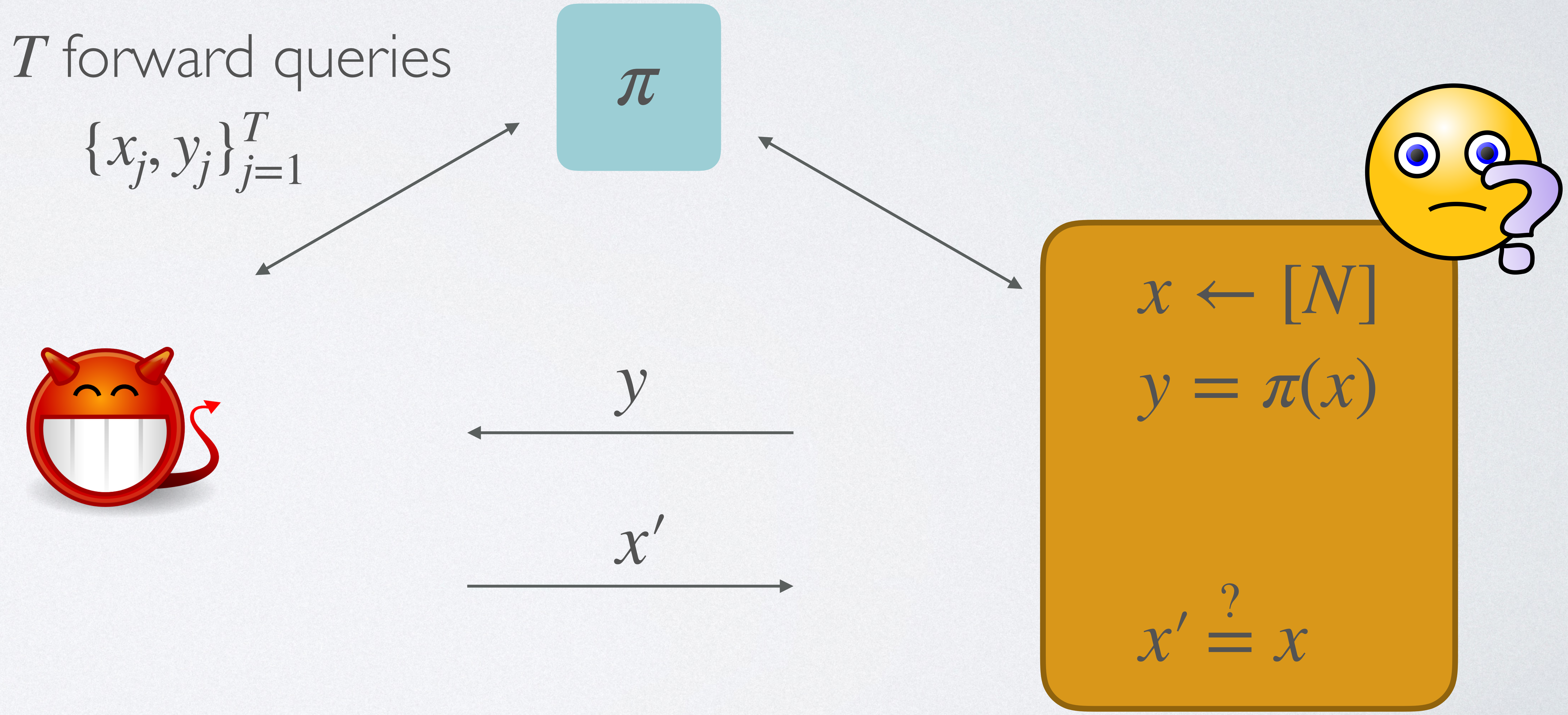

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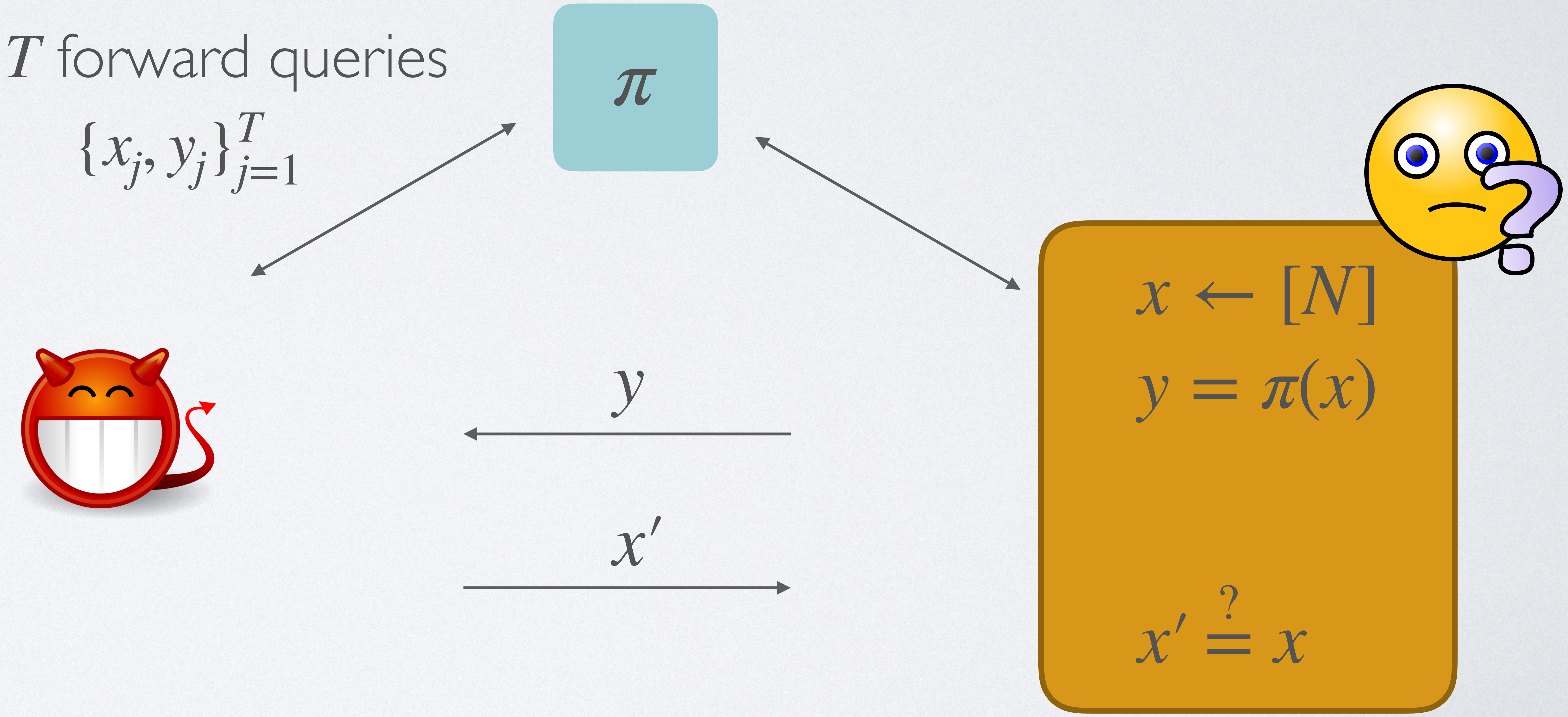
Event **BAD**:

$\exists j : \quad x_j = x$



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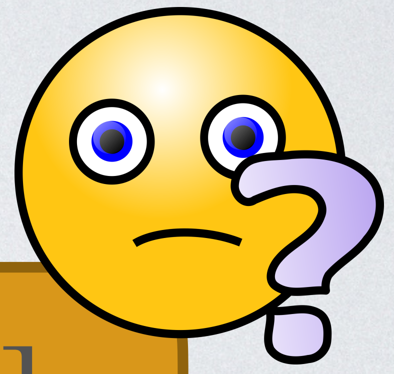
Random permutation $\pi : [N] \to [N]$

T forward queries

$\{x_j, y_j\}_{j=1}^{T}$

π



$x \leftarrow [N]$

$y = \pi(x)$

y

x'

$x' \stackrel{?}{=} x$

Event **BAD**:

$\exists j : \quad x_j = x$

$$\text{P[BAD]} \leq \frac{T}{N}$$

Toy Example: One-Way Permutations

Random permutation $\pi : [N] \to [N]$

Conclusion:

One-Way Permutations secure up to ***N* queries**

Event BAD:

$\exists j : \quad x_j = x$

$\text{P[BAD]} \leq \frac{T}{N}$

x'

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EM Cipher with Random Permutation

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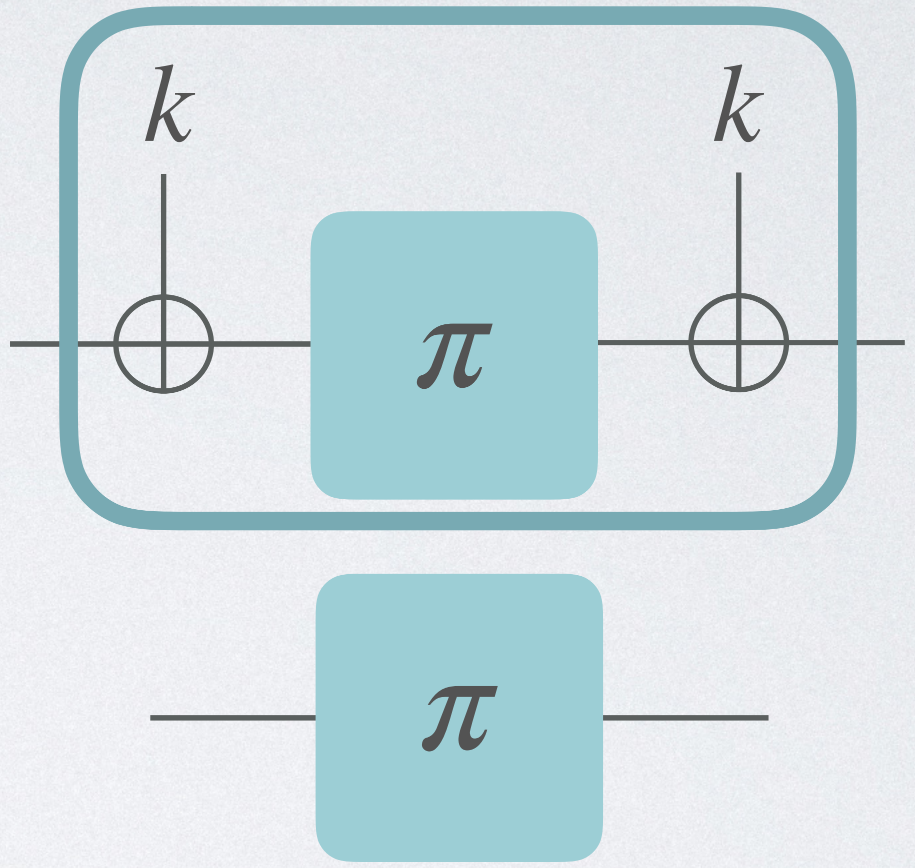
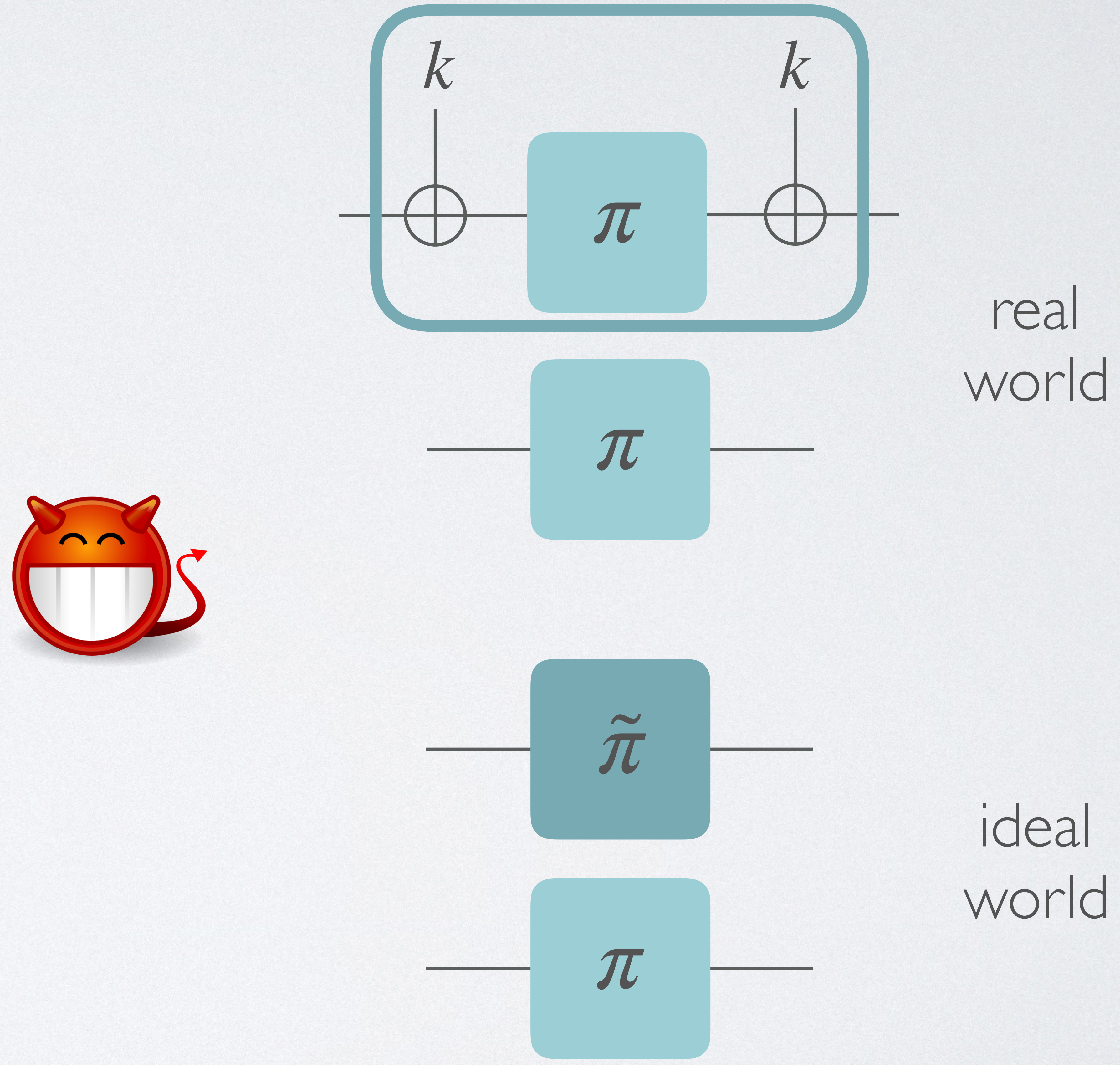


real world

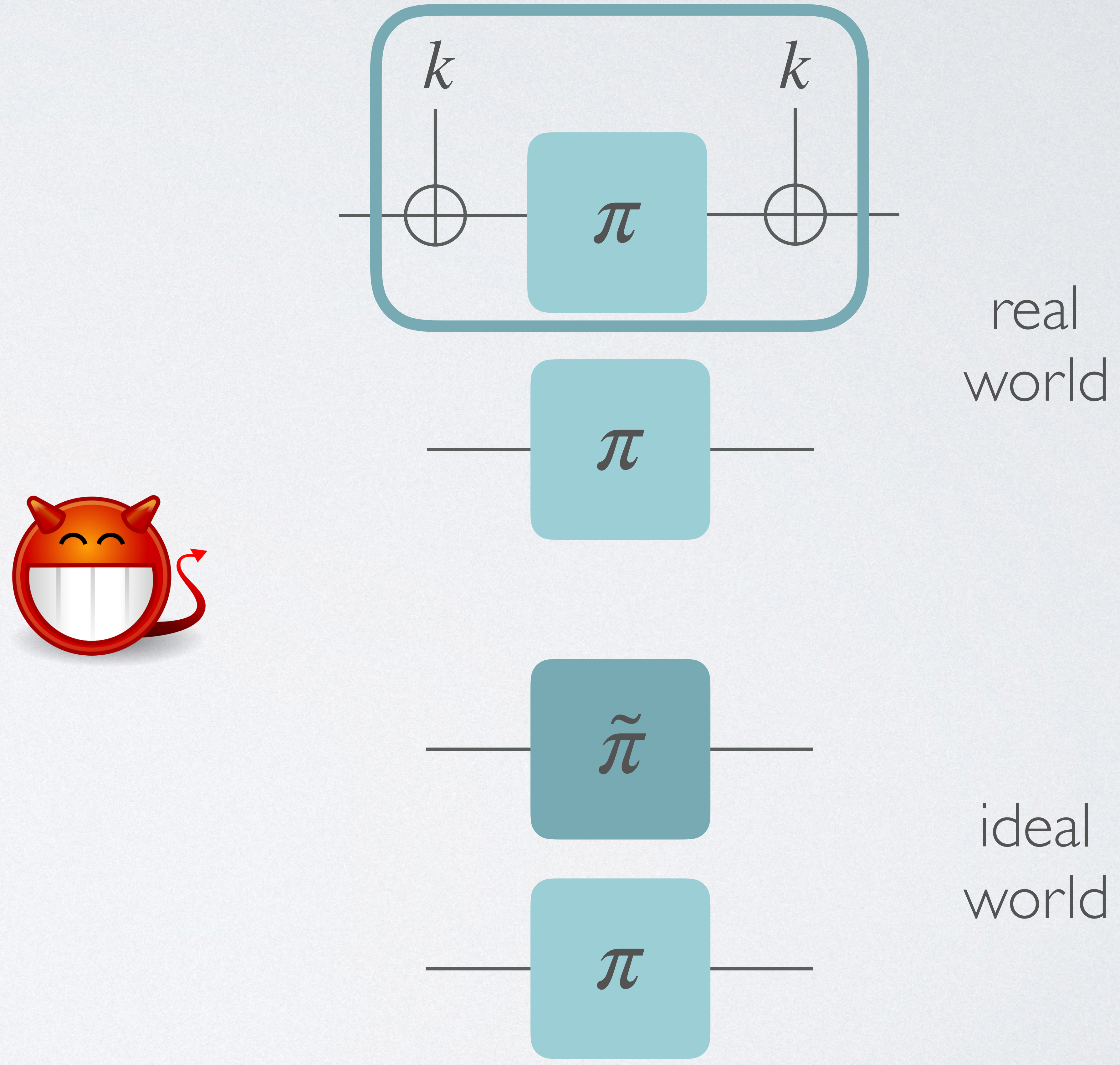
EM Cipher with Random Permutation



EM Cipher with Random Permutation

q construction queries

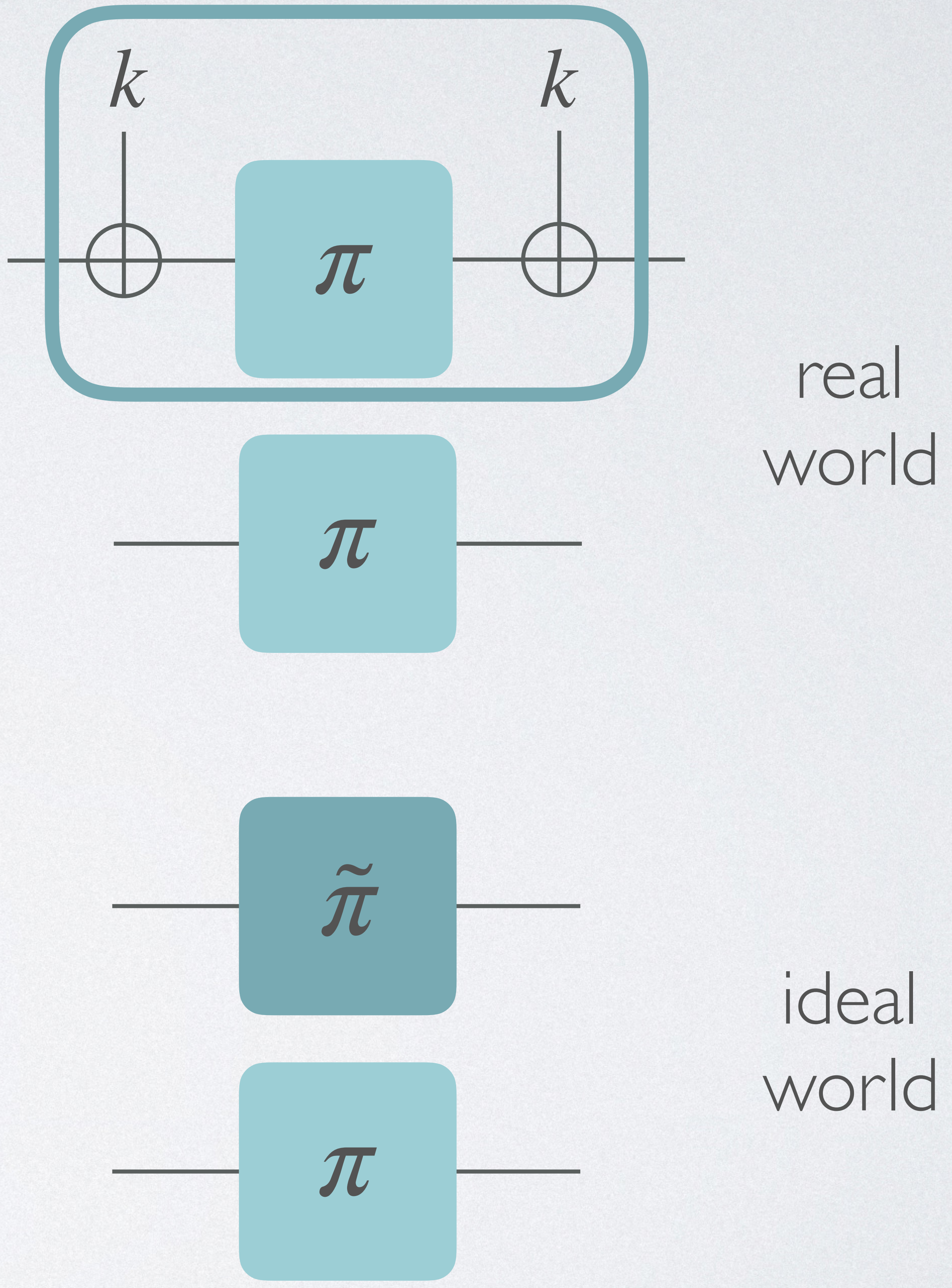
$\{u_i, v_i\}_{i=1}^{q}$



EM Cipher with Random Permutation

q construction queries
$\{u_i, v_i\}_{i=1}^{q}$

T primitive queries
$\{x_j, y_j\}_{j=1}^{T}$



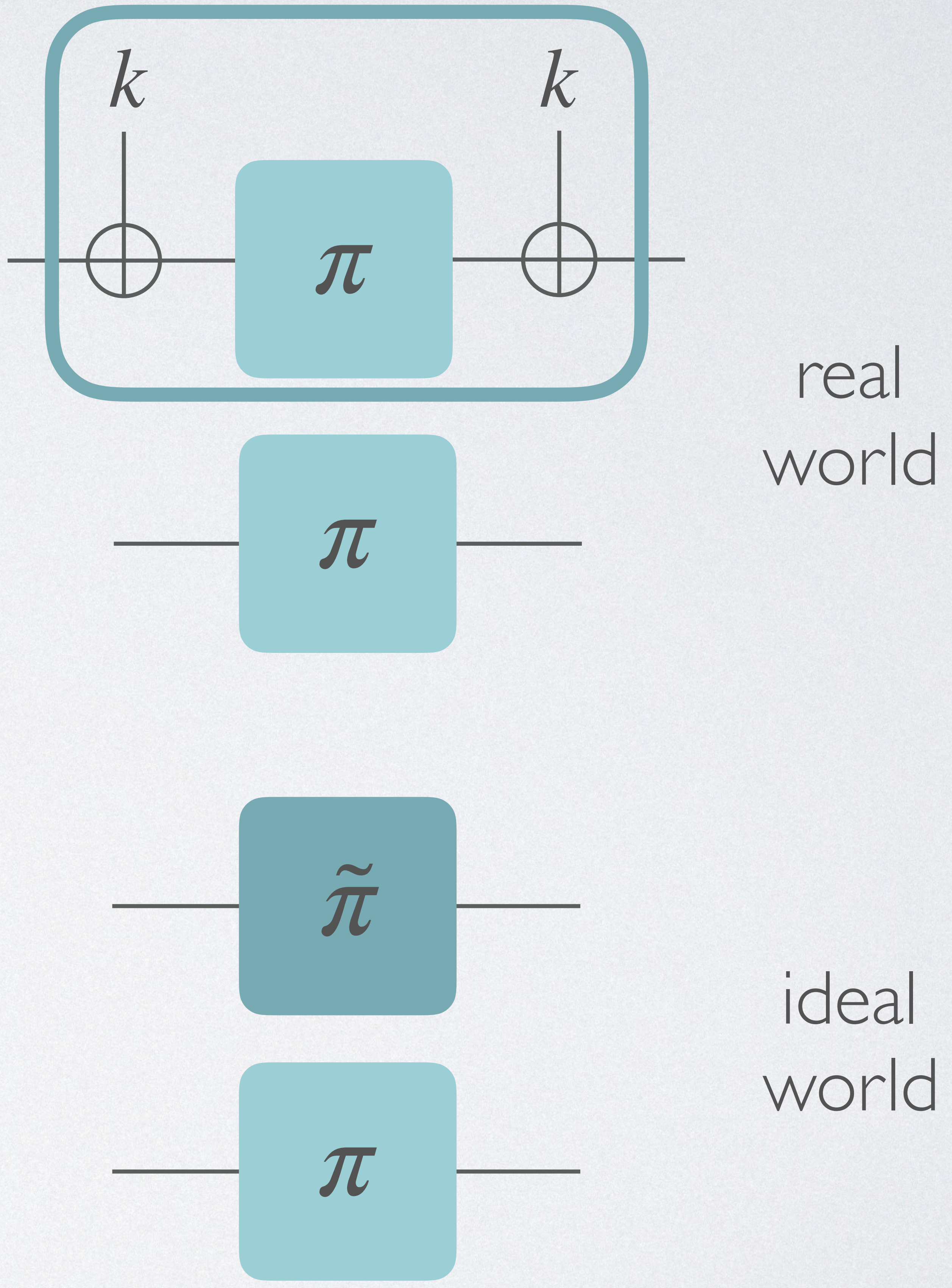
EM Cipher with Random Permutation

q construction queries
$\{u_i, v_i\}_{i=1}^{q}$

T primitive queries
$\{x_j, y_j\}_{j=1}^{T}$

Event BAD:

$\exists i, j : \; u_i \oplus k = x_j \; \vee \; v_i \oplus k = y_j$



EM Cipher with Random Permutation

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T primitive queries

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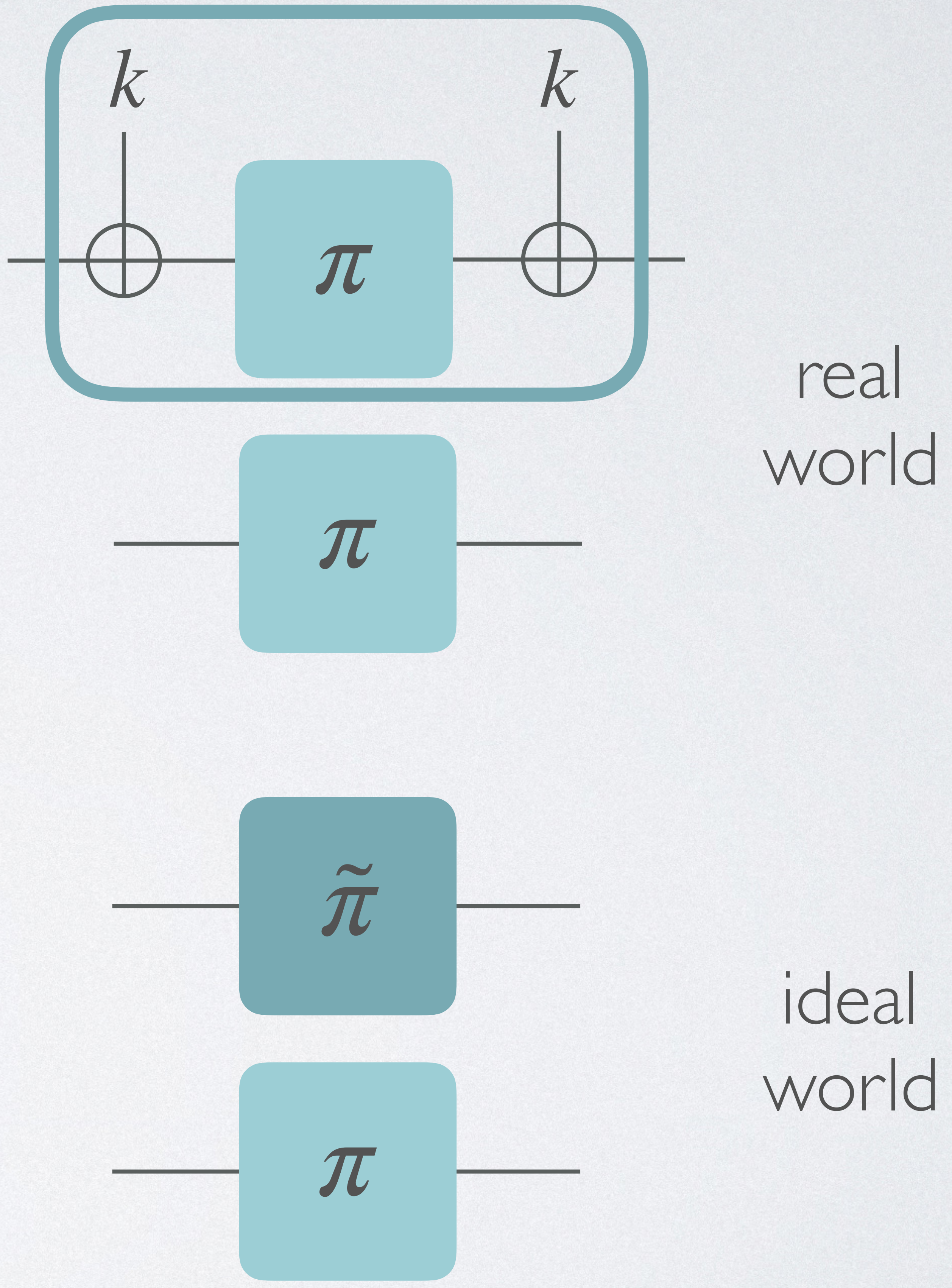


Event BAD:

$$\exists i, j : \; u_i \oplus k = x_j \;\vee\; v_i \oplus k = y_j$$

$$\mathrm{P[BAD]} \leq \frac{qT}{N}$$

EM Cipher with Random Permutation

q construction que

$\{u_i, v_i\}_{i=1}^{q}$

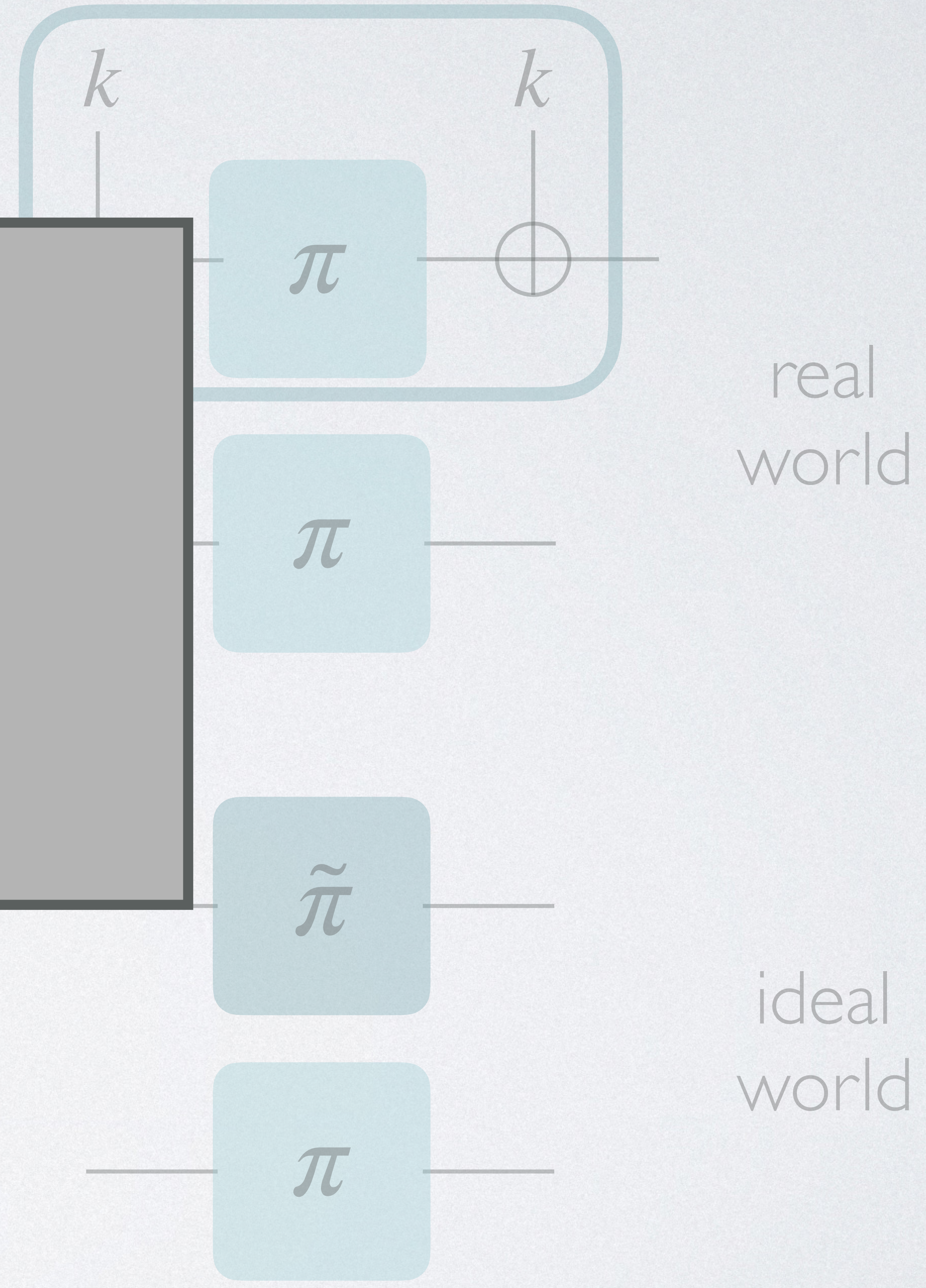
Conclusion:

Even-Mansour secure up to **birthday bound**

Event BAD:

$\exists i, j: \ u_i \oplus k = x_j \vee v_i \oplus k = y_j$

P[BAD] $\leq \frac{qT}{2^n}$



Discrete Logarithms

Rule out generic algorithms via analysis in the
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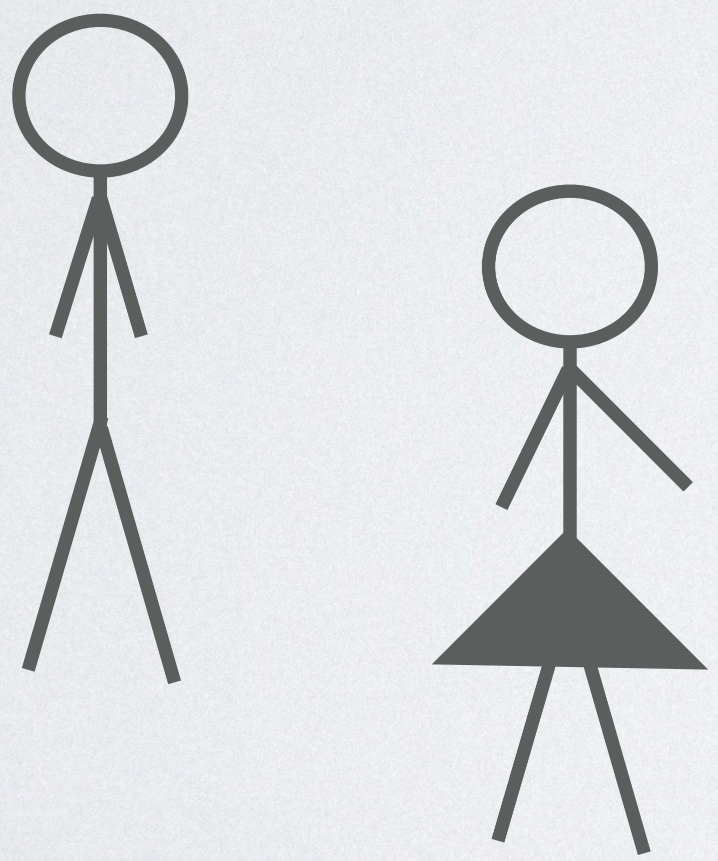

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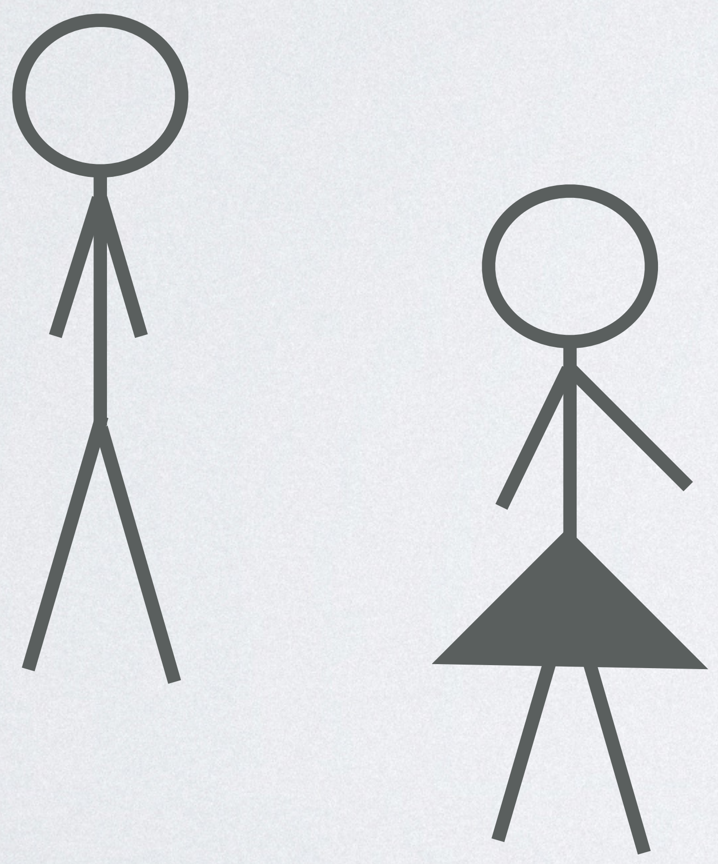

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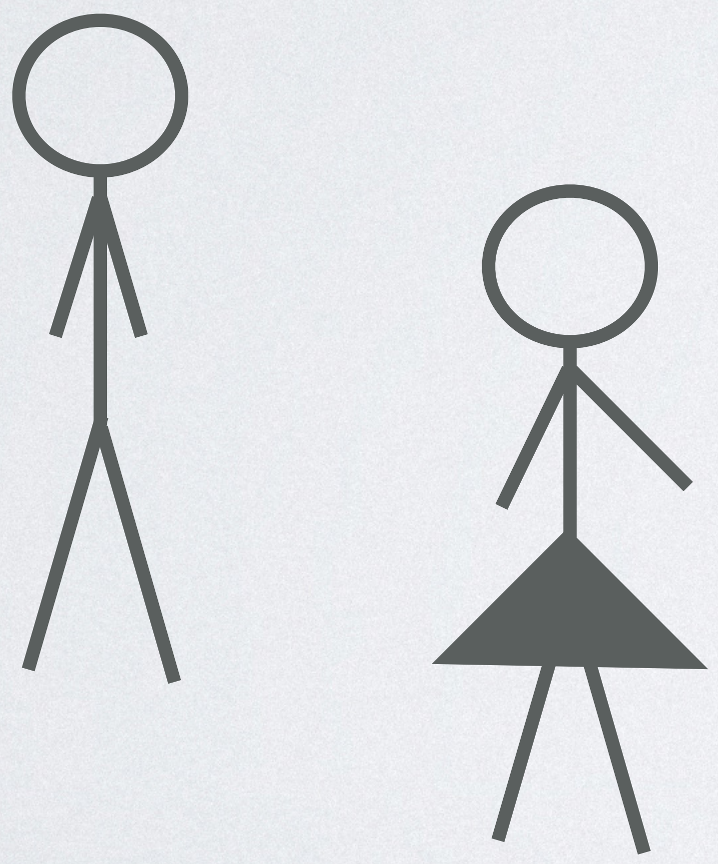

Group operation oracle:

$$\mathcal{O} : \ (\sigma(s), \sigma(s')) \ \mapsto \ \sigma(s + s')$$

Discrete Logarithms in the GGM

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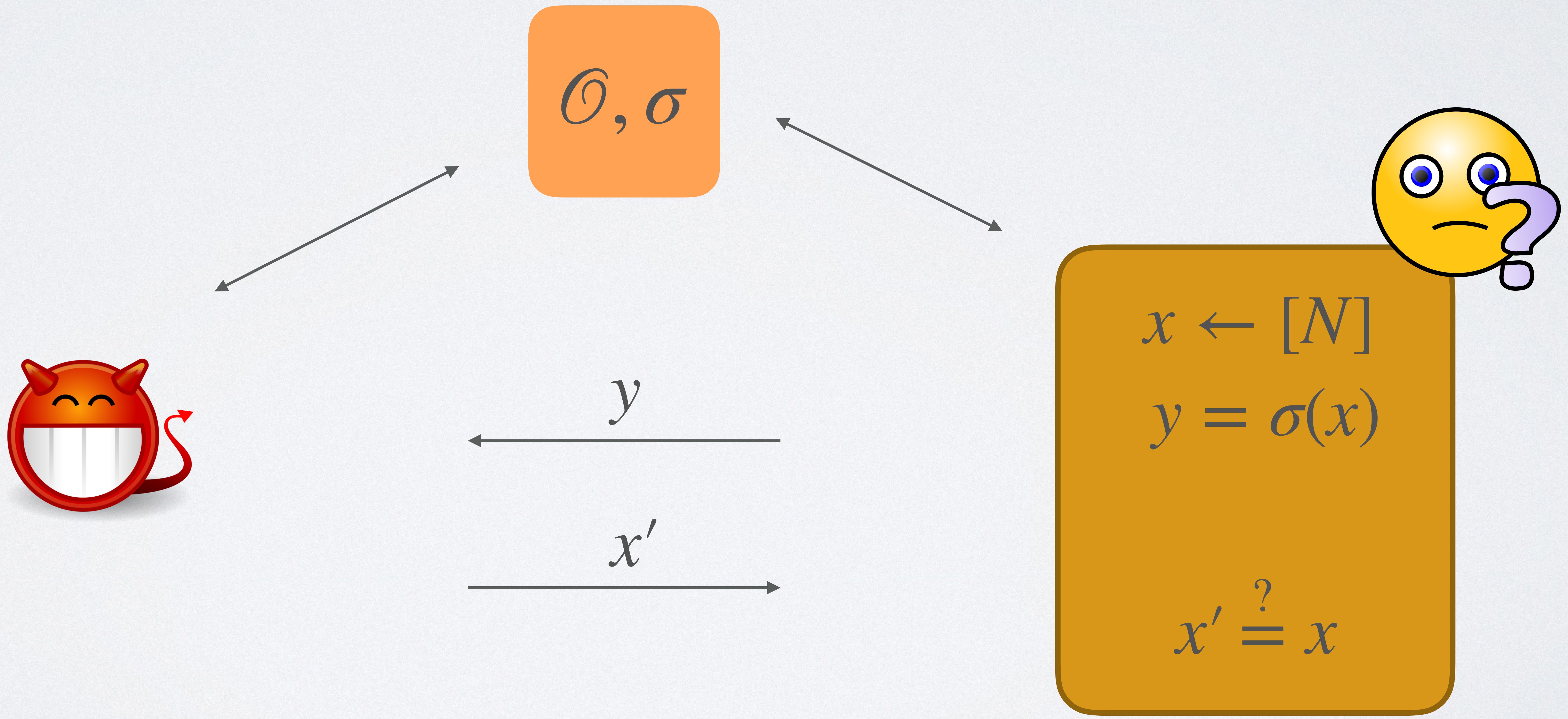

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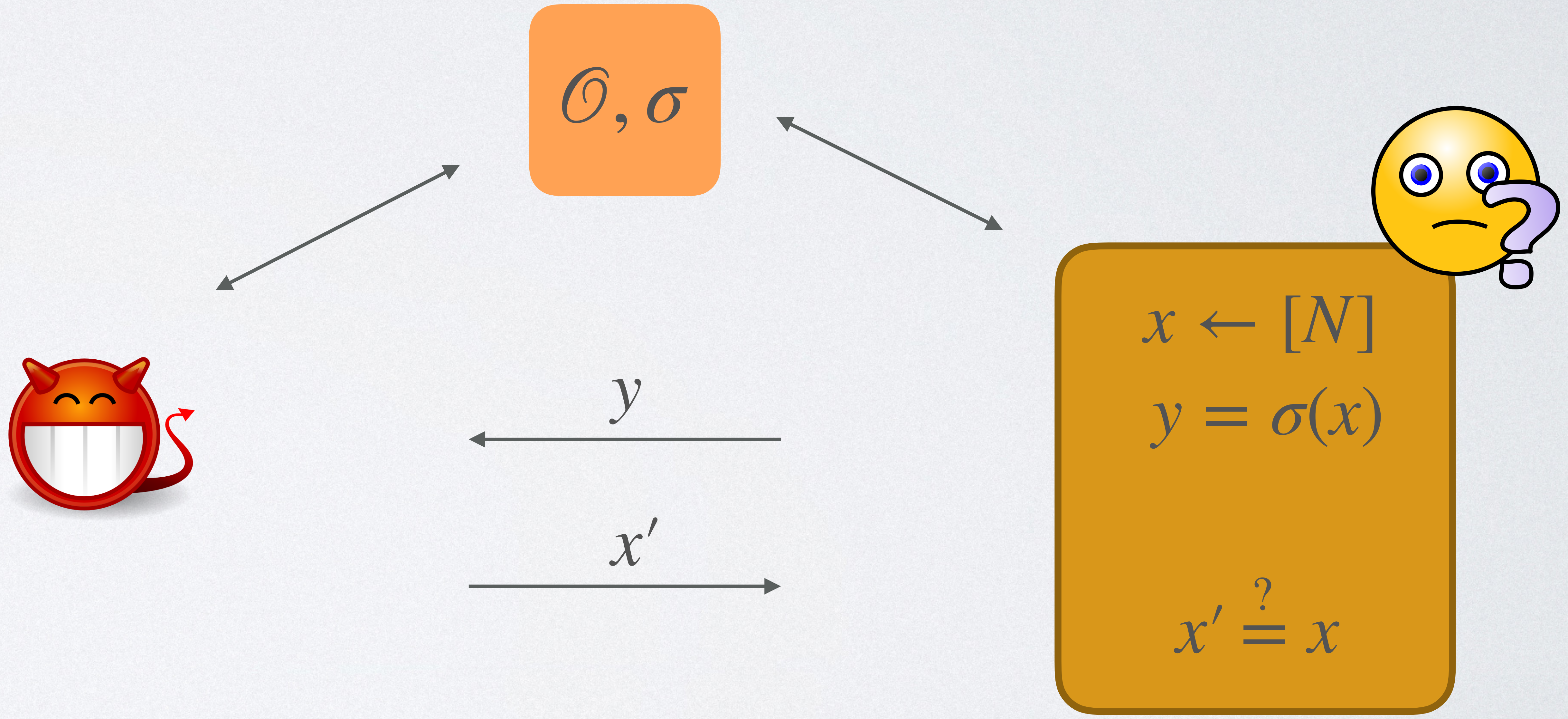

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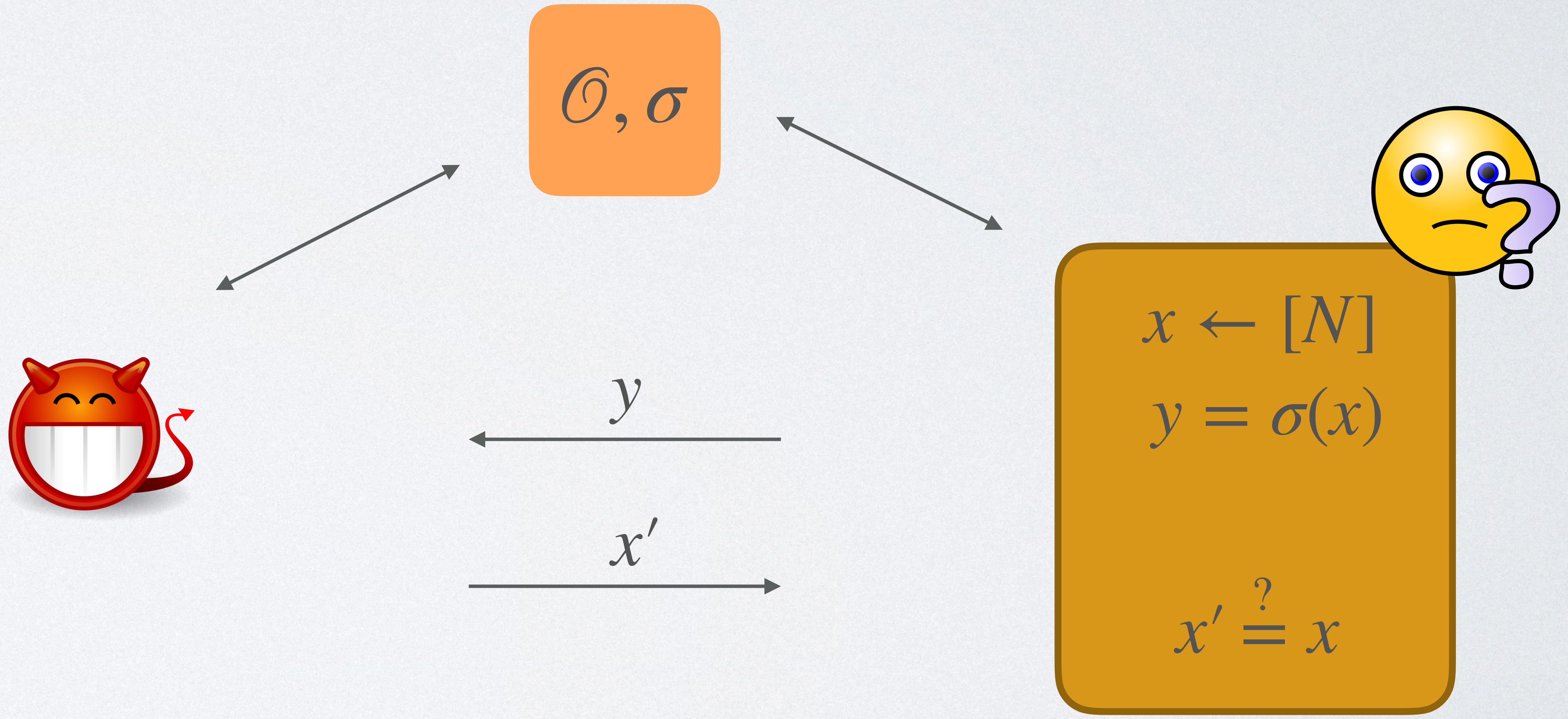

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By making queries to $\mathcal{O}$:

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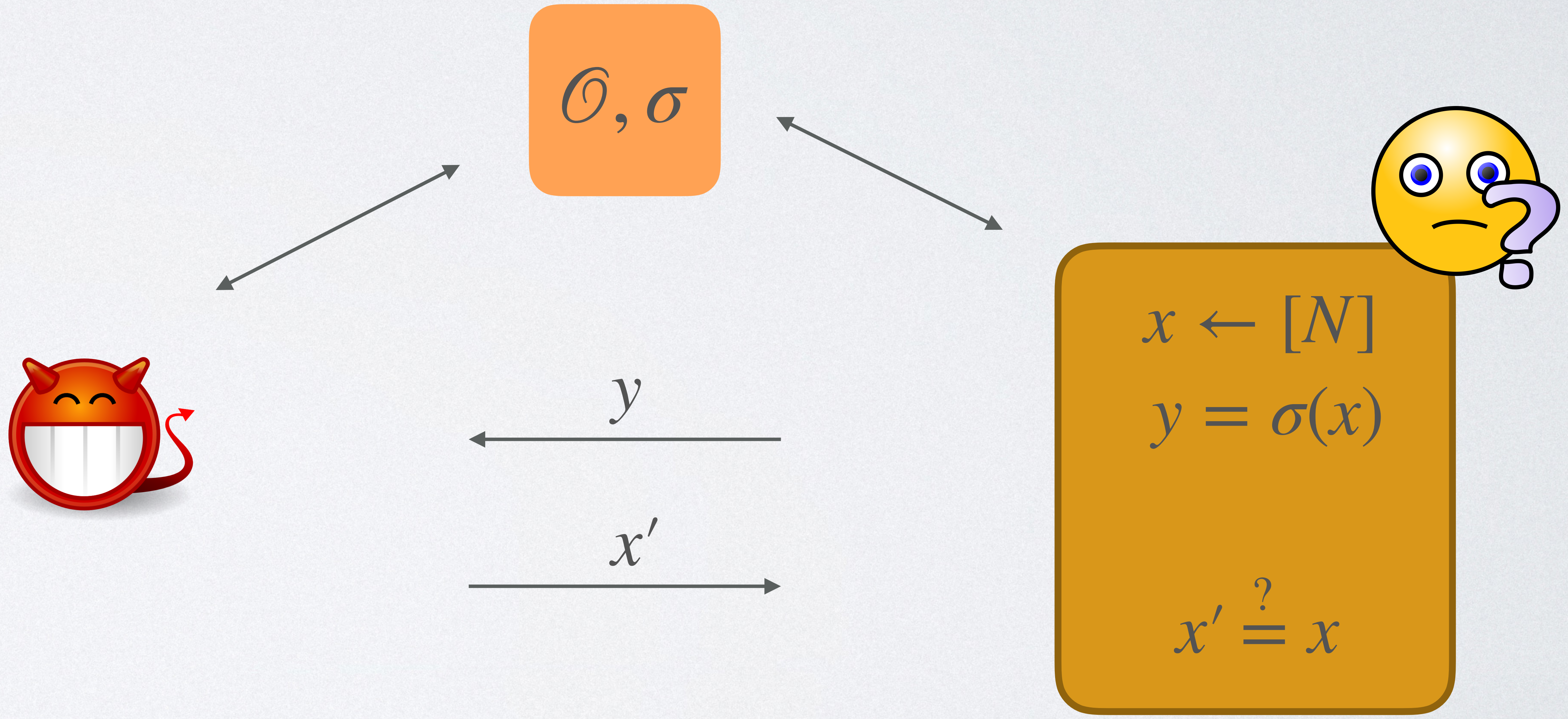
$\mathscr{A}$ "generates" degree-1 polynomials in X



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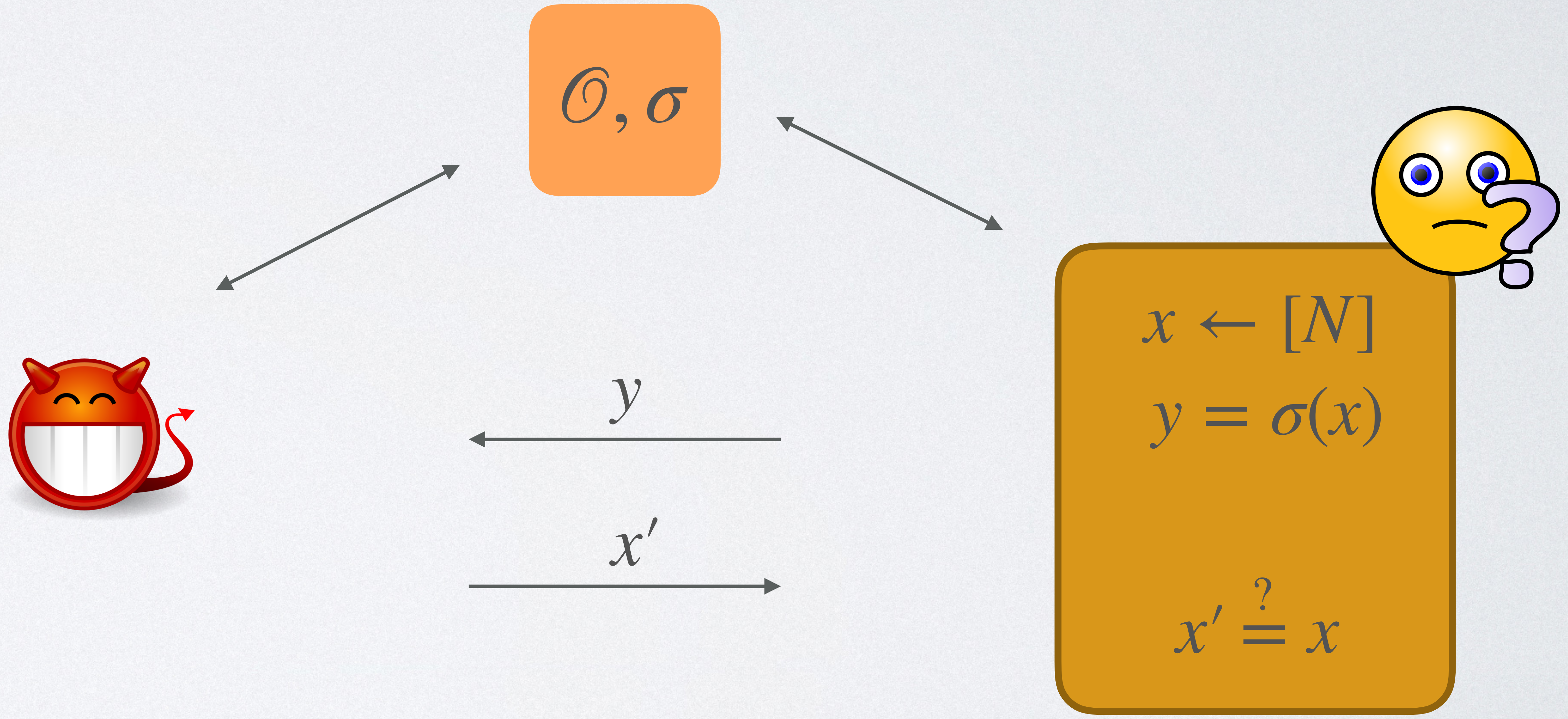

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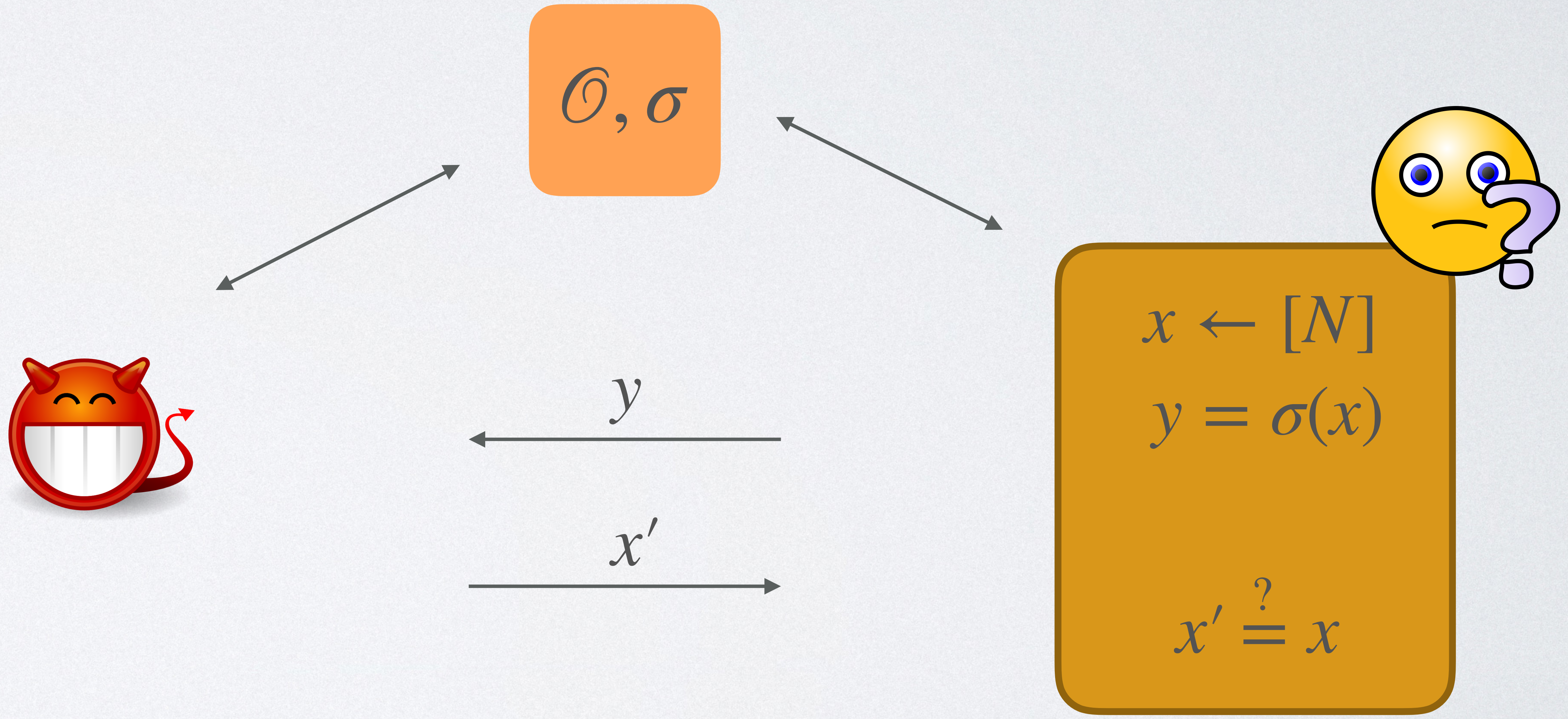
$$\text{P[BAD]} \leq \frac{T^2}{N}$$



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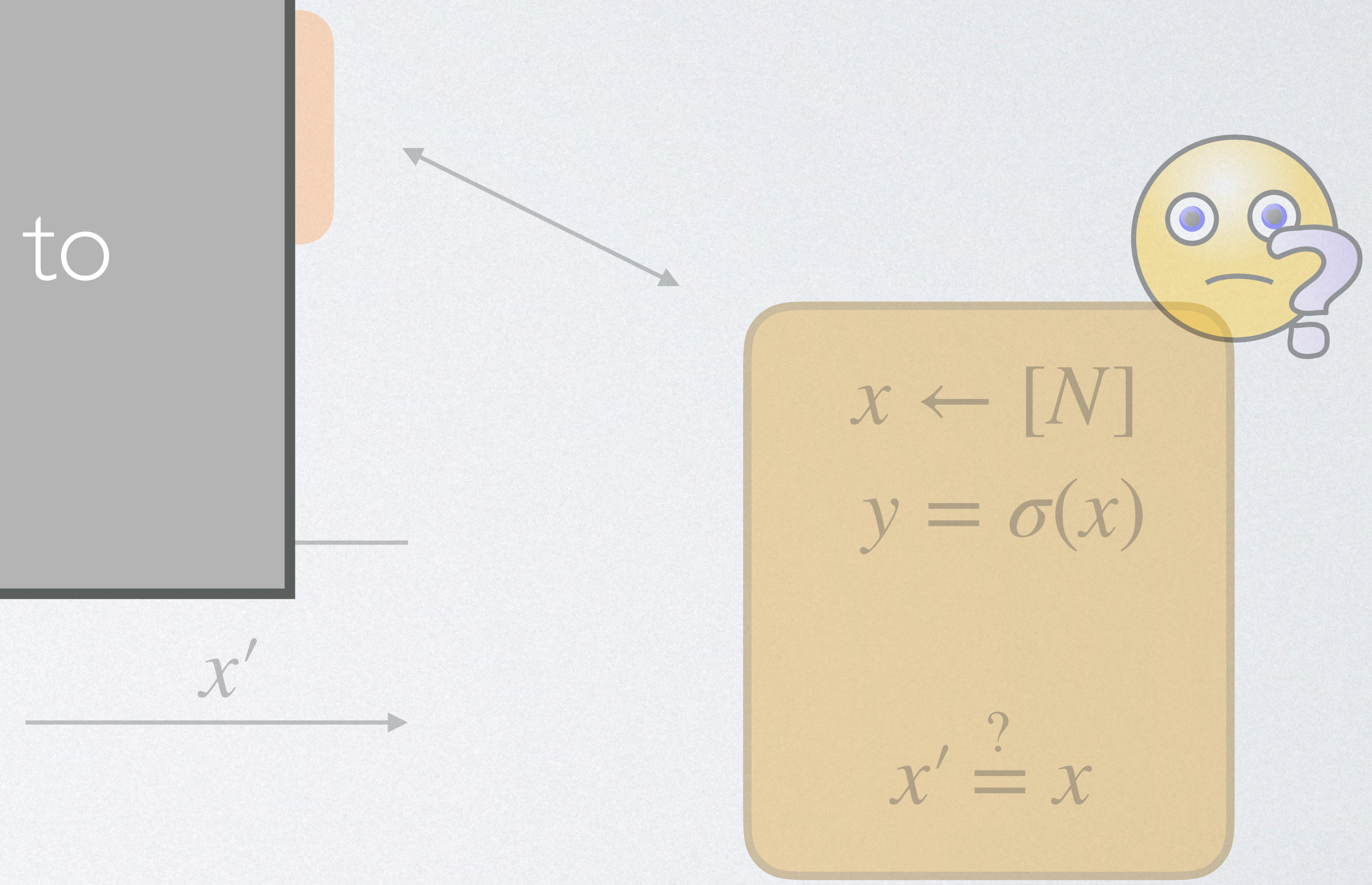
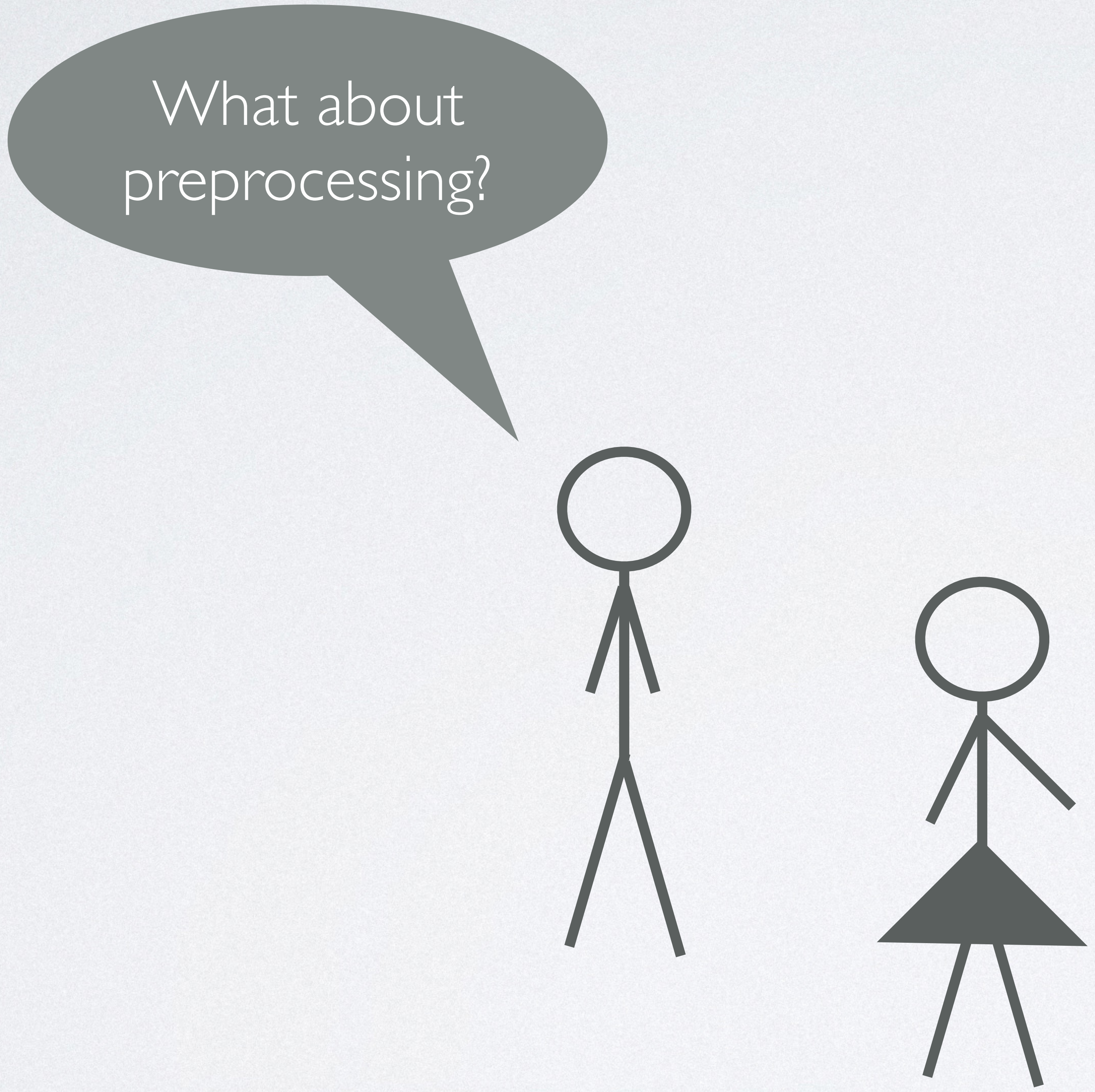


Conclusion:

Discrete logarithm secure up to
birthday bound
in GGM.


What about preprocessing?

Preprocessing Attacks

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In practice:

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In practice:

- security parameter fixed
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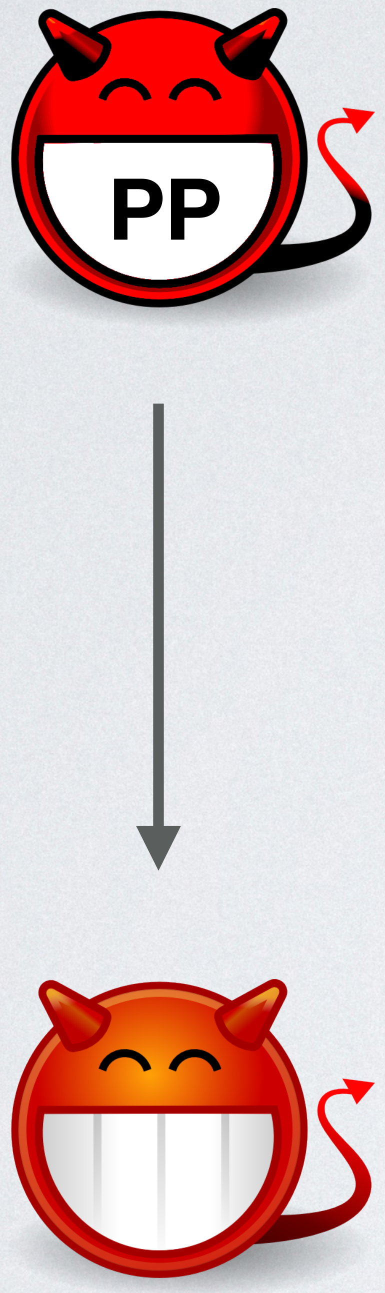
- models non-uniformity

Attacking One-Way Permutations

Hellman '80

Permutation $\pi : [N] \to [N]$



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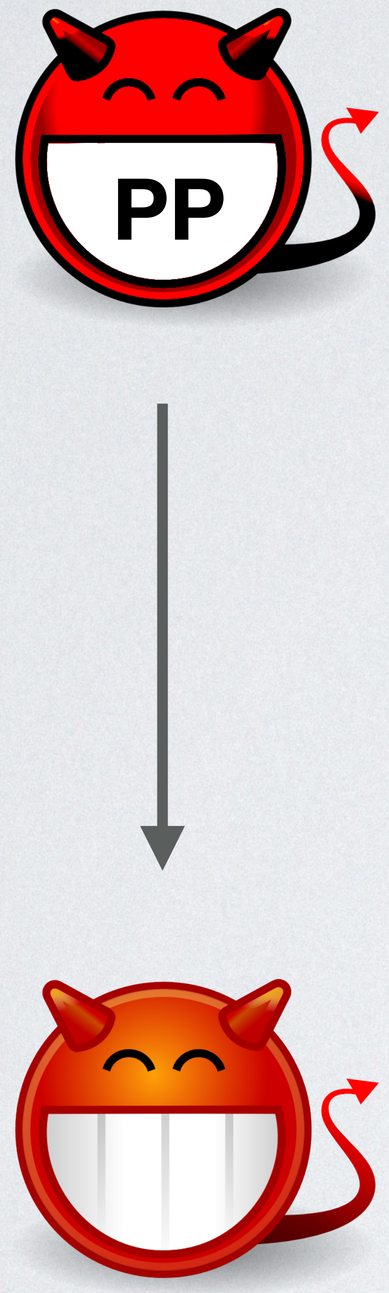
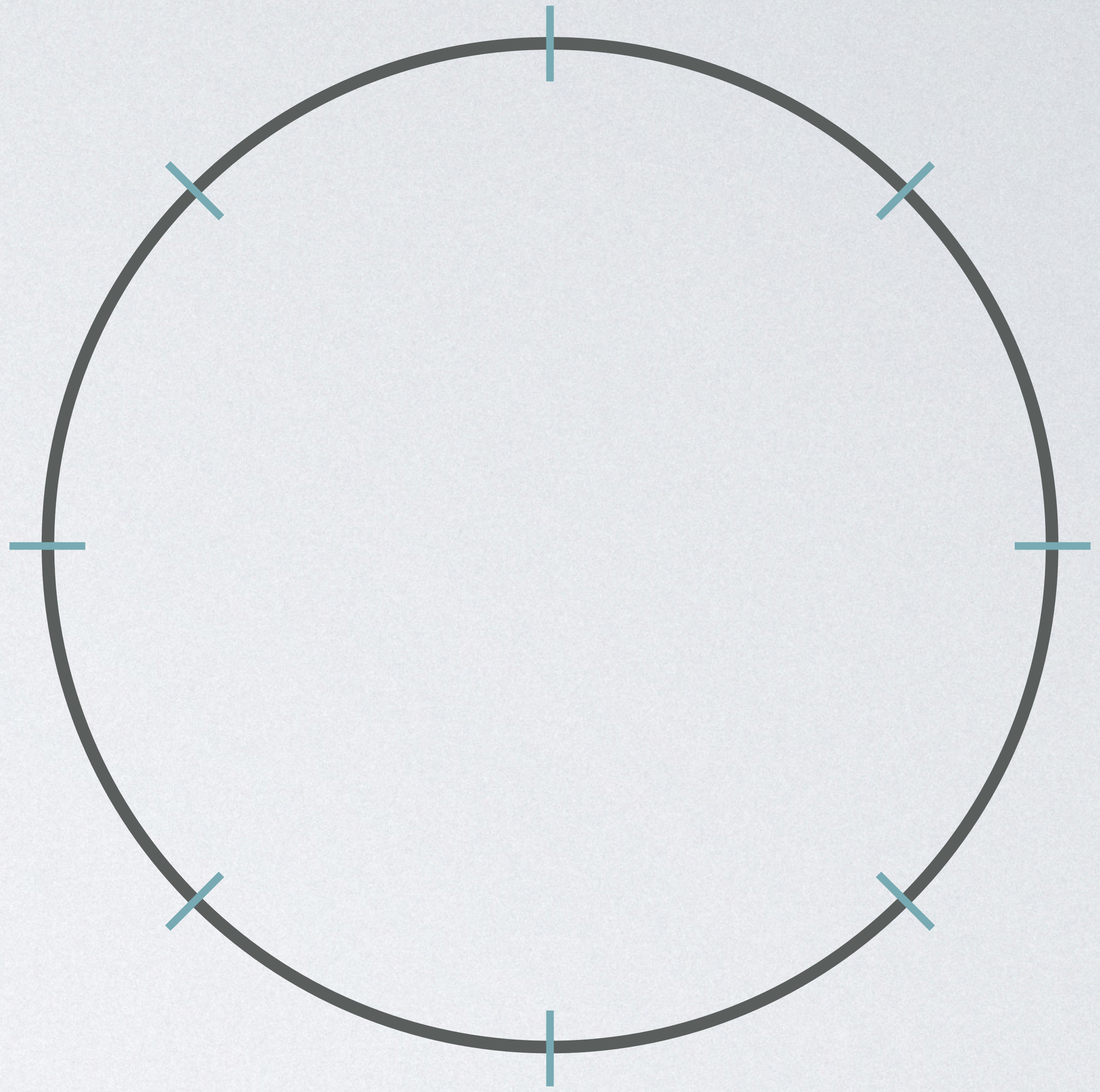


For every cycle of length at least S,
store points x_i at distance N/S



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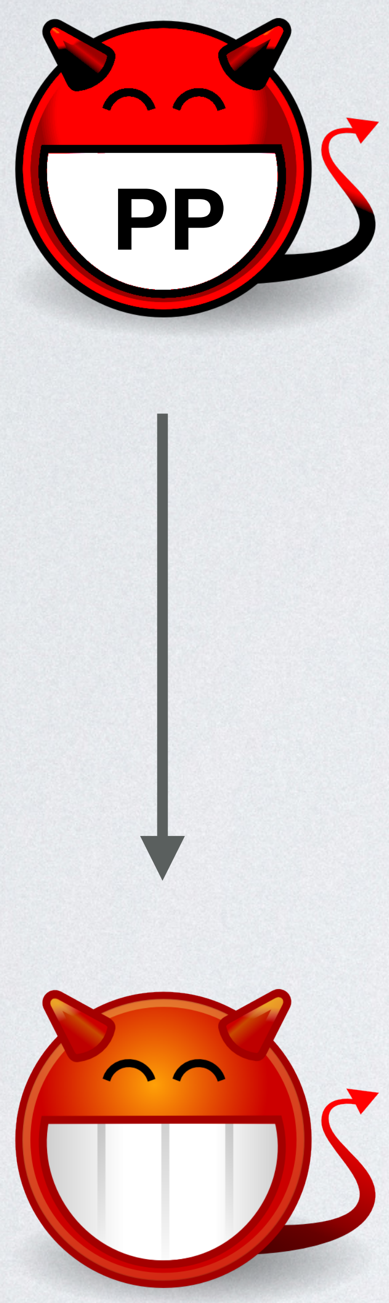

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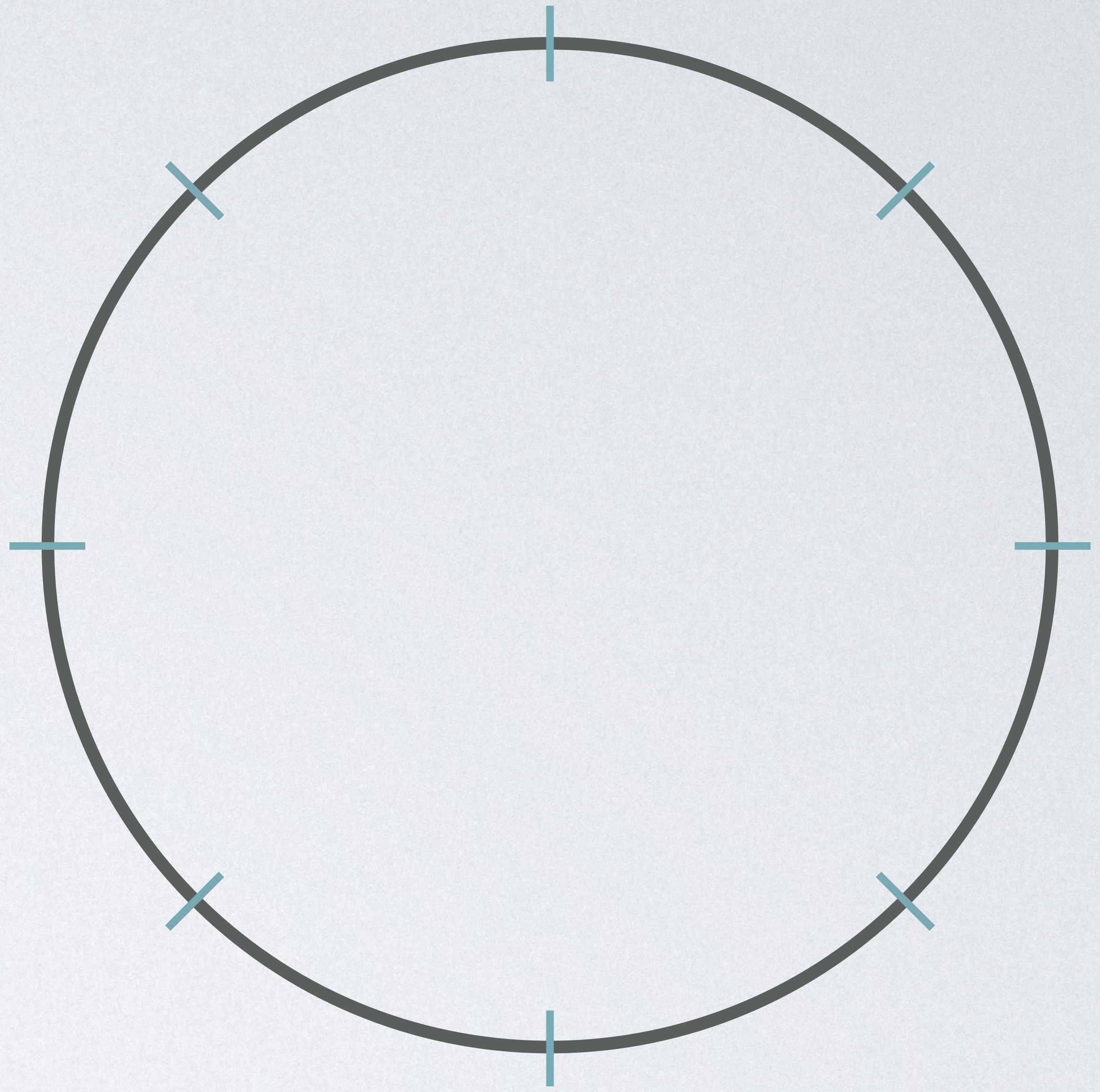

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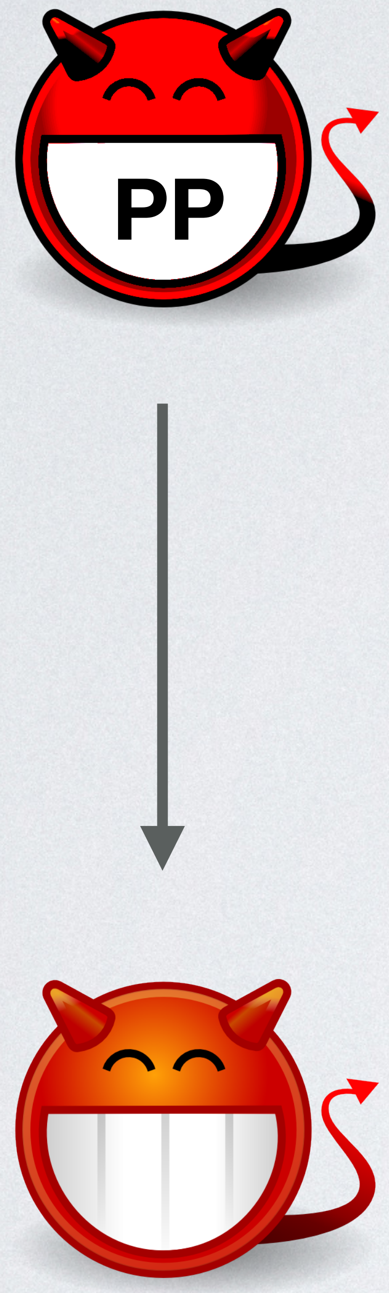

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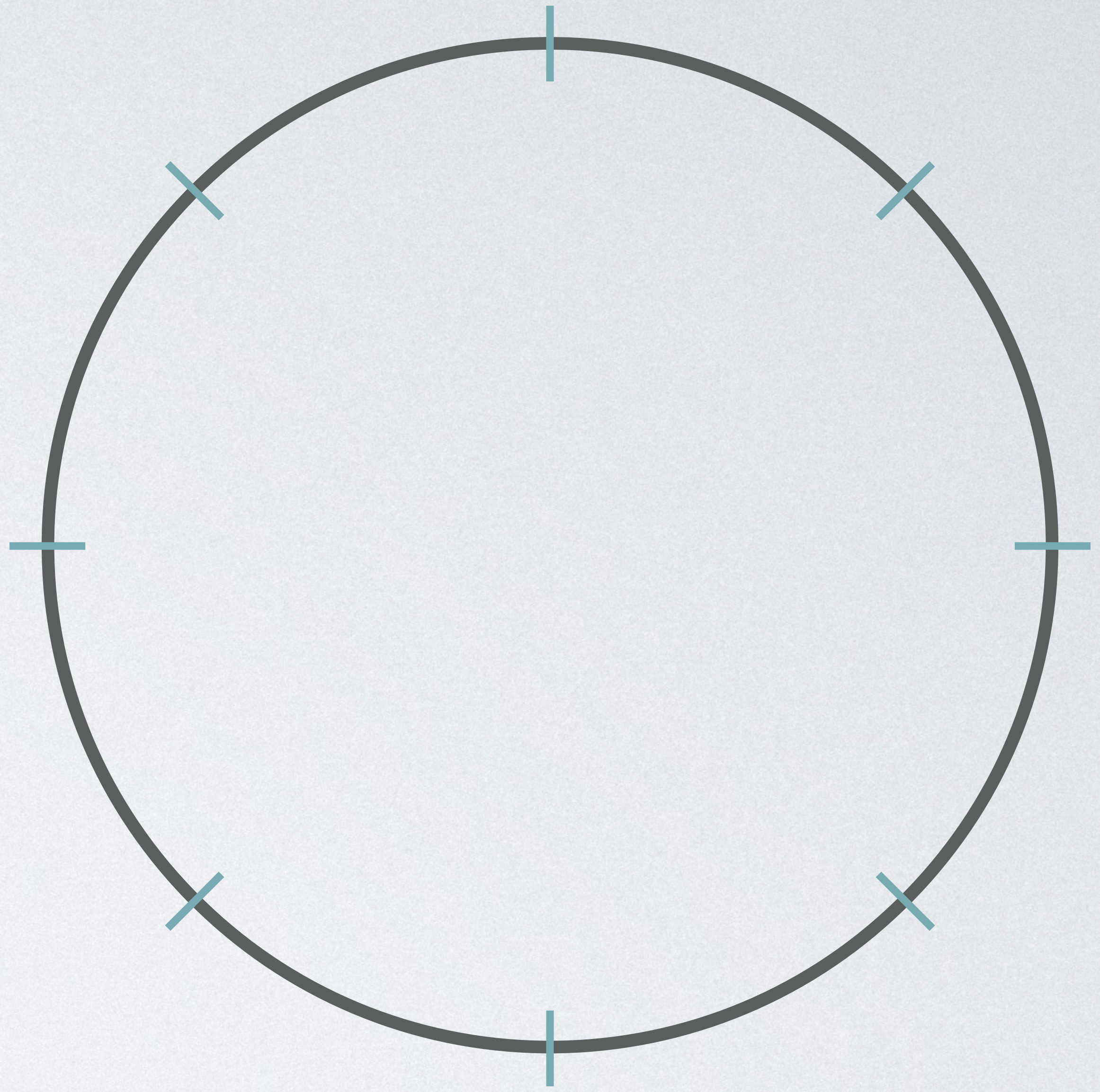

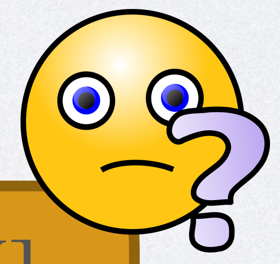

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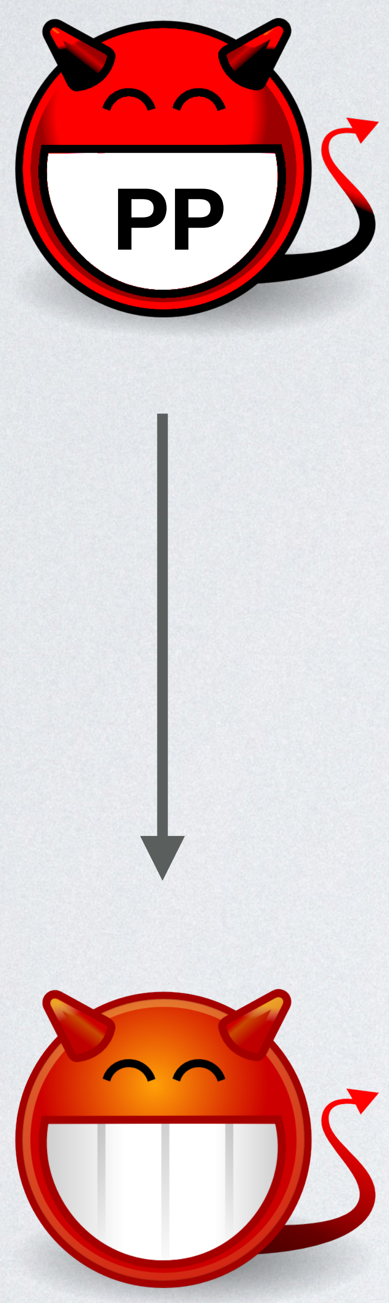

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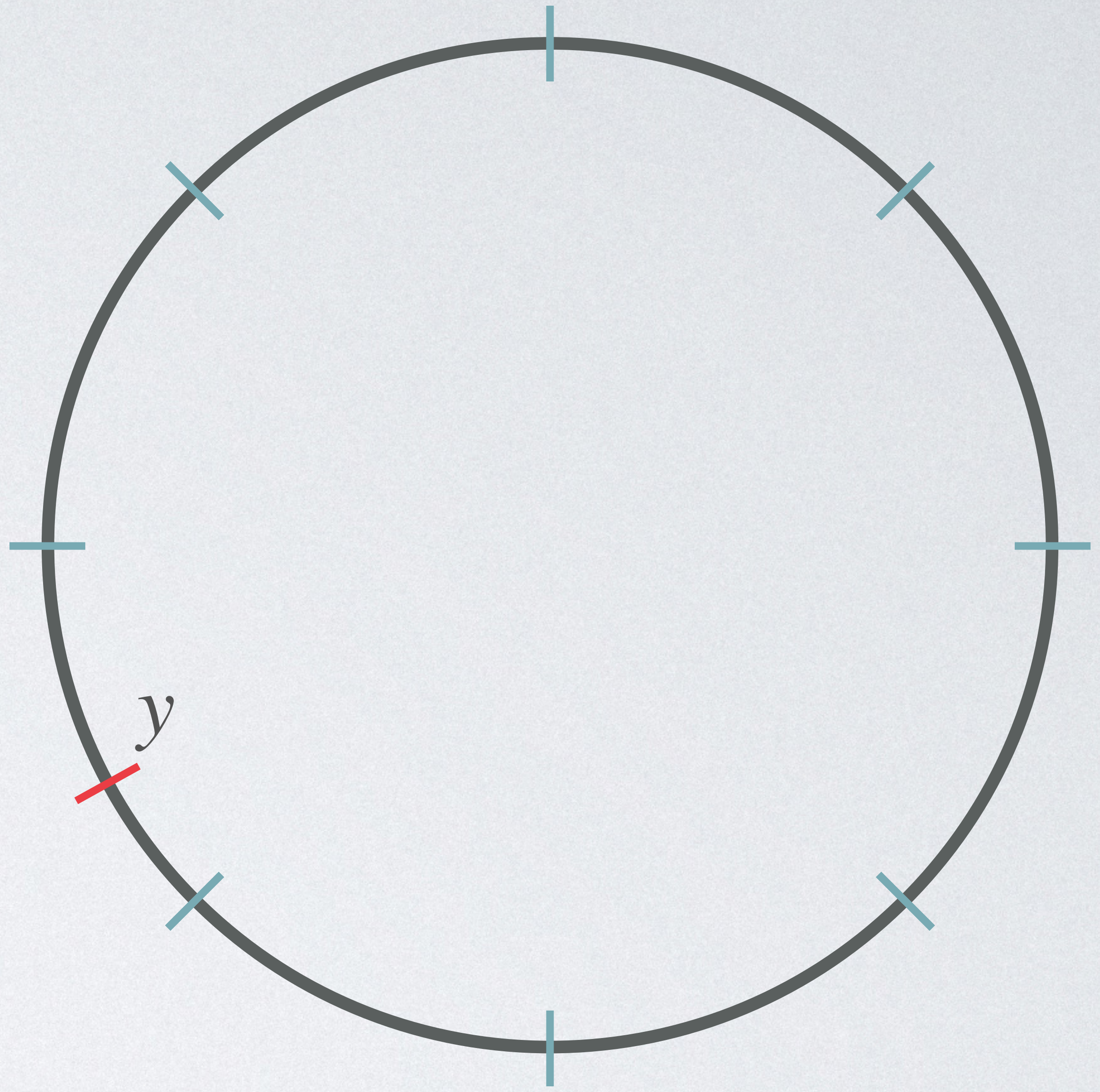

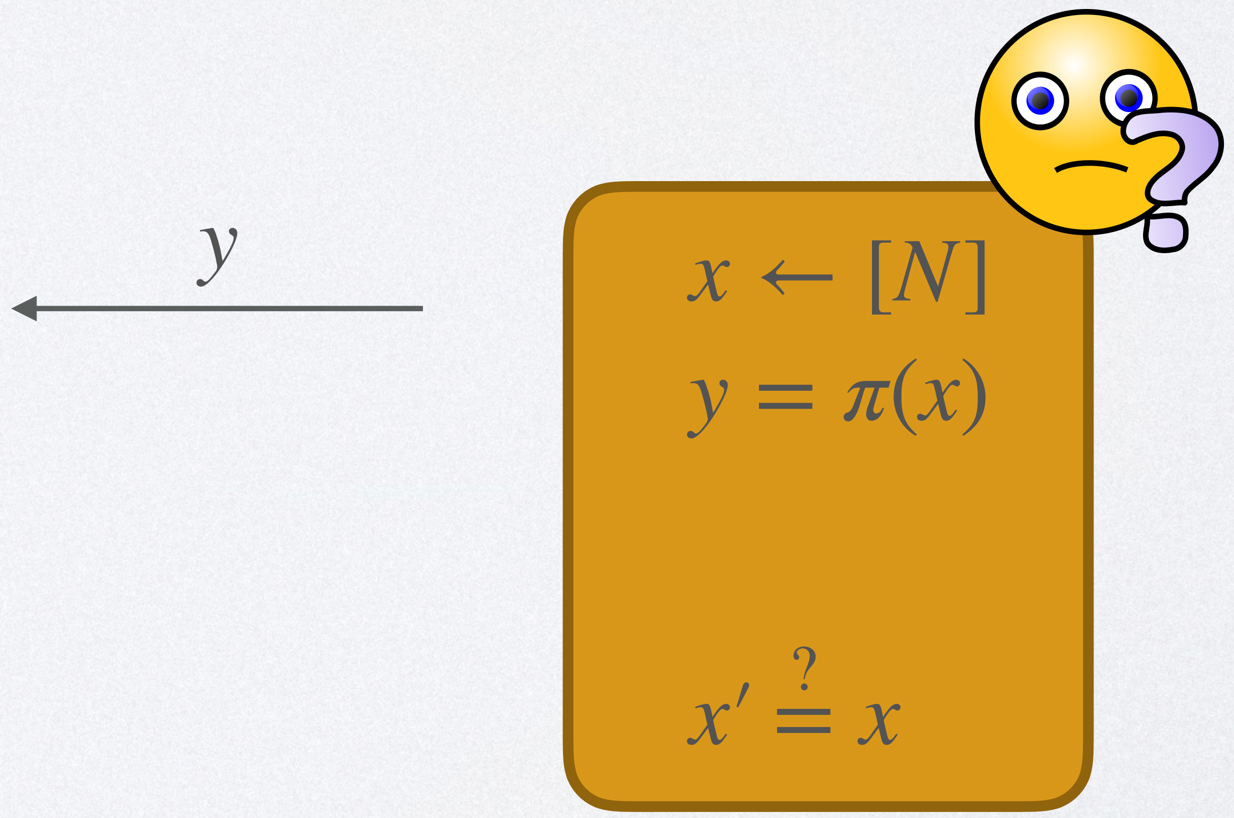

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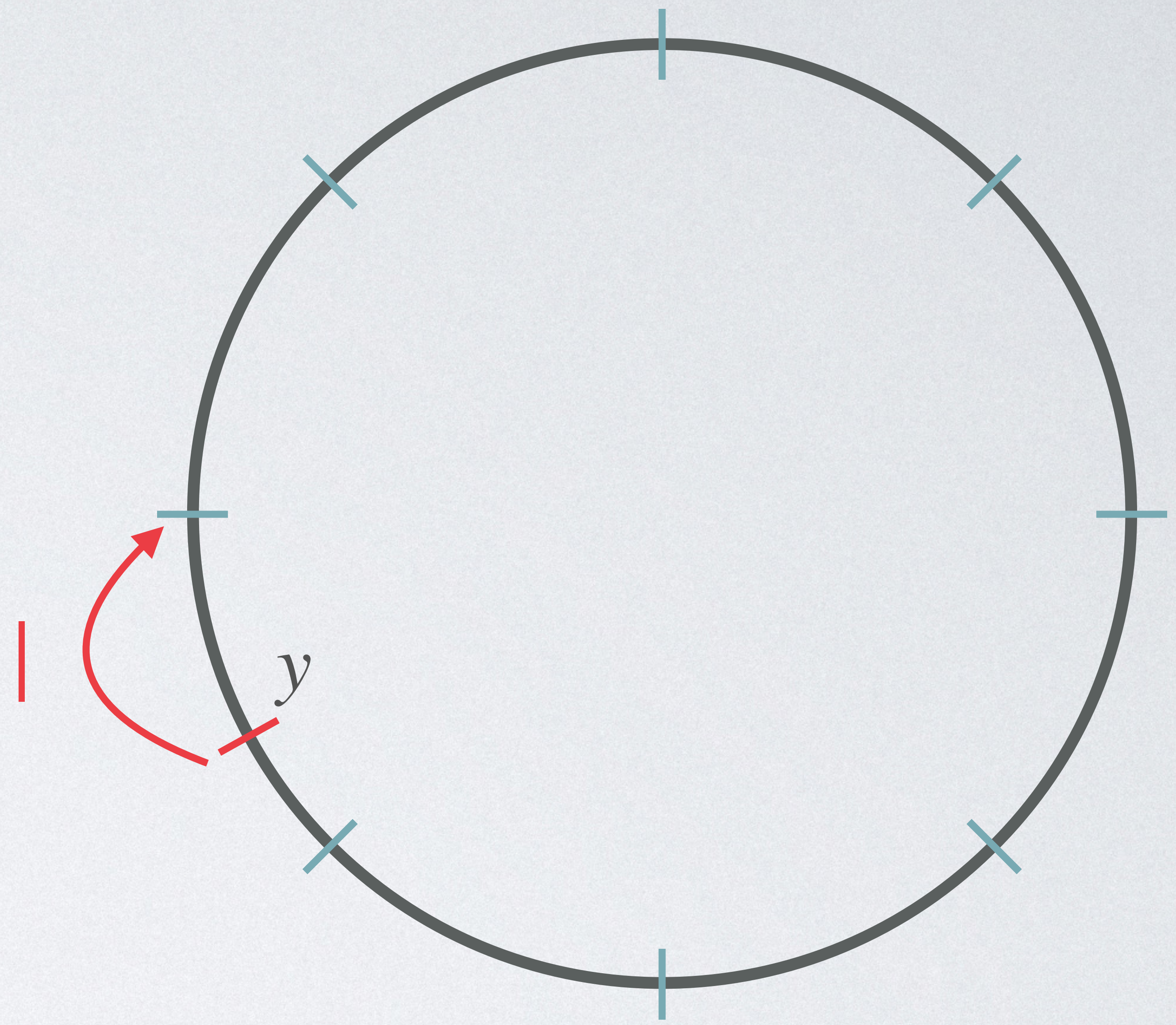

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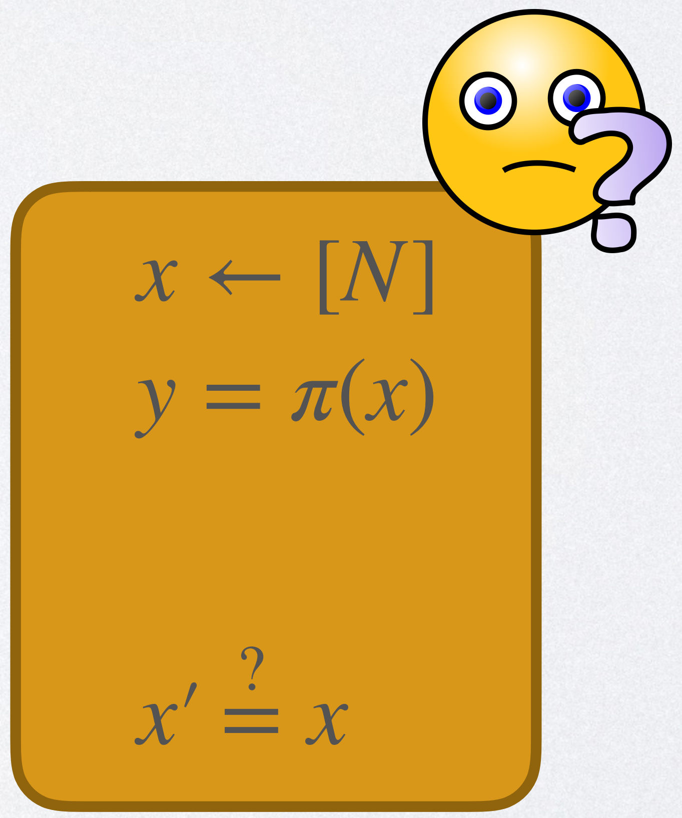
Start at y and apply π until hit x_j,

y ←



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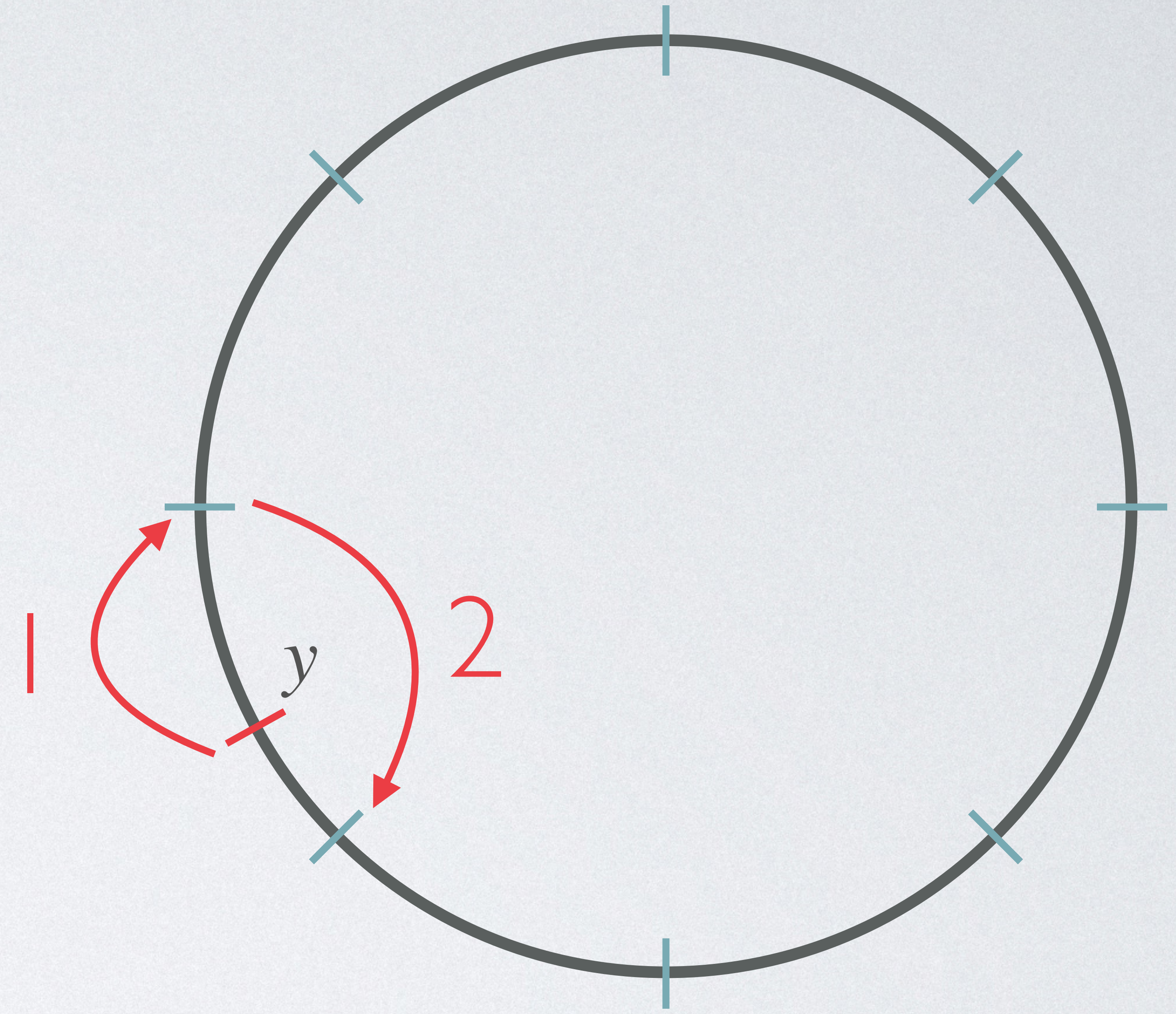

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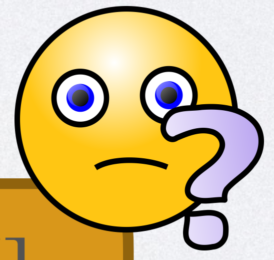
$\xleftarrow{\;y\;}$



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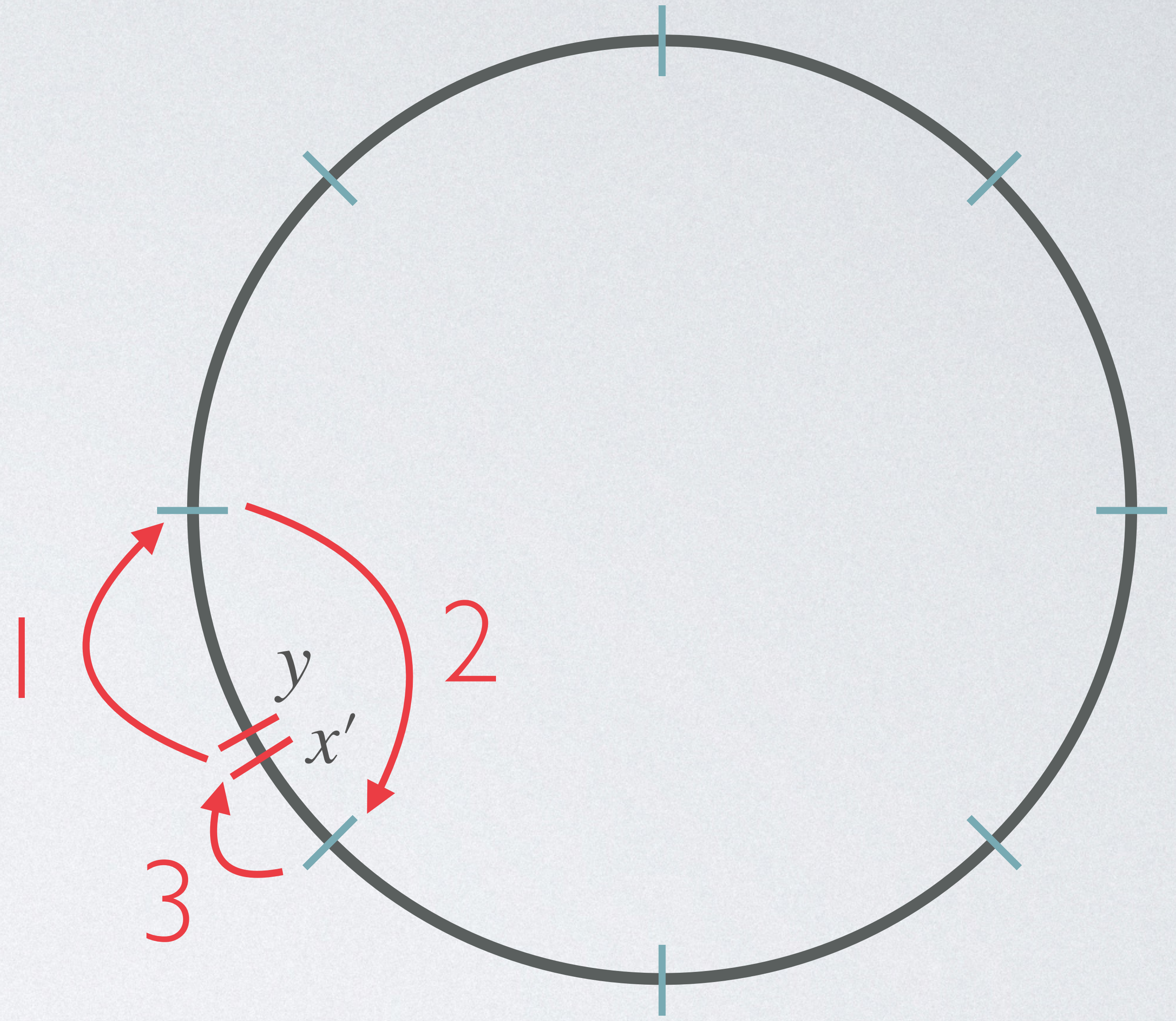

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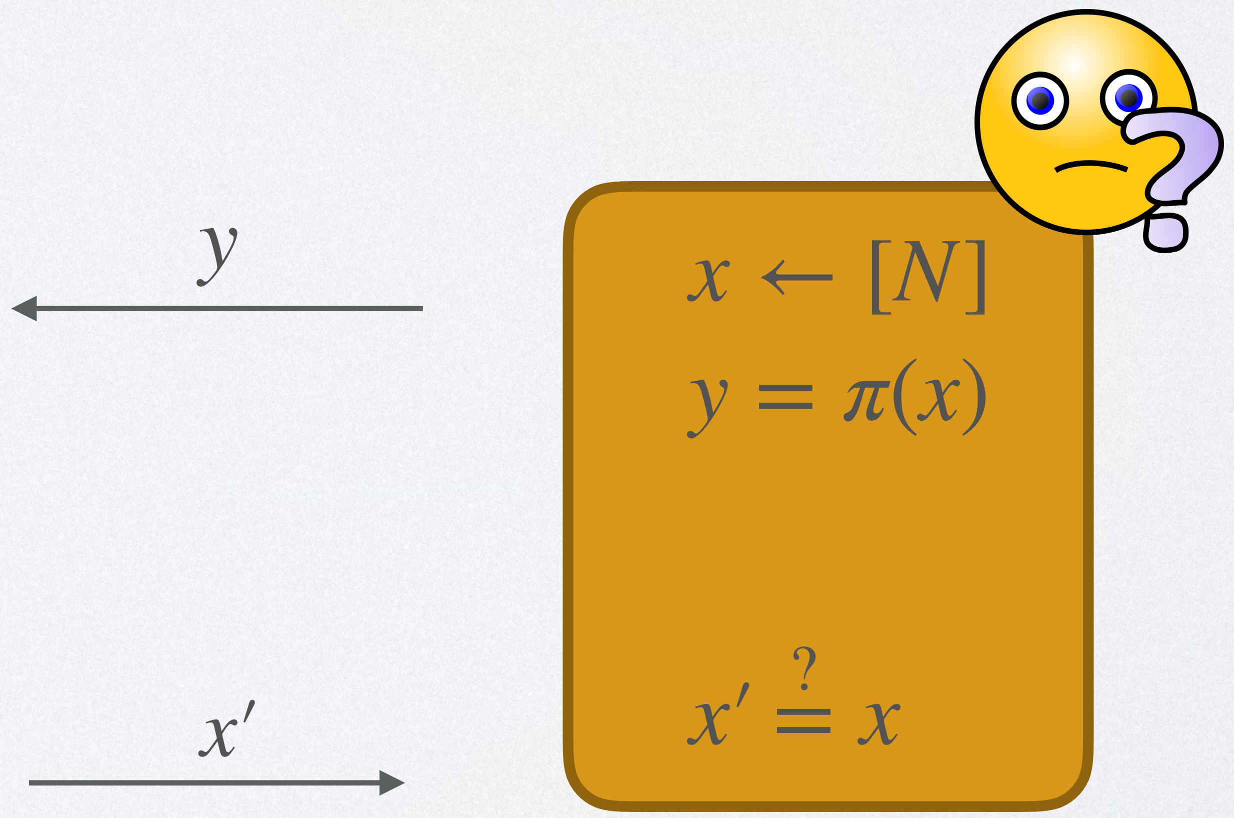

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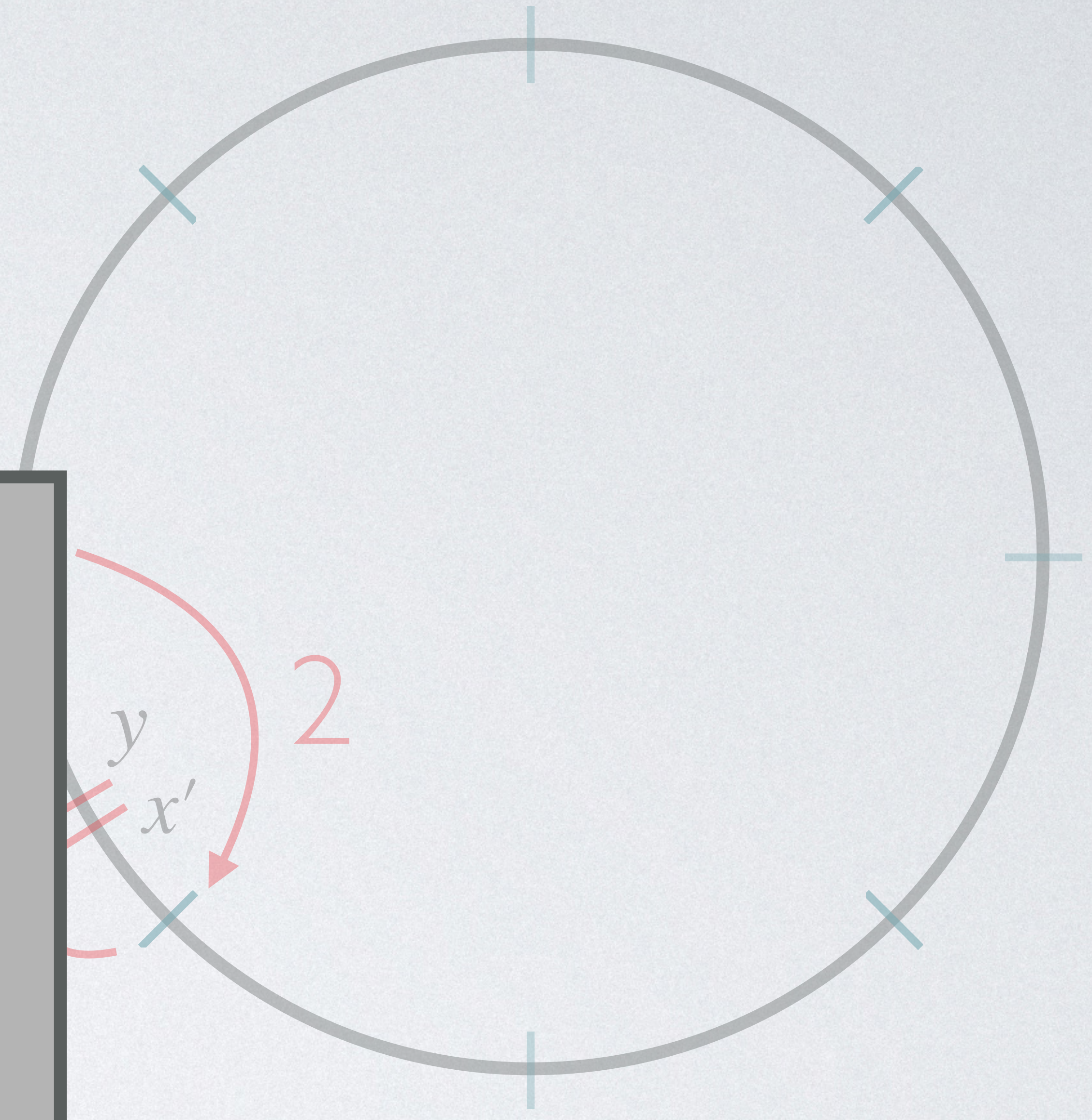
Total complexity for $S = T = \sqrt{N}$: $\sqrt{N}$



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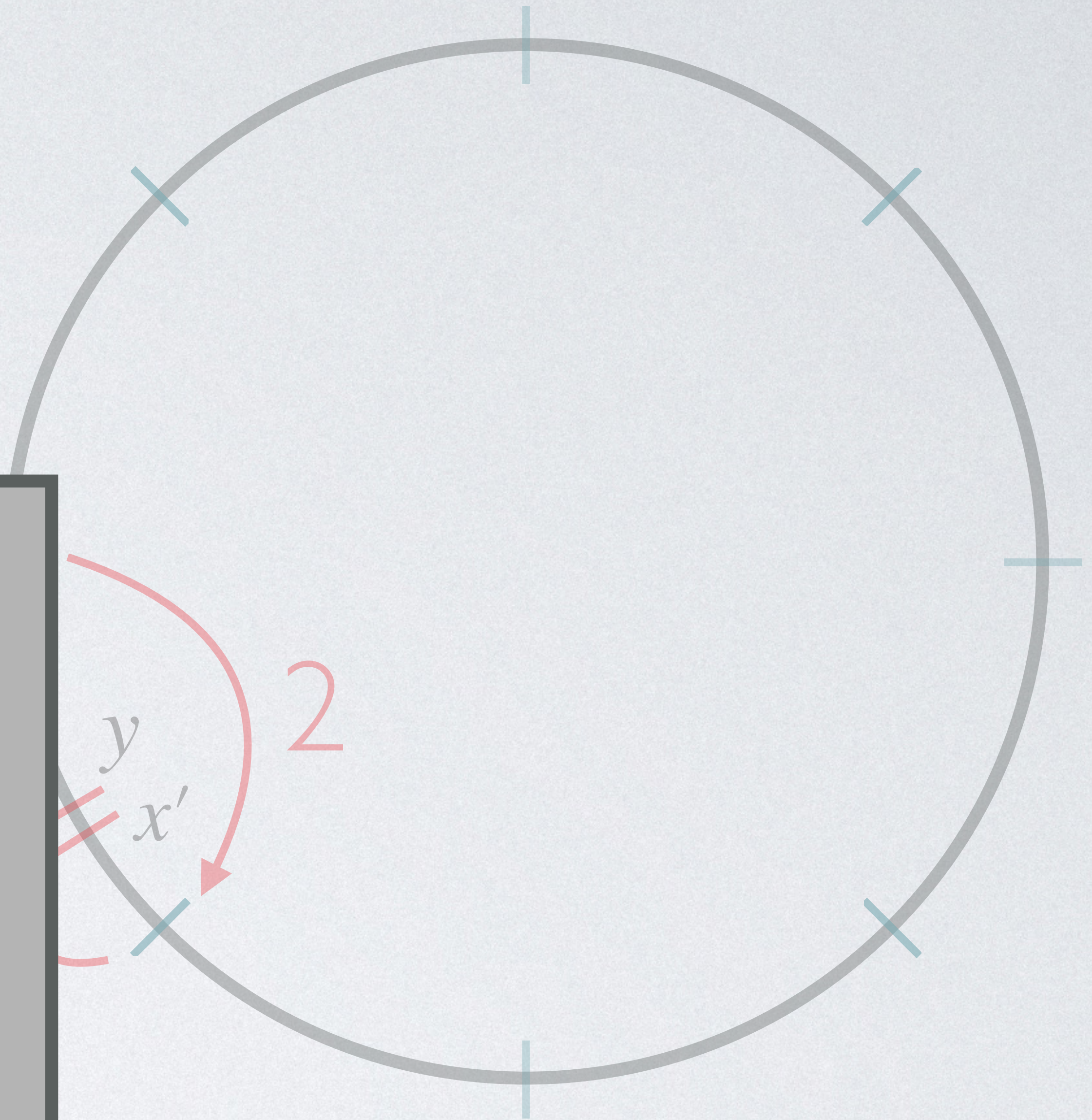

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Analysis in RPM:
security up to N queries

More Preprocessing Attacks

S: Space T: Time

More Preprocessing Attacks

	Bound	Preprocessing Attack	Reference

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OWP	T/N	ST/N	Hellman

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Even Mansour	T^2/N	ST^2/N	Fouque, Joux, Mavromati

S: Space T: Time

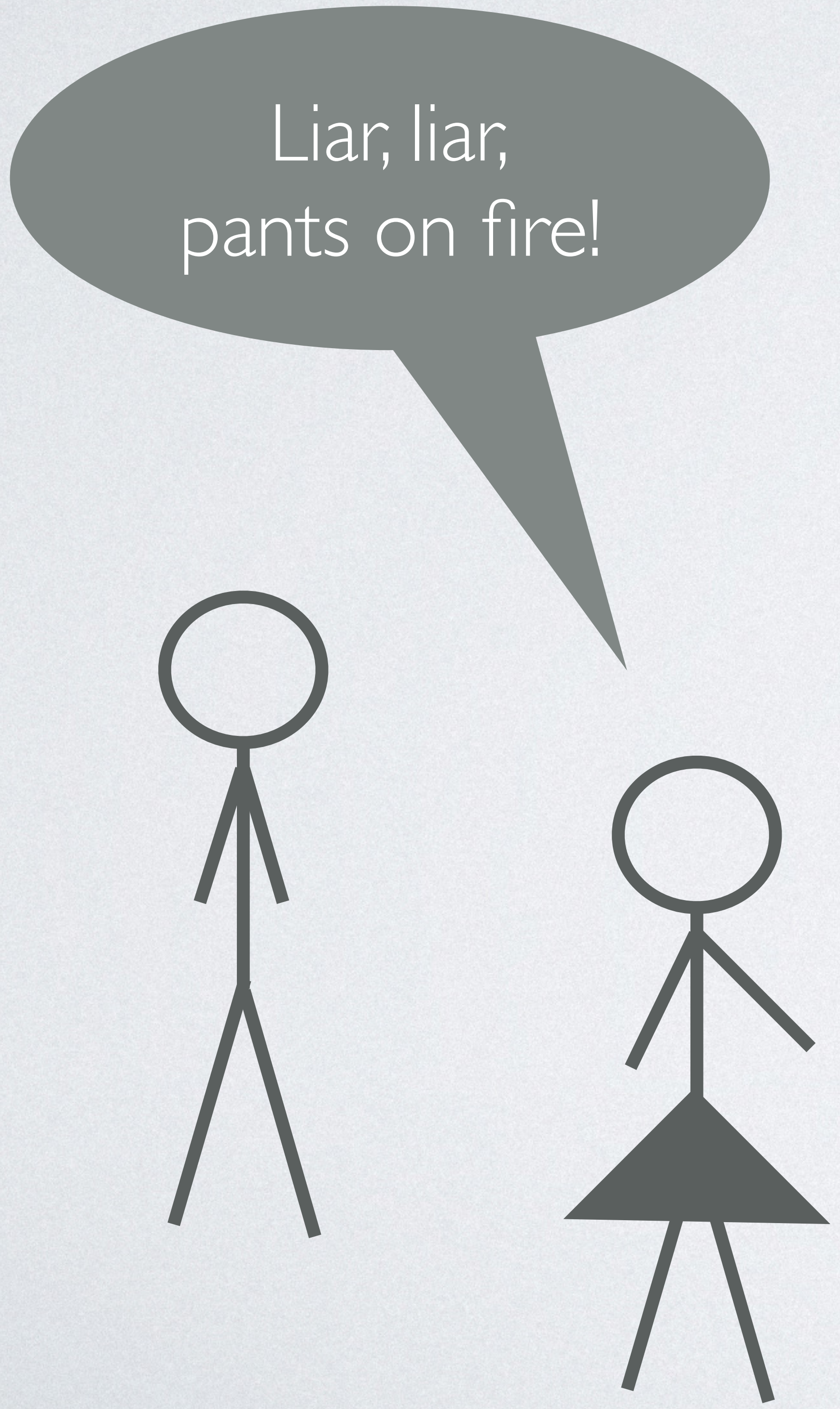
Idealized-Model Methodology

For "natural" applications:

Security in idealized model

=

Security in standard model
using best possible instantiation



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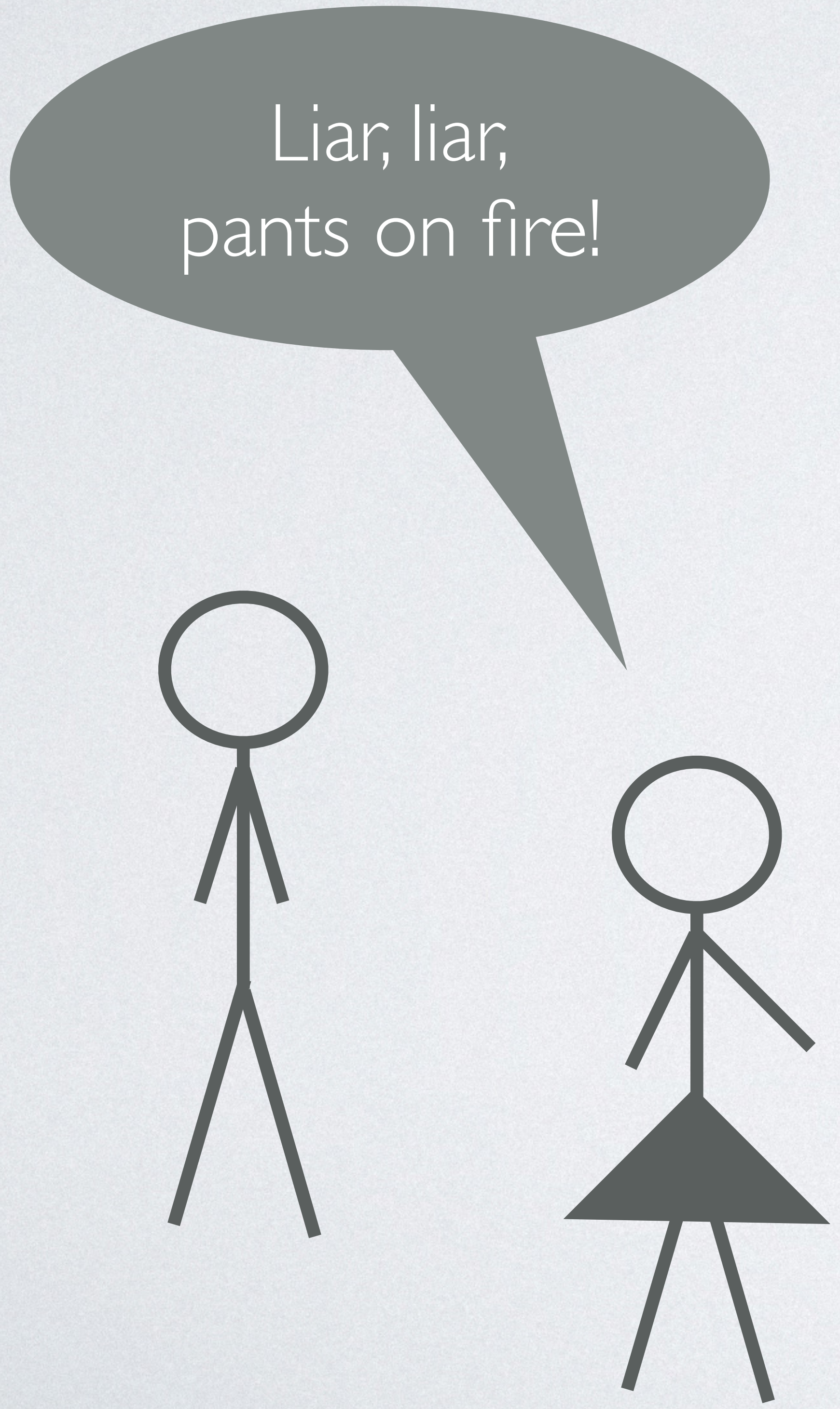

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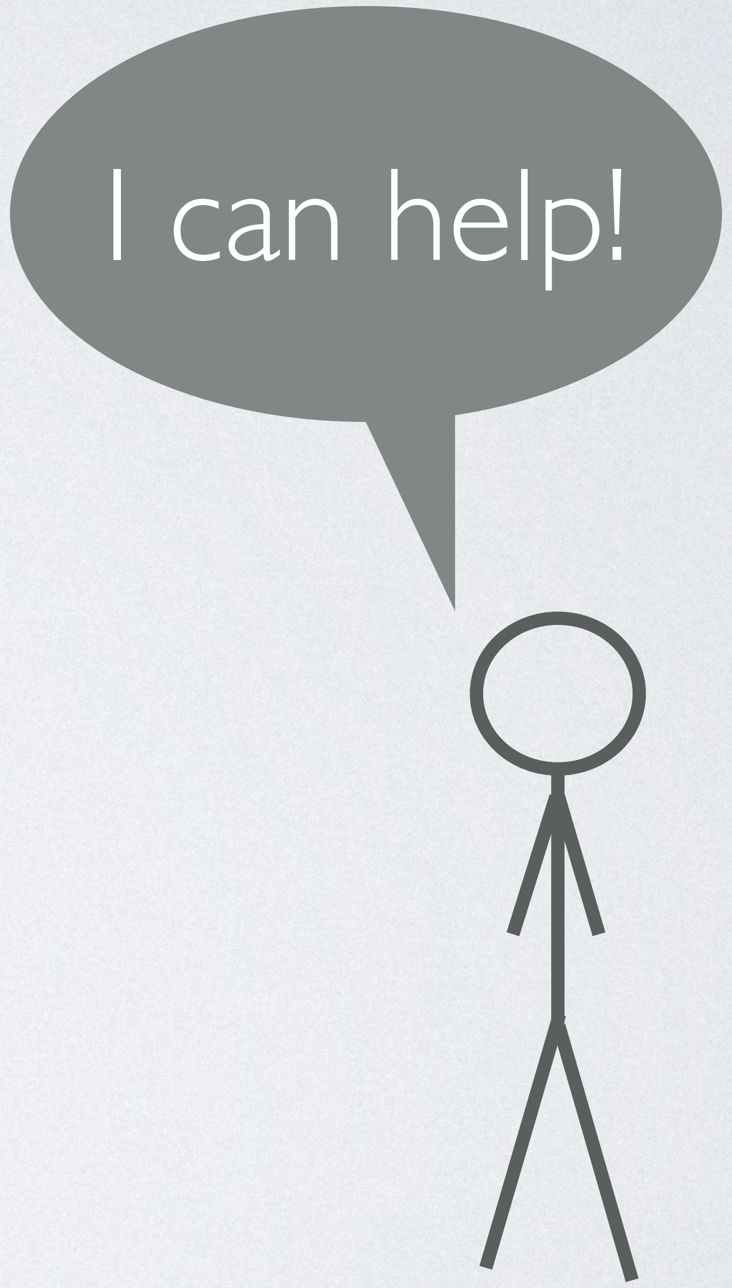
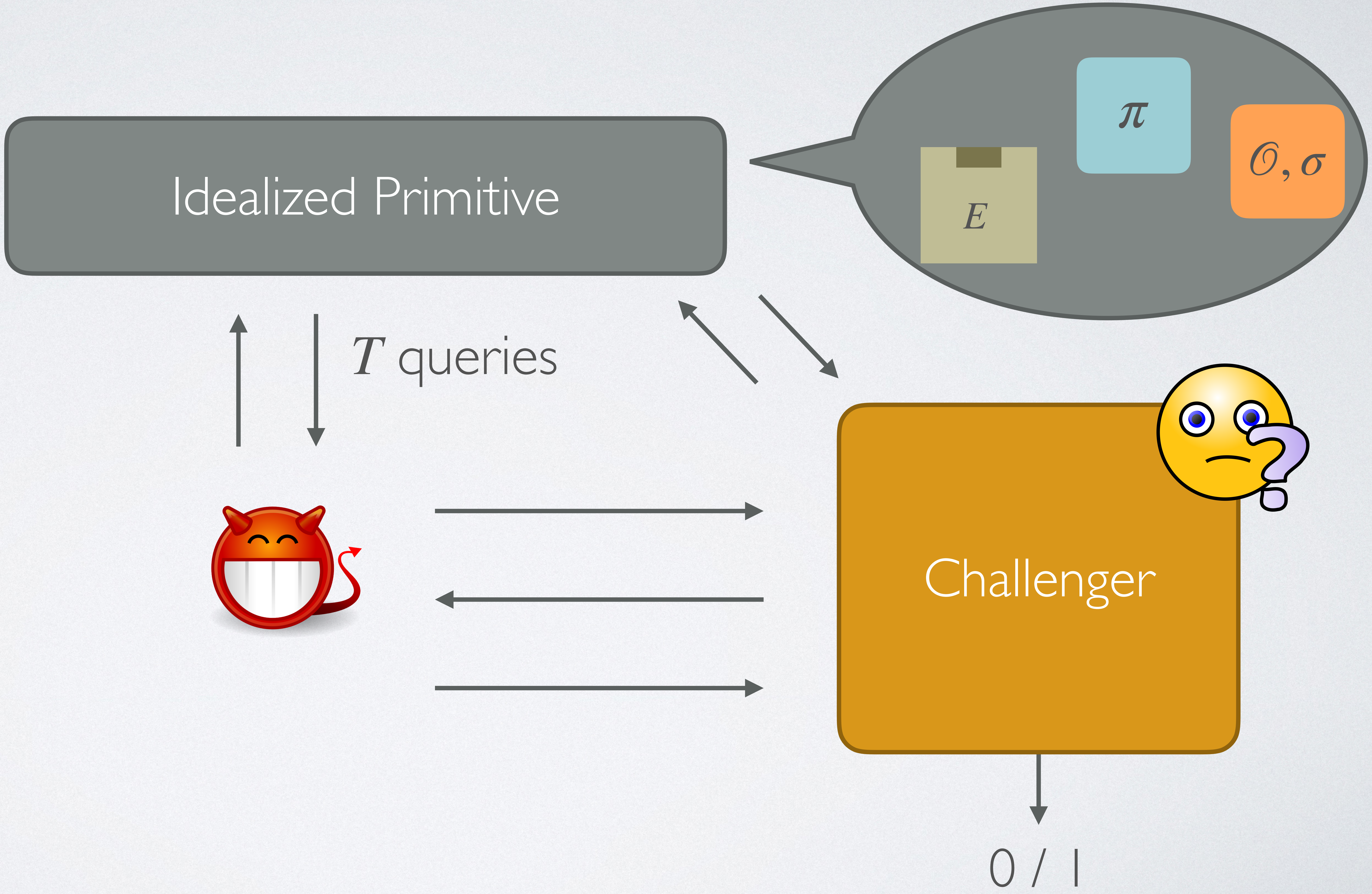


Unruh

Auxiliary-Input (AI) Model

Unruh '07



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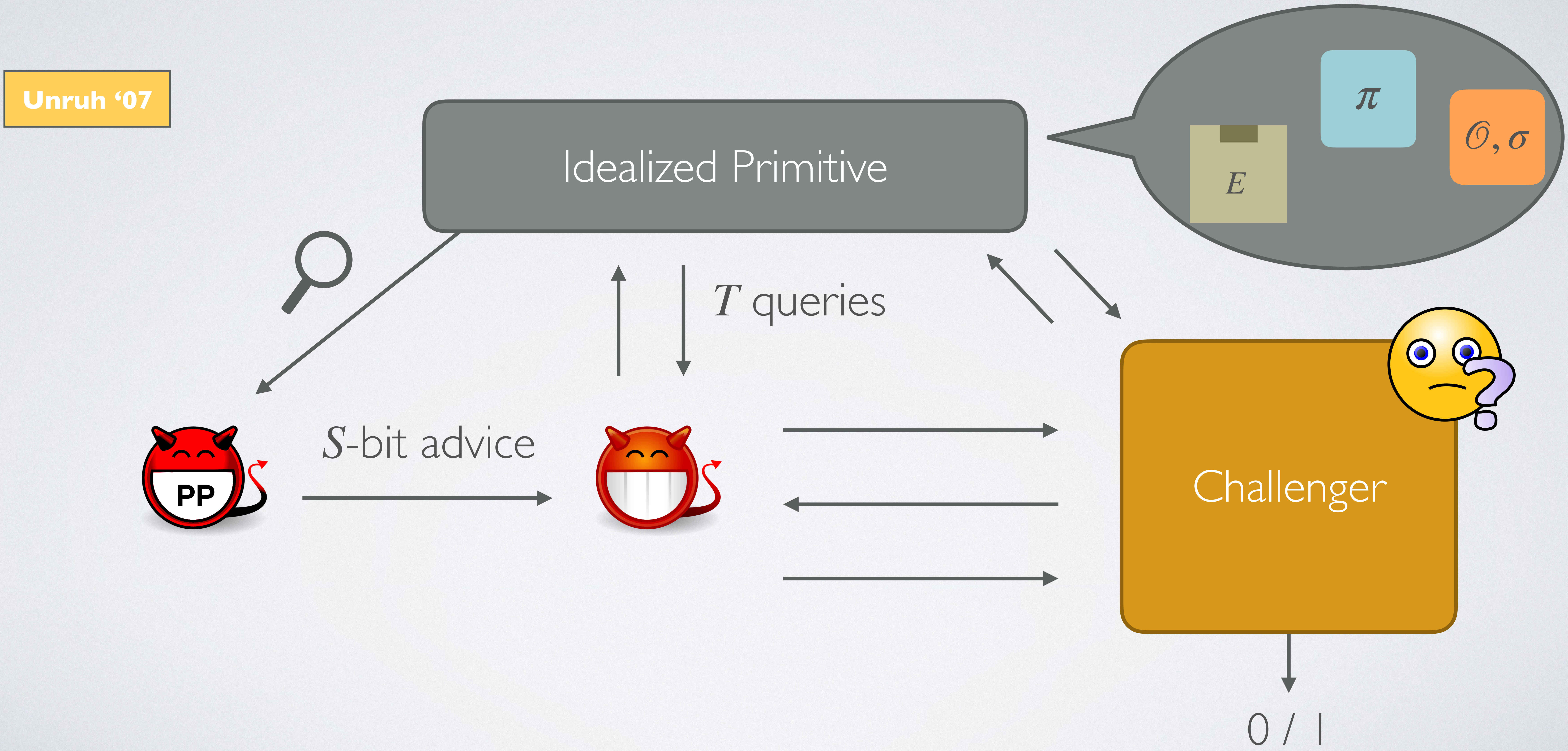

Auxiliary-Input Idealized-Model Methodology

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=

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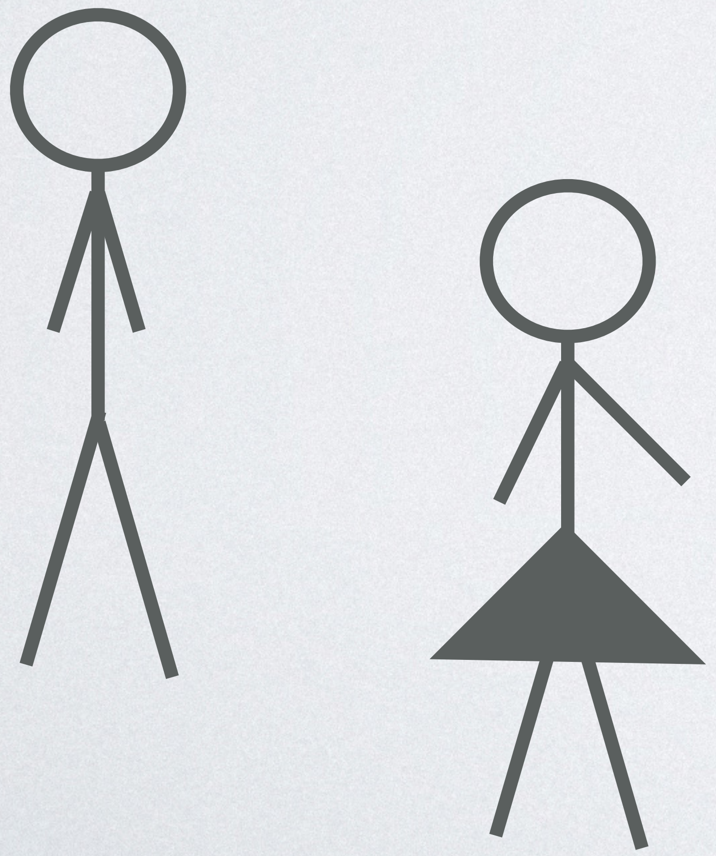

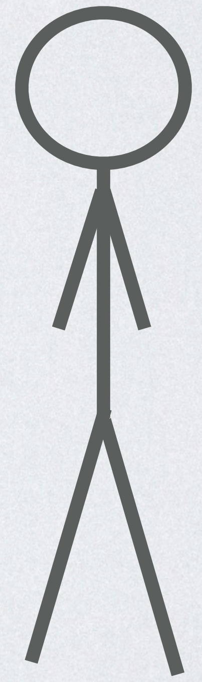

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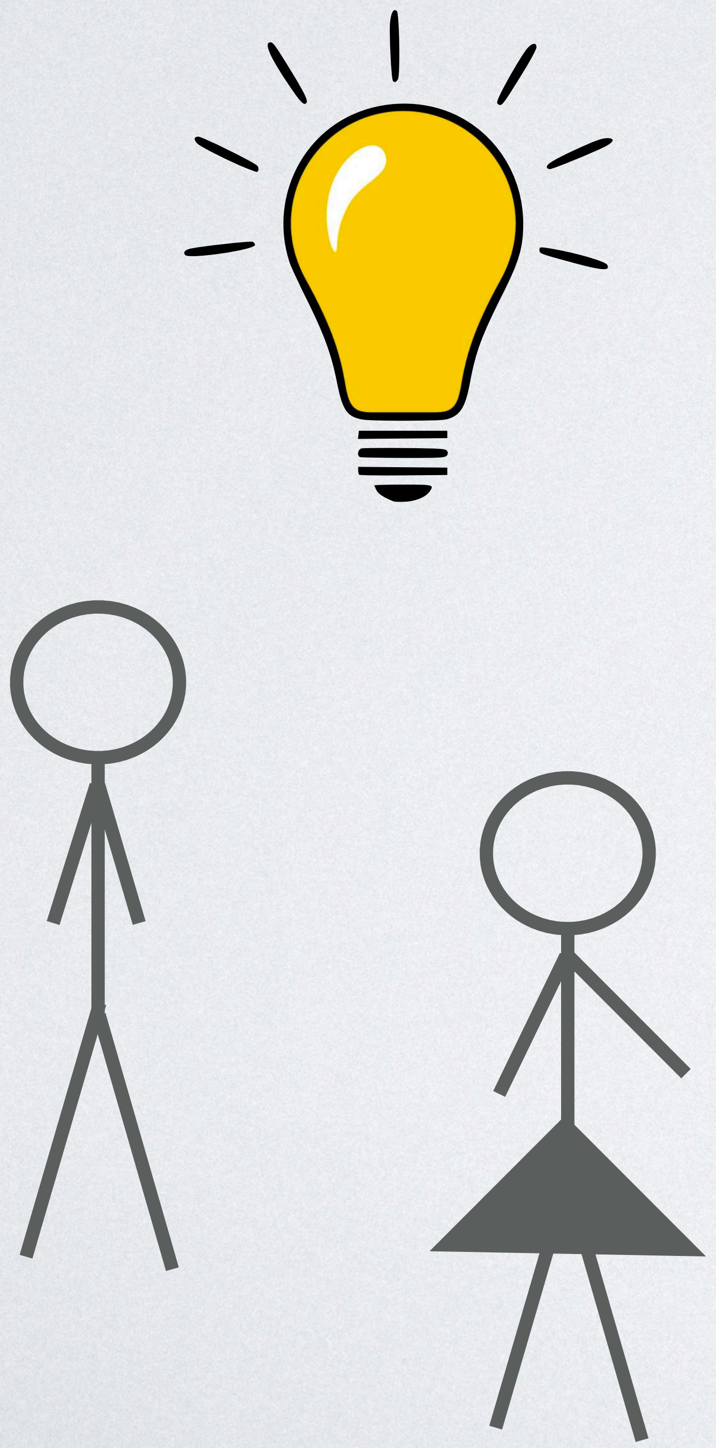

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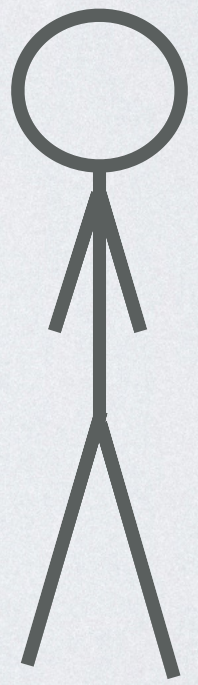
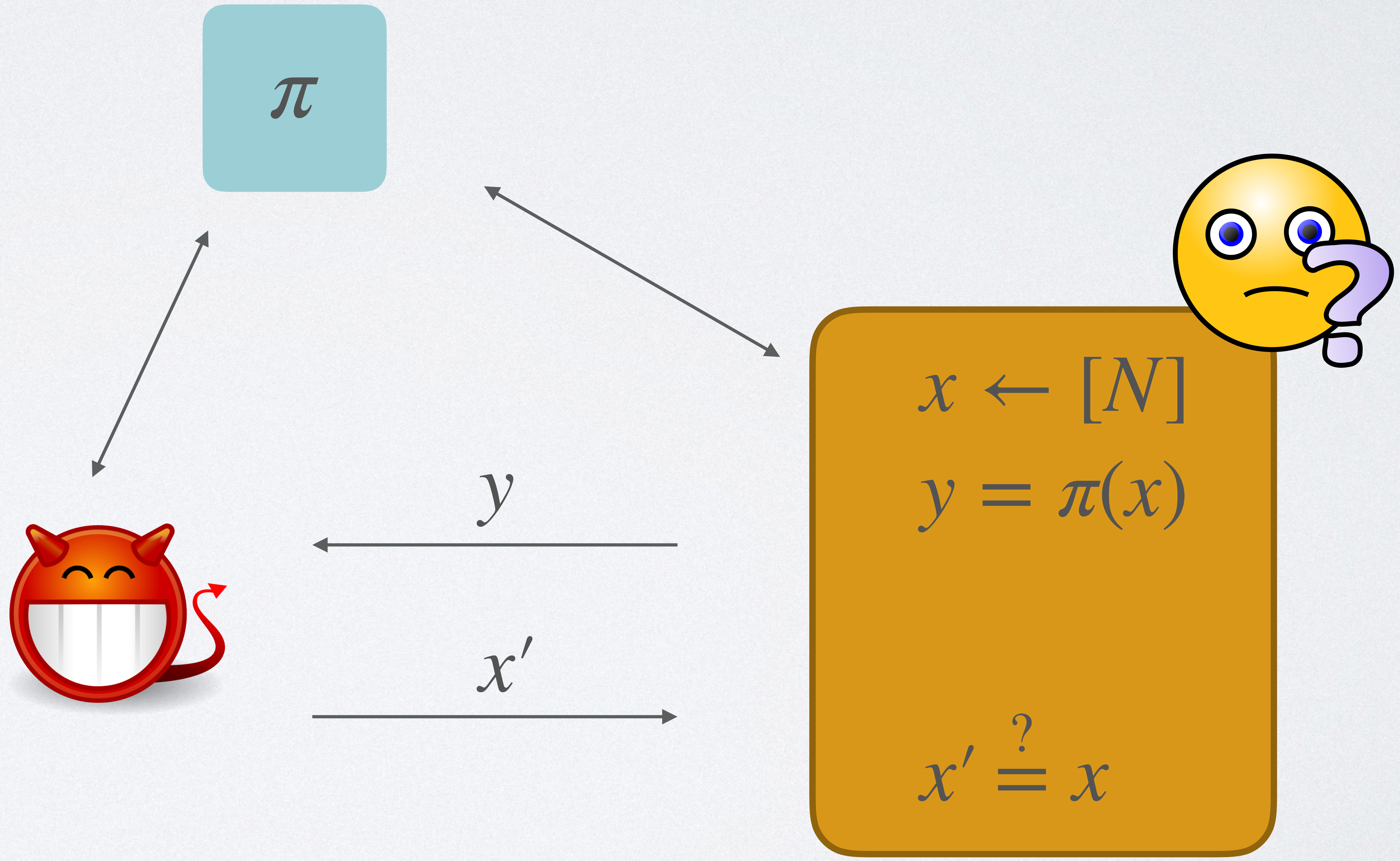


Toy Example: One-Way Permutations

Random permutation $\pi : [N] \to [N]$



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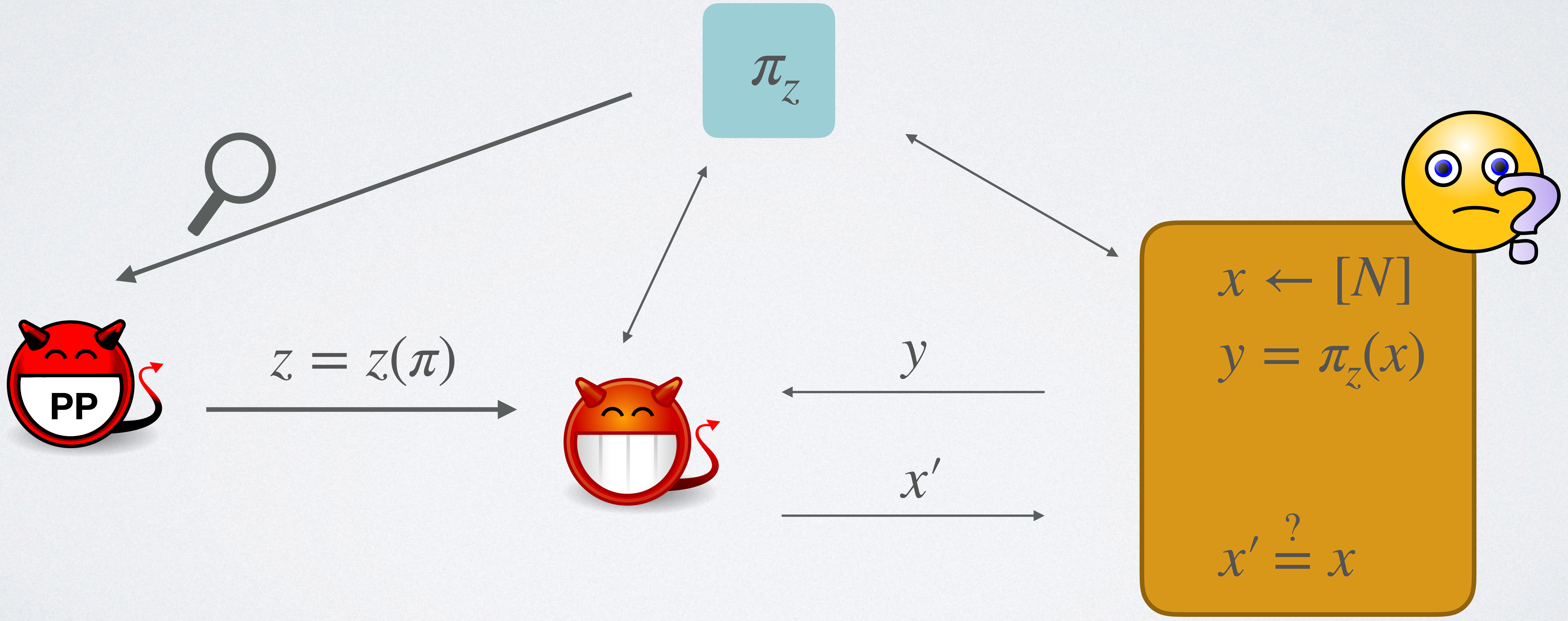

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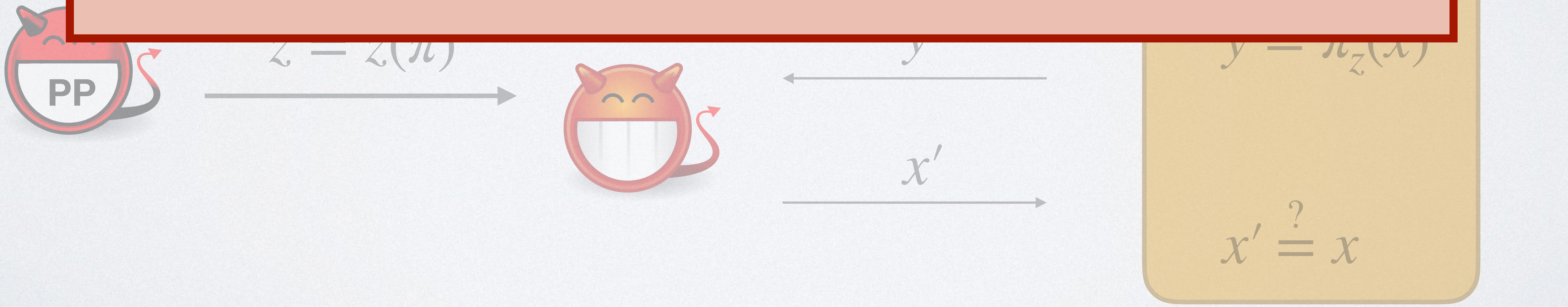
Conditioned on z, distribution of π may be ugly:

– Distribution of coordinates unclear

– Dependence of coordinates unclear



Security analysis with auxiliary information seems hard…

AI and the Random-Oracle Model

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Reference	Technique	Difficulty	Applicability	Bounds	Computational
Unruh '07	Presampling	Easy	Generic	Loose	Limited

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Before this work:

No **non-uniform** bounds known for any symmetric primitives

AI and the Generic-Group Model

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Corrigan-Gibbs, Kogan '18	Compression	Hard	DL, CDH, DDH, …	Tight

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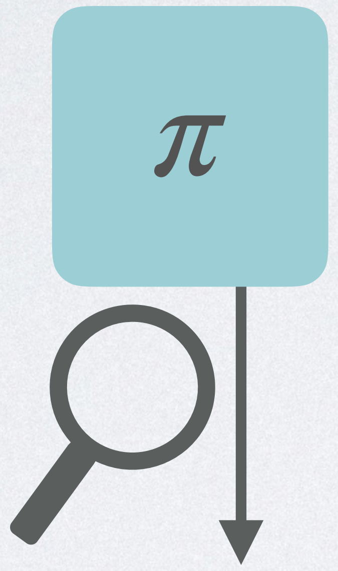
Presampling Technique

- Analyze constructions in much simpler so-called **Bit-Fixing (BF) Model**
- Use **generic connection** between AI model and BF model to get AI model bound

Bit-Fixing: Random Permutations

$$\pi : [N] \to [N]$$



AI-RPM: Leak arbitrary S-bit advice about entire function table

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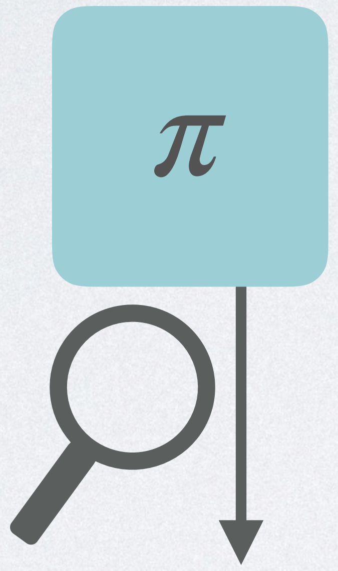

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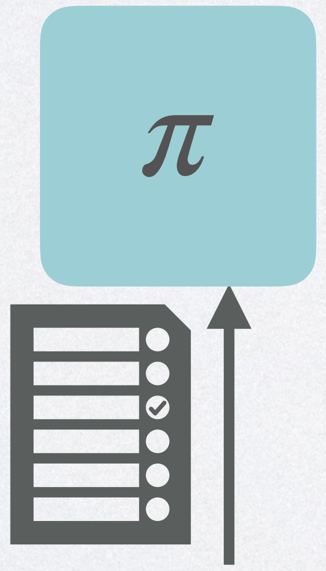


BF-RPM: Prefix arbitrary P coordinates (no collisions)

Bit-Fixing: Ideal Ciphers

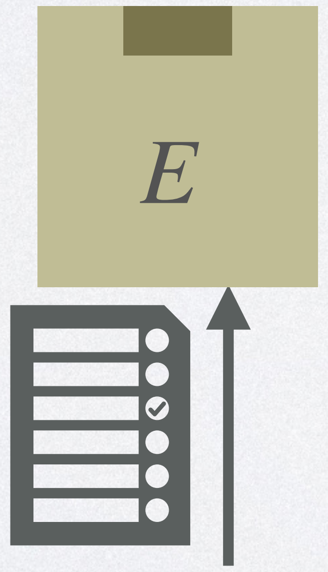
$$E : [K] \times [N] \rightarrow [N]$$

AI-ICM: Leak arbitrary S-bit advice about entire function table

Bit-Fixing: Ideal Ciphers

$$E : [K] \times [N] \to [N]$$

AI-ICM: Leak arbitrary S-bit advice about entire function table



BF-ICM: Prefix arbitrary P coordinates (no collisions for each key)

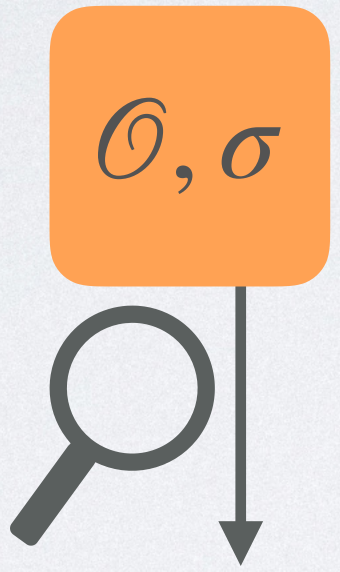
Bit-Fixing: Generic Groups

$$\sigma : [N] \to [M]$$

AI-GGM: Leak arbitrary S-bit advice about entire function table of σ

Bit-Fixing: Generic Groups

$$\sigma : [N] \to [M]$$



AI-GGM: Leak arbitrary S-bit advice about entire function table of σ

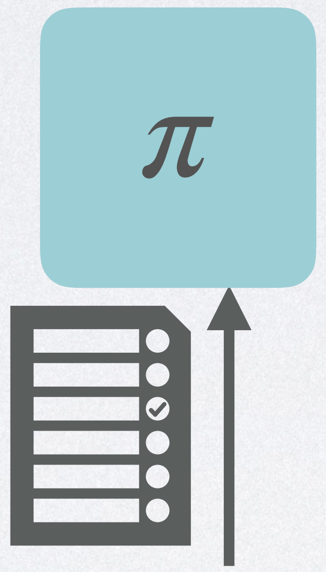
BF-GGM: Prefix arbitrary P coordinates of σ (no collisions)

Bit-Fixing to Auxiliary Input

Theorem:

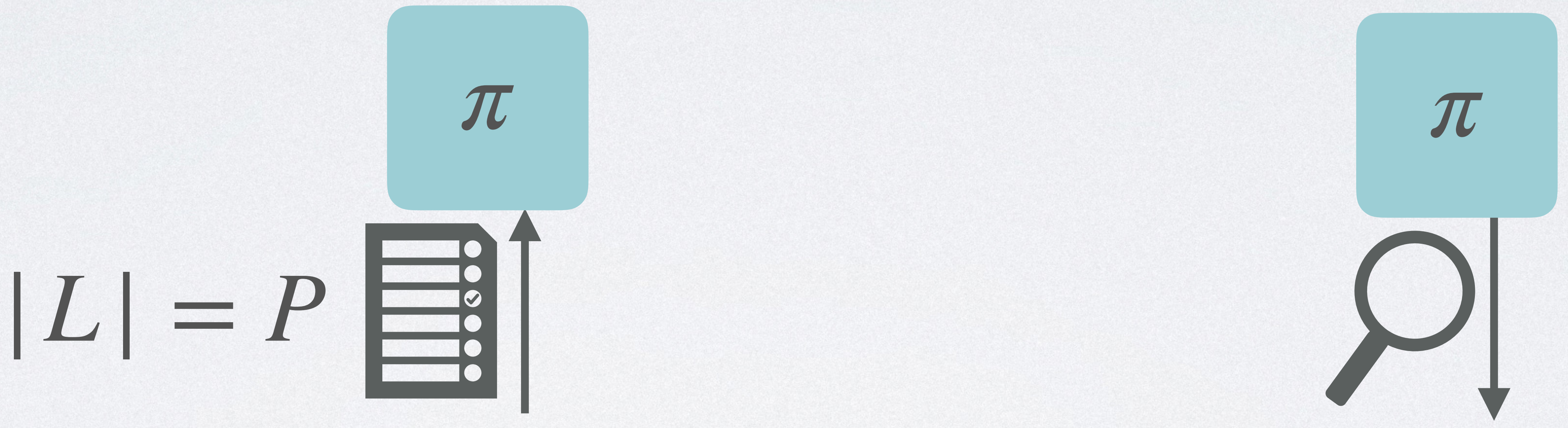
Bit-Fixing to Auxiliary Input

$|L| = P$



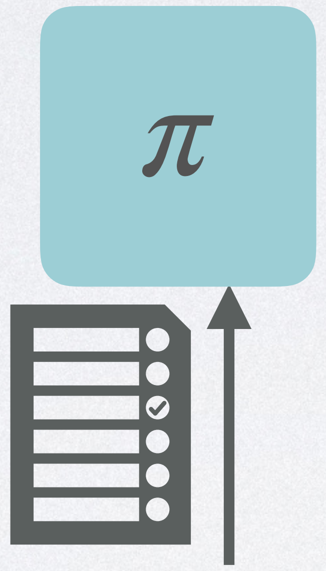
Theorem: (S, T, ε)-secure

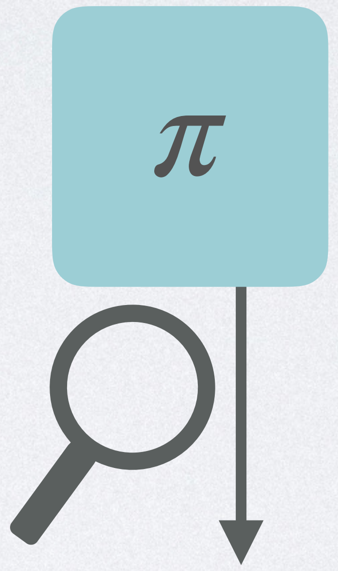
Bit-Fixing to Auxiliary Input



Theorem: (S, T, ε)-secure $\implies$ (S, T, ε')-secure

Bit-Fixing to Auxiliary Input

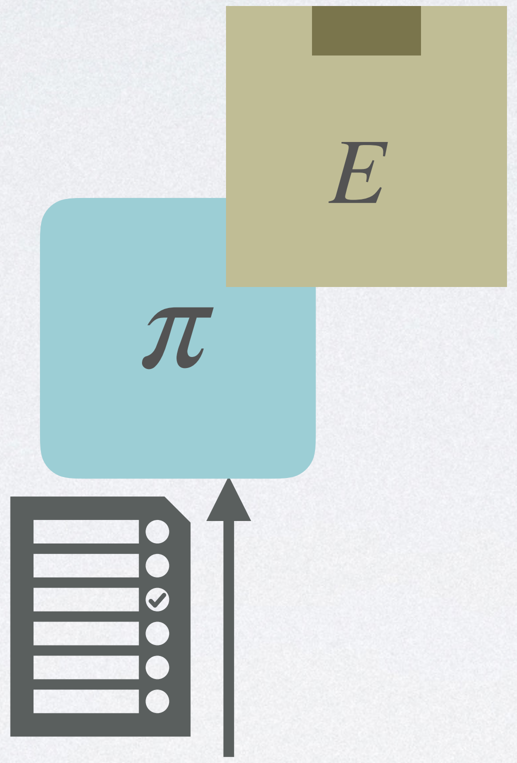
$|L| = P$ 

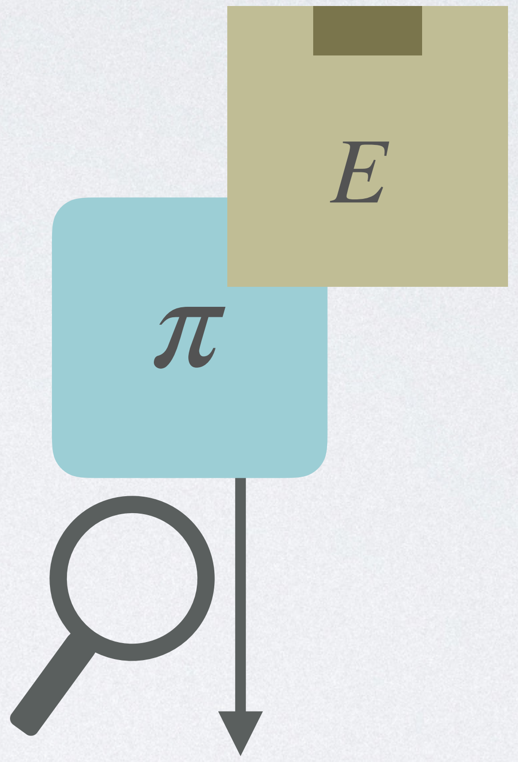


Theorem: (S, T, ε)-secure $\Longrightarrow$ (S, T, ε')-secure

where $\varepsilon' \leq \varepsilon + \frac{ST}{P}$

Bit-Fixing to Auxiliary Input

$|L| = P$ 



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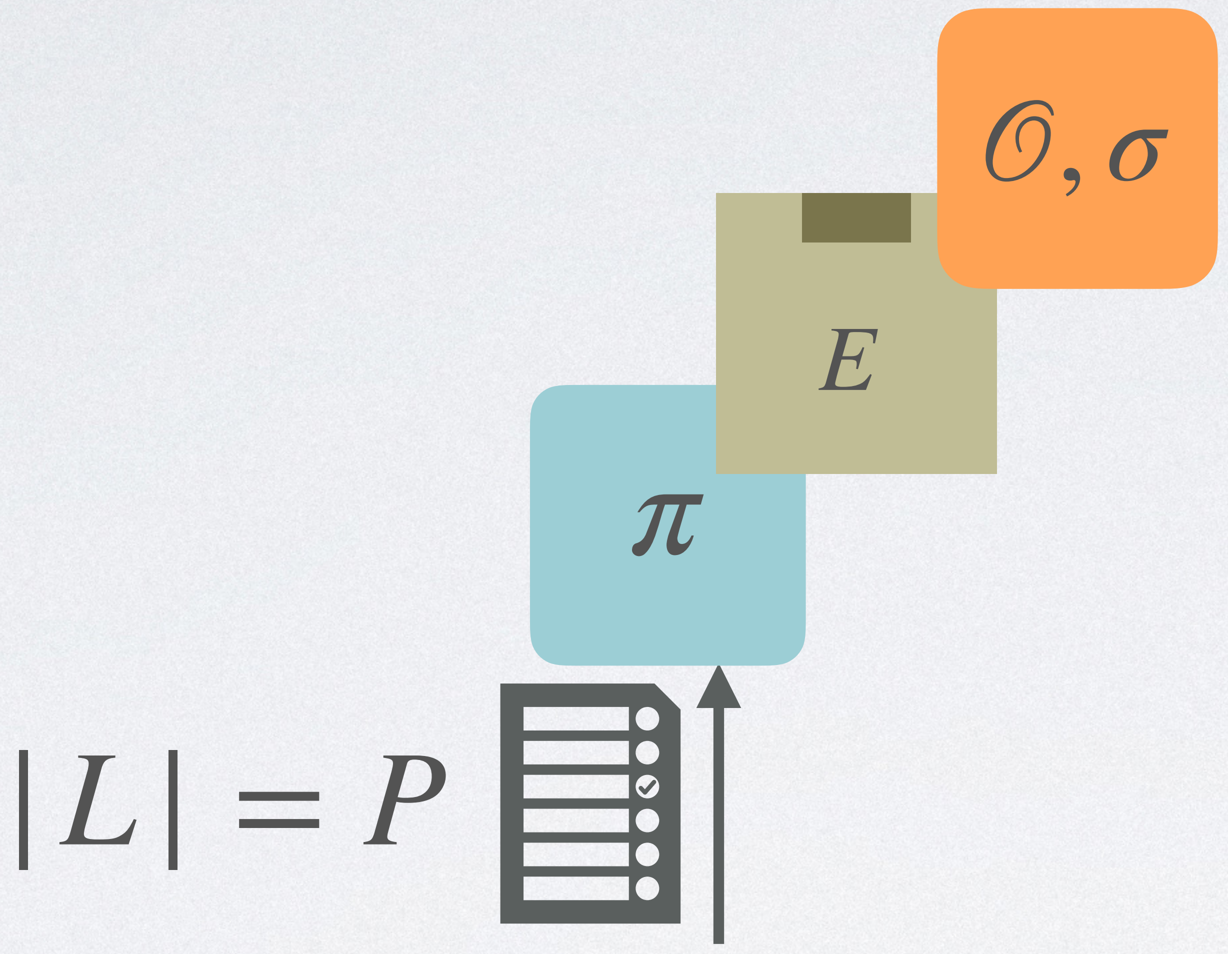
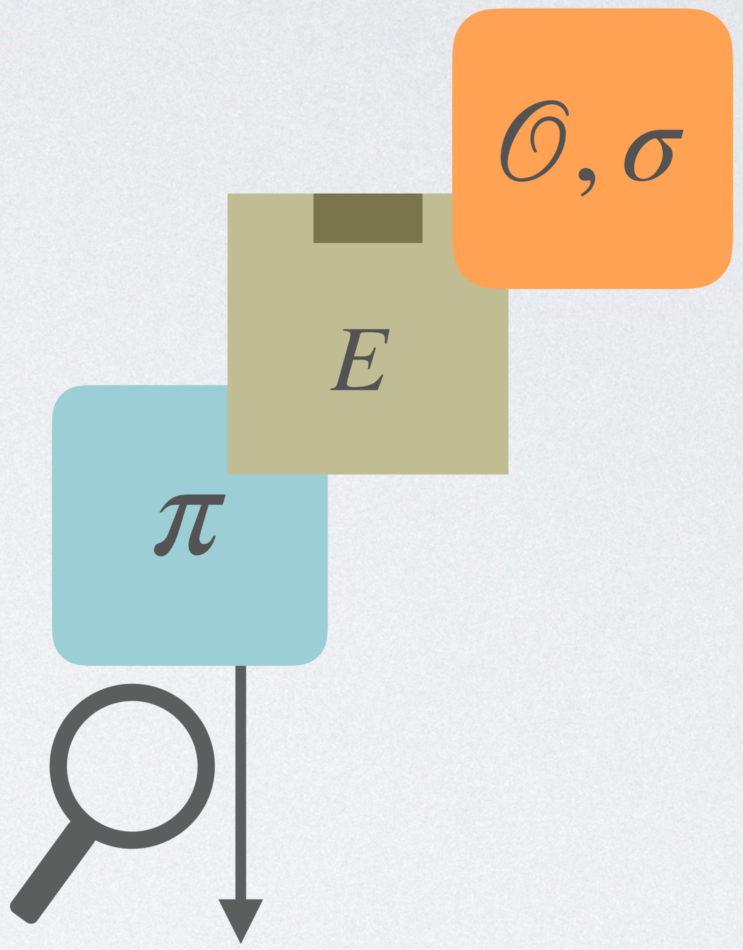
Bit-Fixing to Auxiliary Input

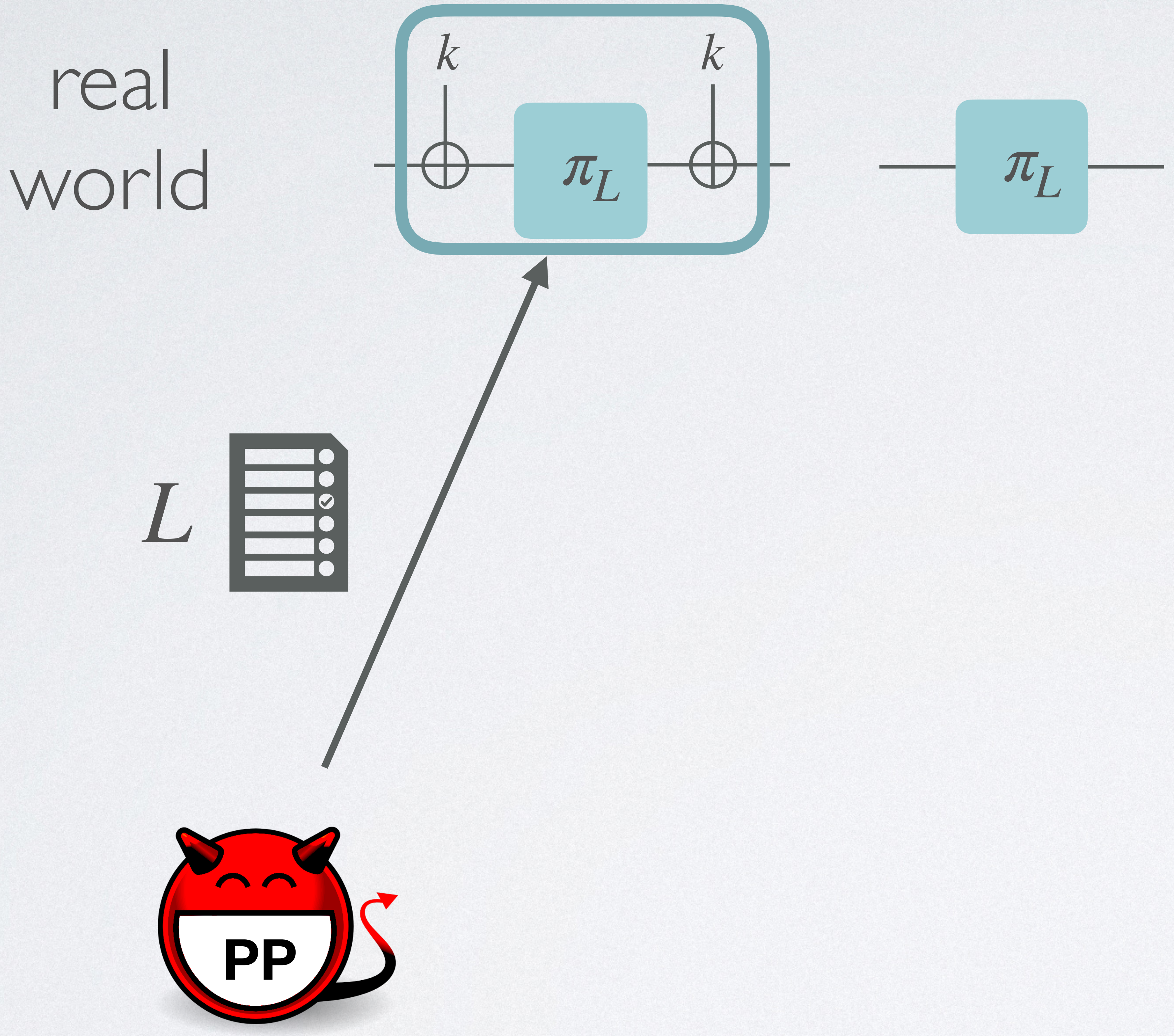




Theorem: (S, T, ε)-secure $\implies$ (S, T, ε')-secure

where $\varepsilon' \leq \varepsilon + \dfrac{ST}{P}$

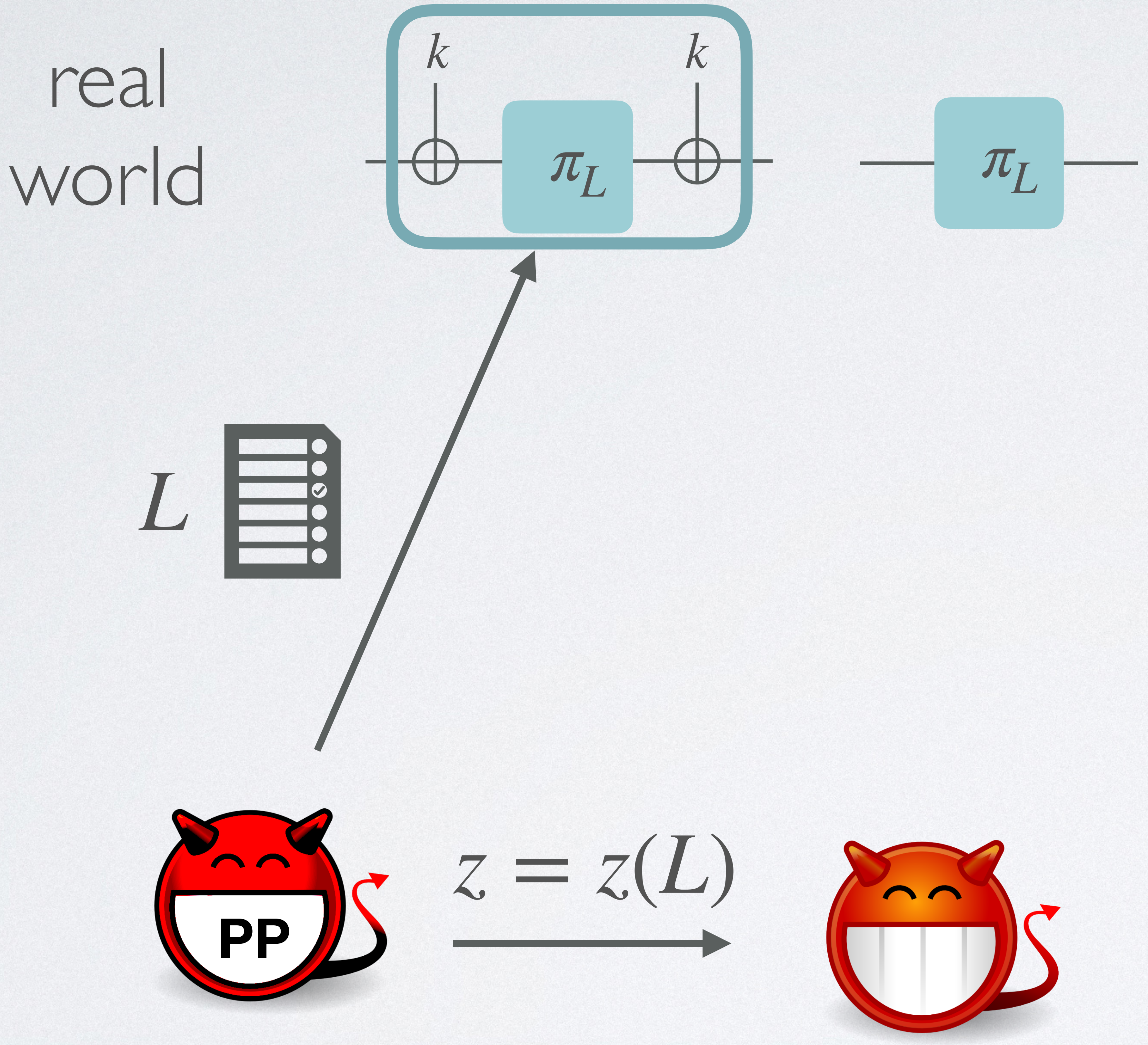
EM Cipher in BF Model



P prefixed coordinates

$\{\tilde{x}_\ell, \tilde{y}_\ell\}_{\ell=1}^{L}$

EM Cipher in BF Model



P prefixed coordinates

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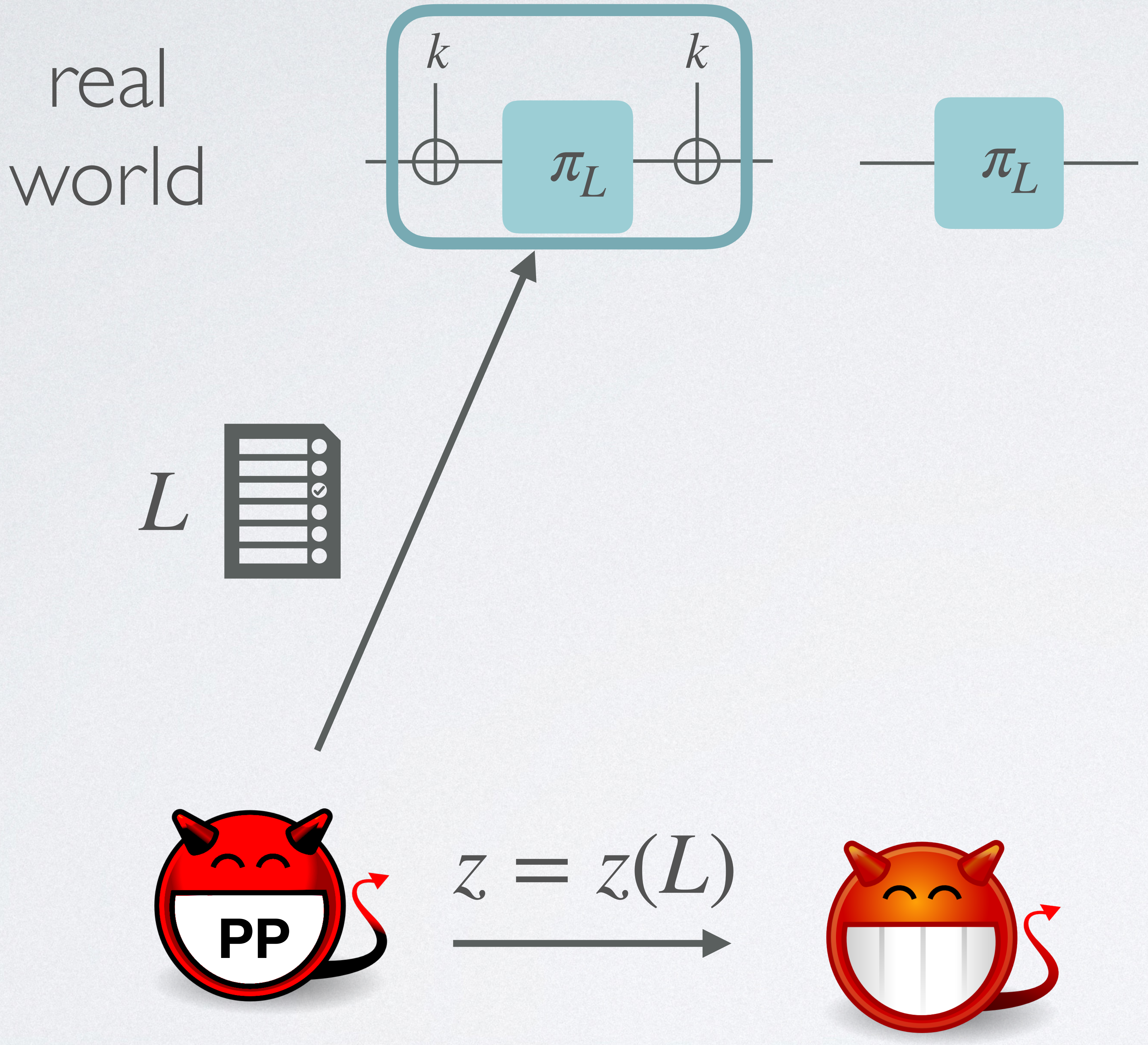
EM Cipher in BF Model



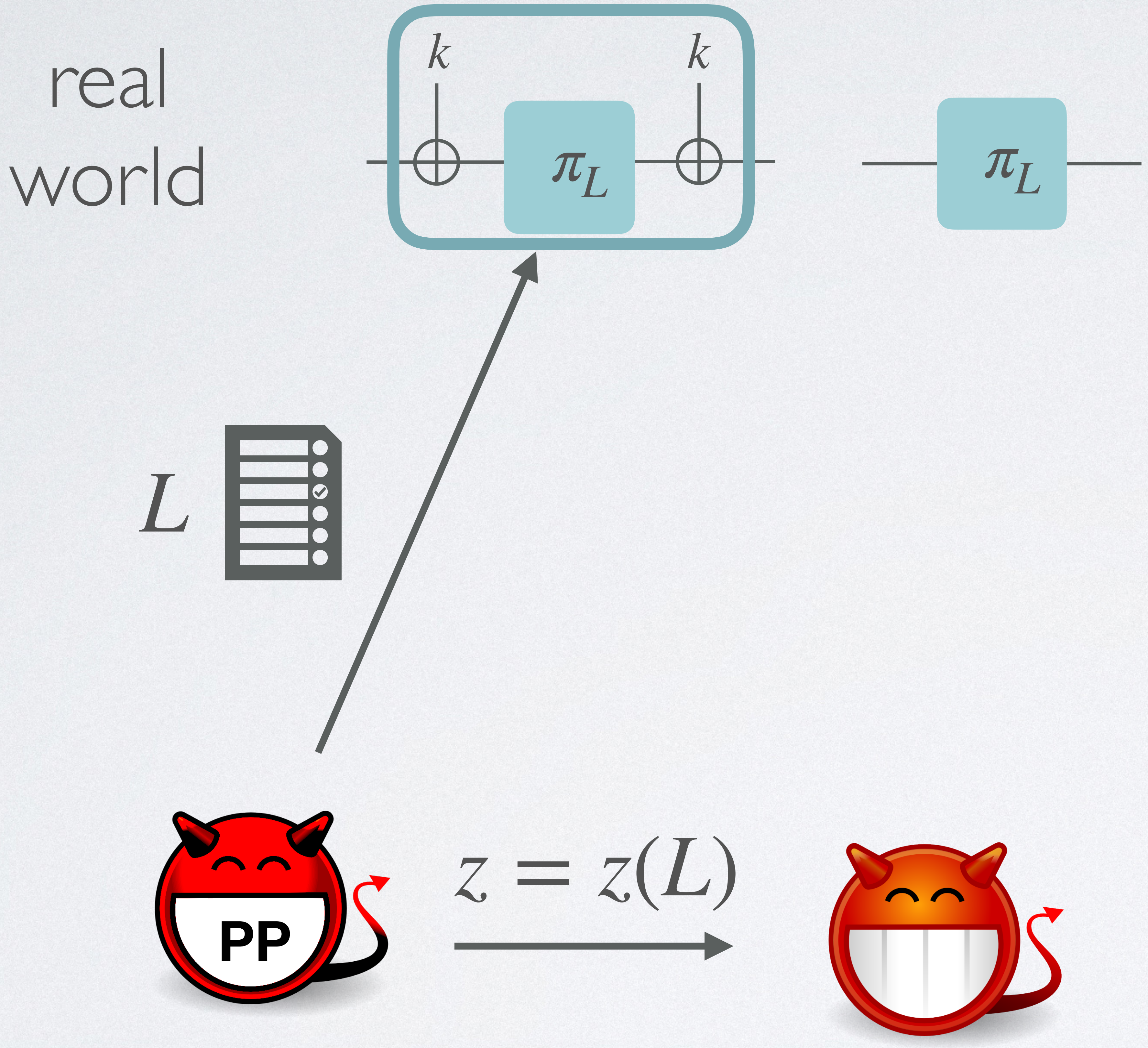
P prefixed coordinates

$\{\tilde{x}_\ell, \tilde{y}_\ell\}_{\ell=1}^{L}$

q construction queries

$\{u_i, v_i\}_{i=1}^{q}$

EM Cipher in BF Model



P prefixed coordinates

$\{\tilde{x}_\ell, \tilde{y}_\ell\}_{\ell=1}^{L}$

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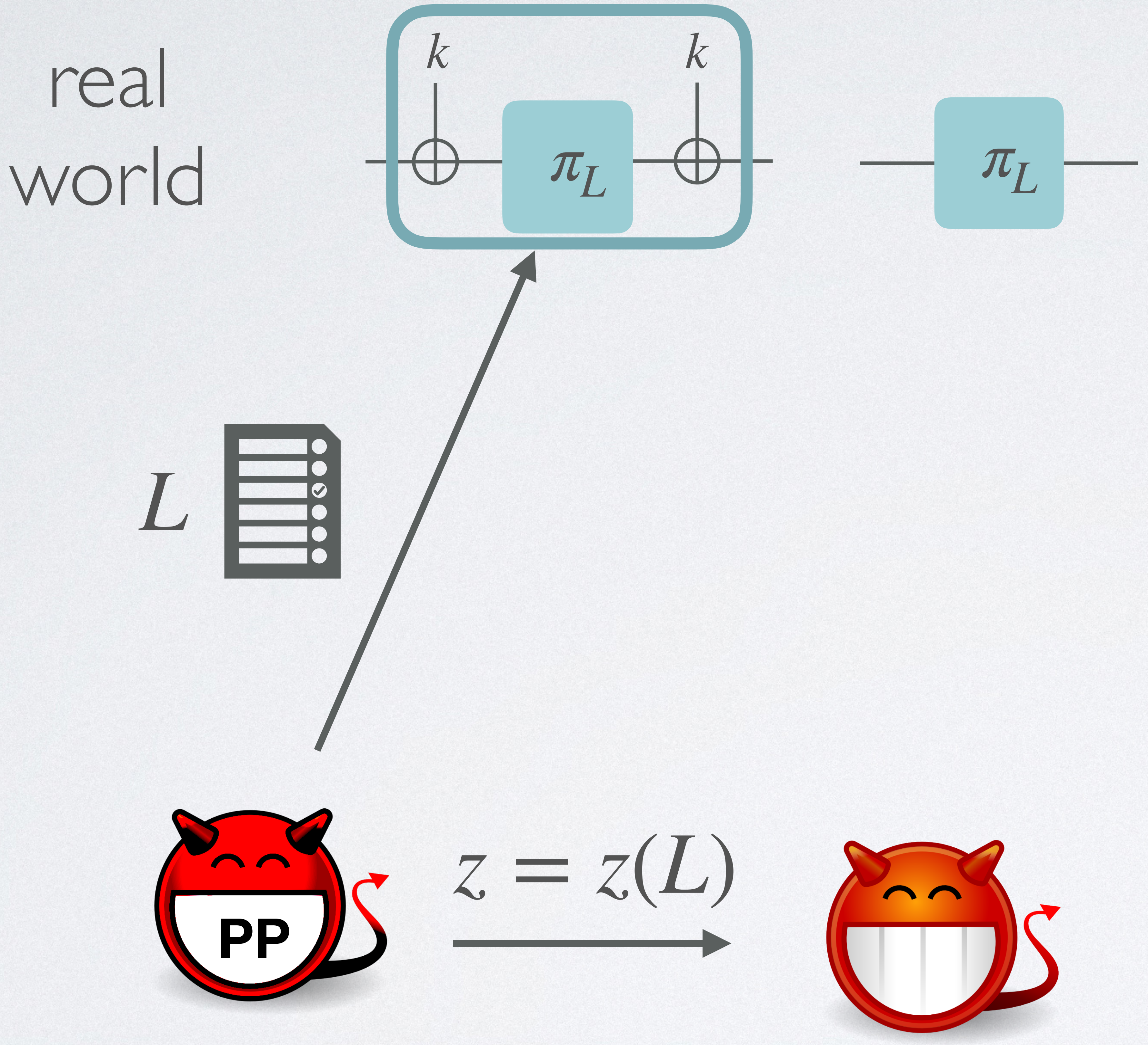
$\{u_i, v_i\}_{i=1}^{q}$

T primitive queries

$\{x_j, y_j\}_{j=1}^{T}$

EM Cipher in BF Model



P prefixed coordinates

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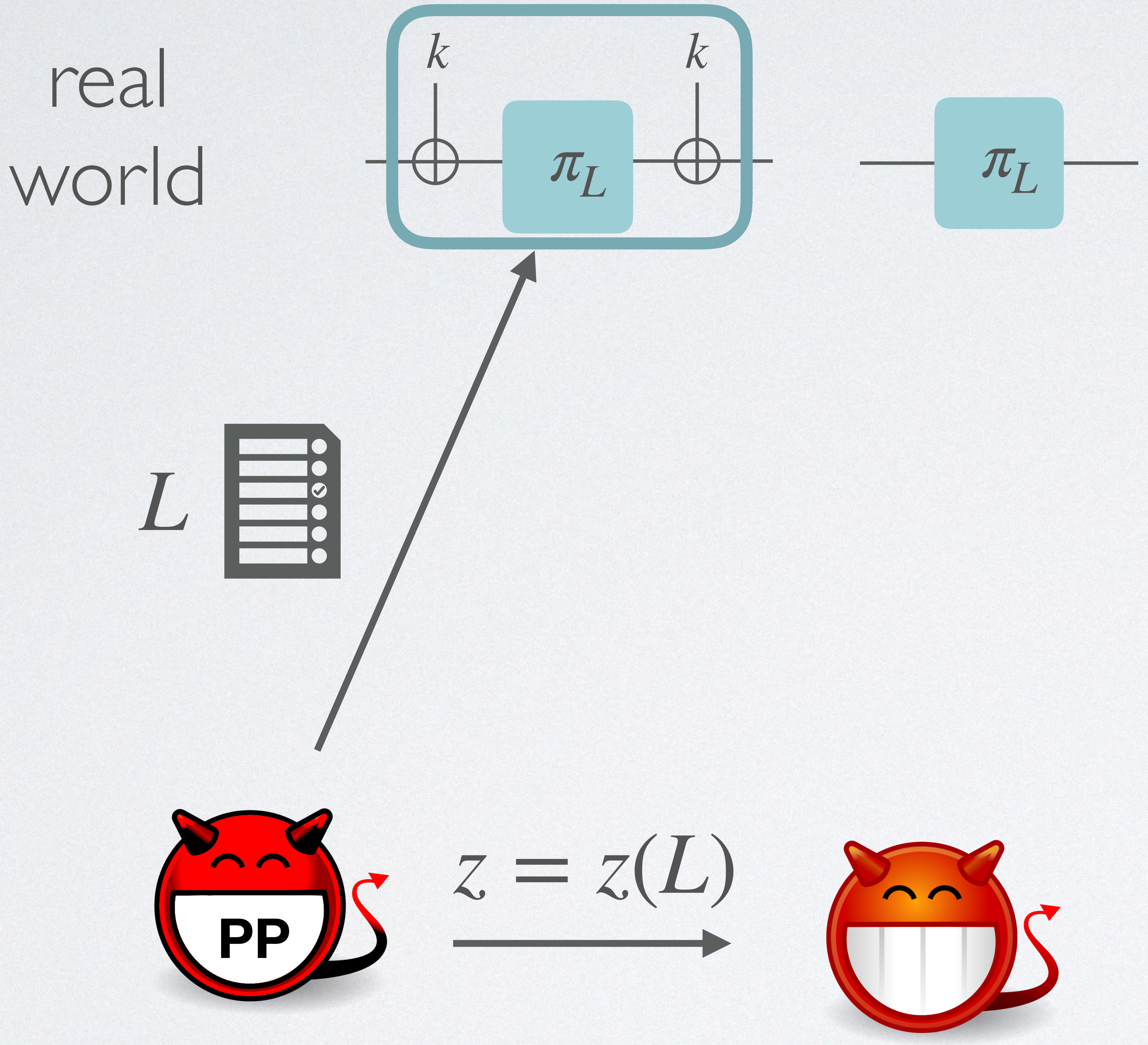
T primitive queries

$\{x_j, y_j\}_{j=1}^{T}$

Event **BAD**:

$\exists i, j : \; u_i \oplus k = x_j \; \vee \; v_i \oplus k = y_j$

EM Cipher in BF Model



P prefixed coordinates

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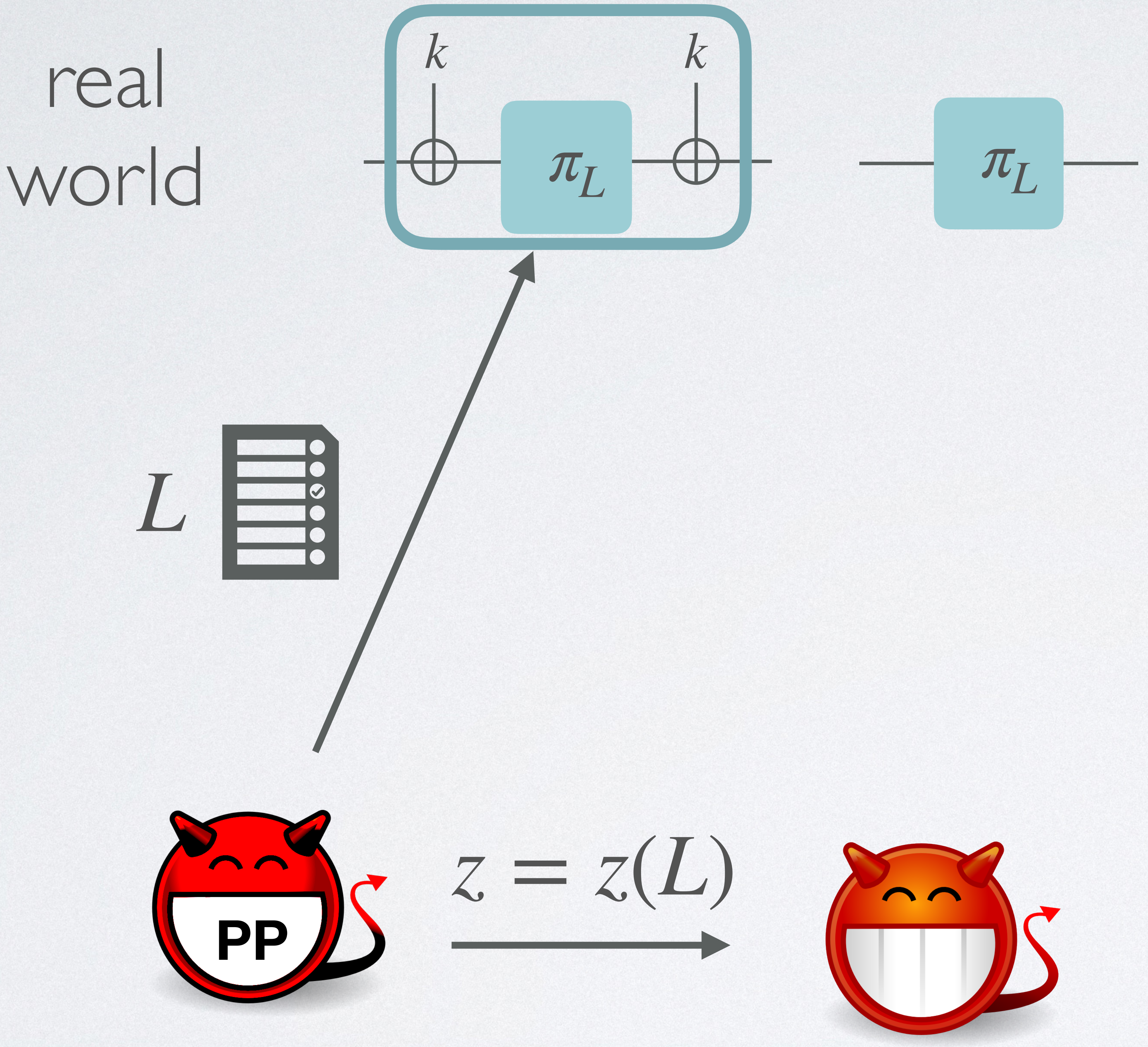
$$\exists i, j : \; u_i \oplus k = \tilde{x}_j \;\vee\; v_i \oplus k = \tilde{y}_j$$

EM Cipher in BF Model



P prefixed coordinates

$$\{\tilde{x}_\ell, \tilde{y}_\ell\}_{\ell=1}^{L}$$

q construction queries

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T primitive queries

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Event **BAD**:

$$\exists i, j: \; u_i \oplus k = x_j \; \vee \; v_i \oplus k = y_j$$

$$\exists i, j: \; u_i \oplus k = \tilde{x}_j \; \vee \; v_i \oplus k = \tilde{y}_j$$

$$\text{P[BAD]} \leq \frac{qT}{2^n} + \frac{qP}{2^n}$$

EM Cipher in BF Model

Bound in BF-RPM: $$\frac{qT}{2^n} + \frac{qP}{2^n}$$

EM Cipher in BF Model

Bound in BF-RPM: $$\frac{qT}{2^n} + \frac{qP}{2^n}$$

Bound in AI-RPM: $$\frac{qT}{2^n} + \frac{qP}{2^n} + \frac{ST}{P}$$

EM Cipher in BF Model

Bound in BF-RPM: $$\frac{qT}{2^n} + \frac{qP}{2^n}$$

Bound in AI-RPM: $$\frac{qT}{2^n} + \boxed{\frac{qP}{2^n} + \frac{ST}{P}}$$

EM Cipher in BF Model

Bound in BF-RPM: $\frac{qT}{2^n} + \frac{qP}{2^n}$

Bound in AI-RPM: $\frac{qT}{2^n} + \boxed{\frac{qP}{2^n} + \frac{ST}{P}} \longrightarrow P \approx \sqrt{STN/q}$

EM Cipher in BF Model

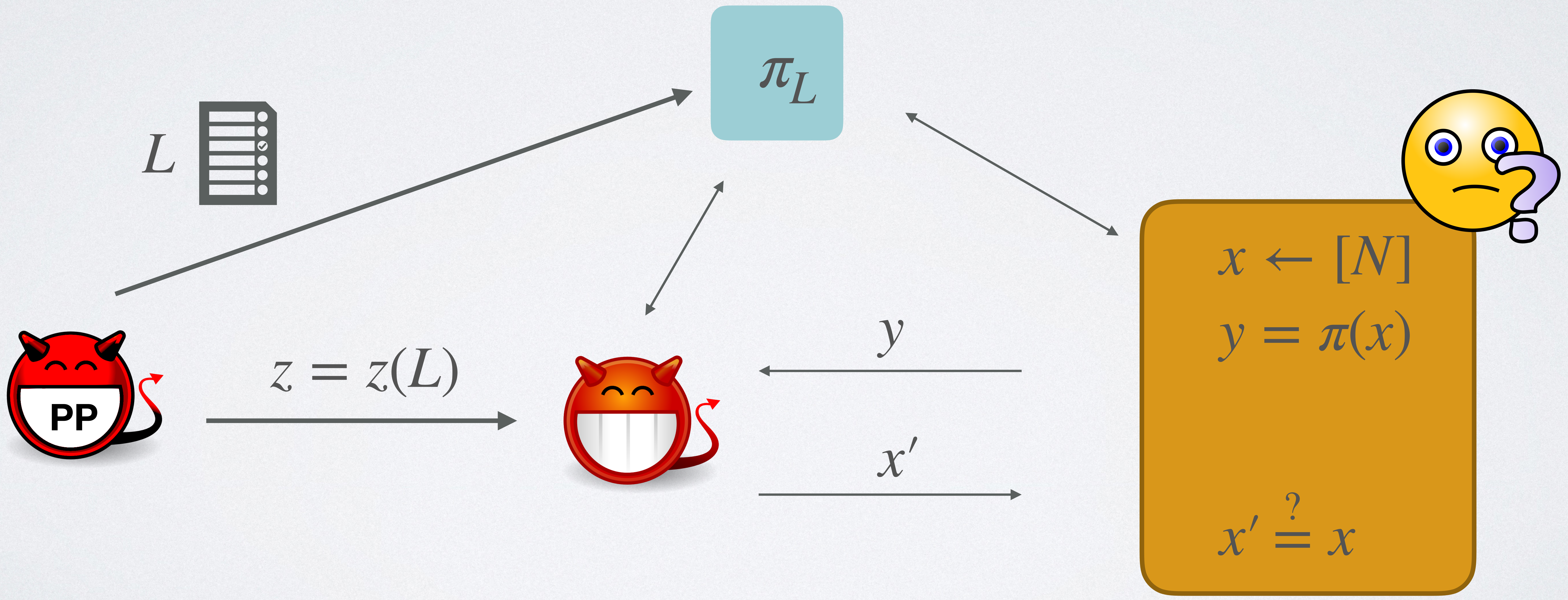
Bound in BF-RPM: $\frac{qT}{2^n} + \frac{qP}{2^n}$

Bound in AI-RPM: $\frac{qT}{2^n} + \boxed{\frac{qP}{2^n} + \frac{ST}{P}} \longrightarrow P \approx \sqrt{STN/q}$

$$= \frac{qT}{2^n} + \sqrt{\frac{STq}{N}}$$

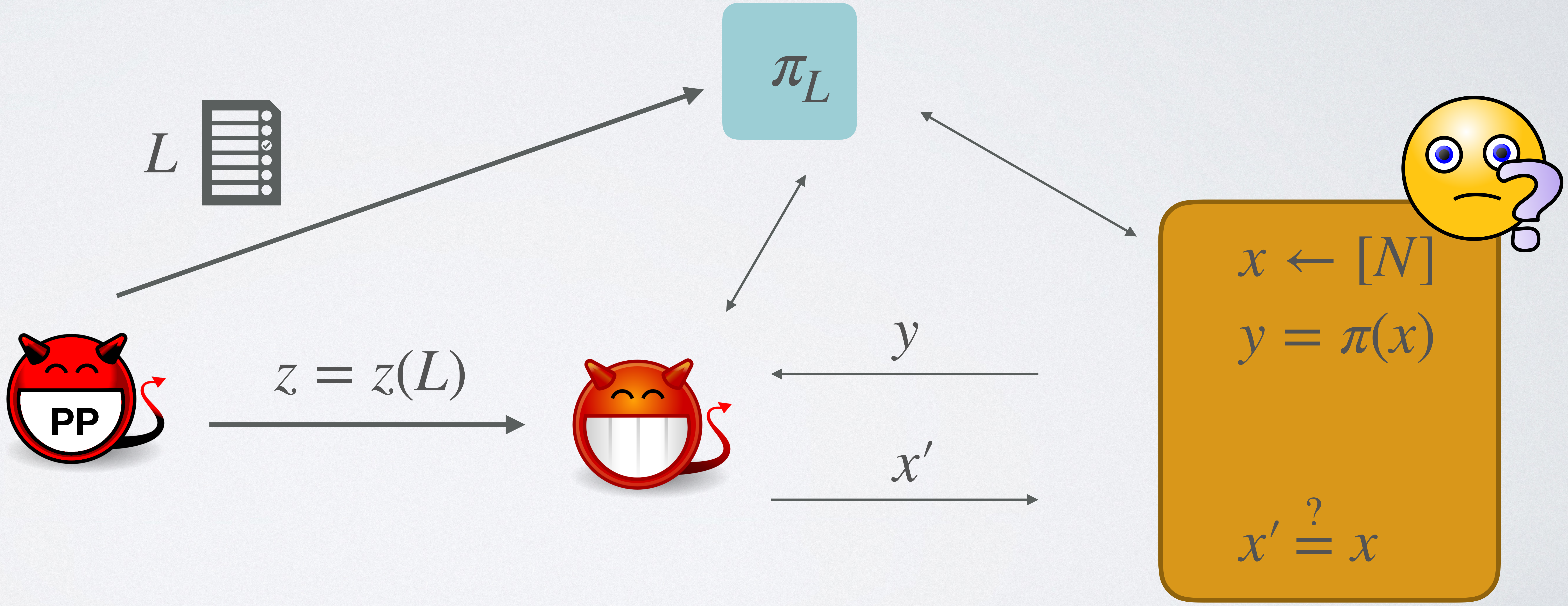
OWP in BF Model

Random permutation $\pi : [N] \to [N]$



OWP in BF Model

Prefixed random permutation $\pi_L : [N] \to [N]$



OWP in BF Model

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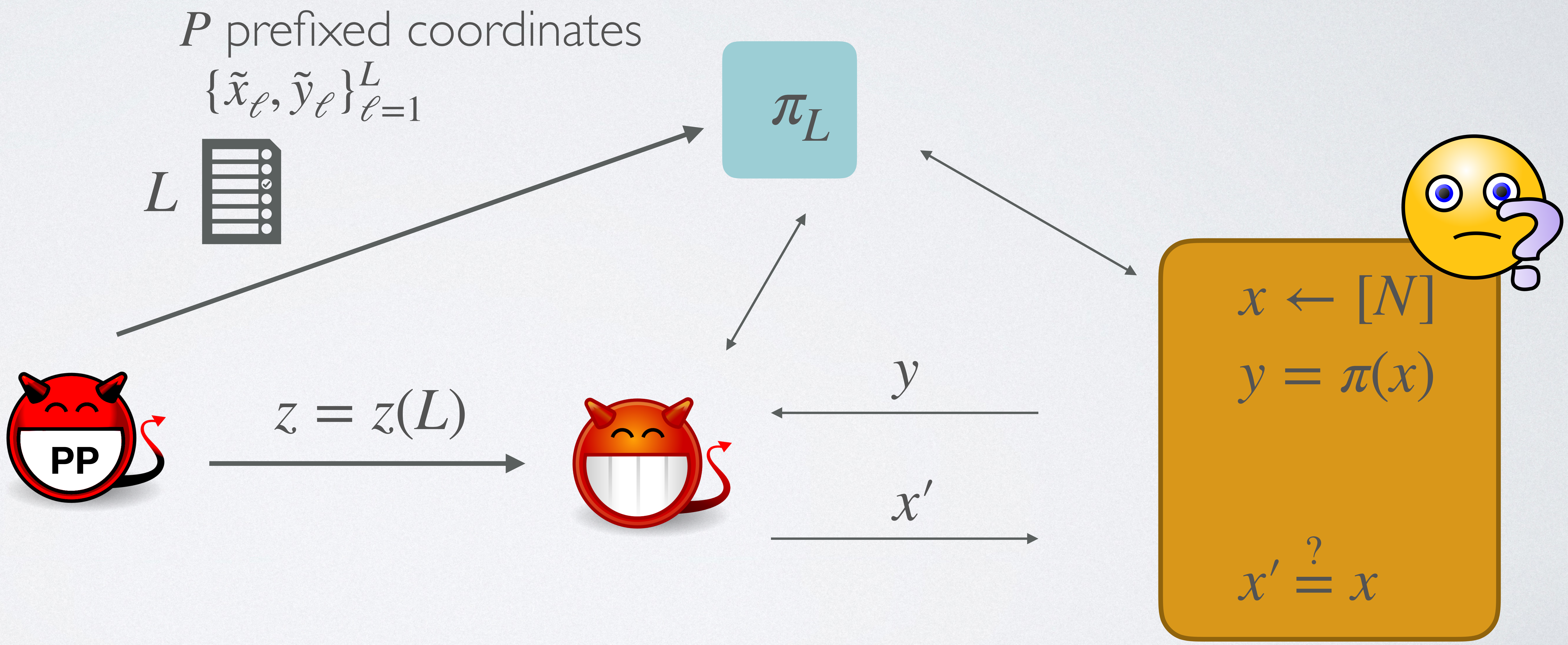

OWP in BF Model

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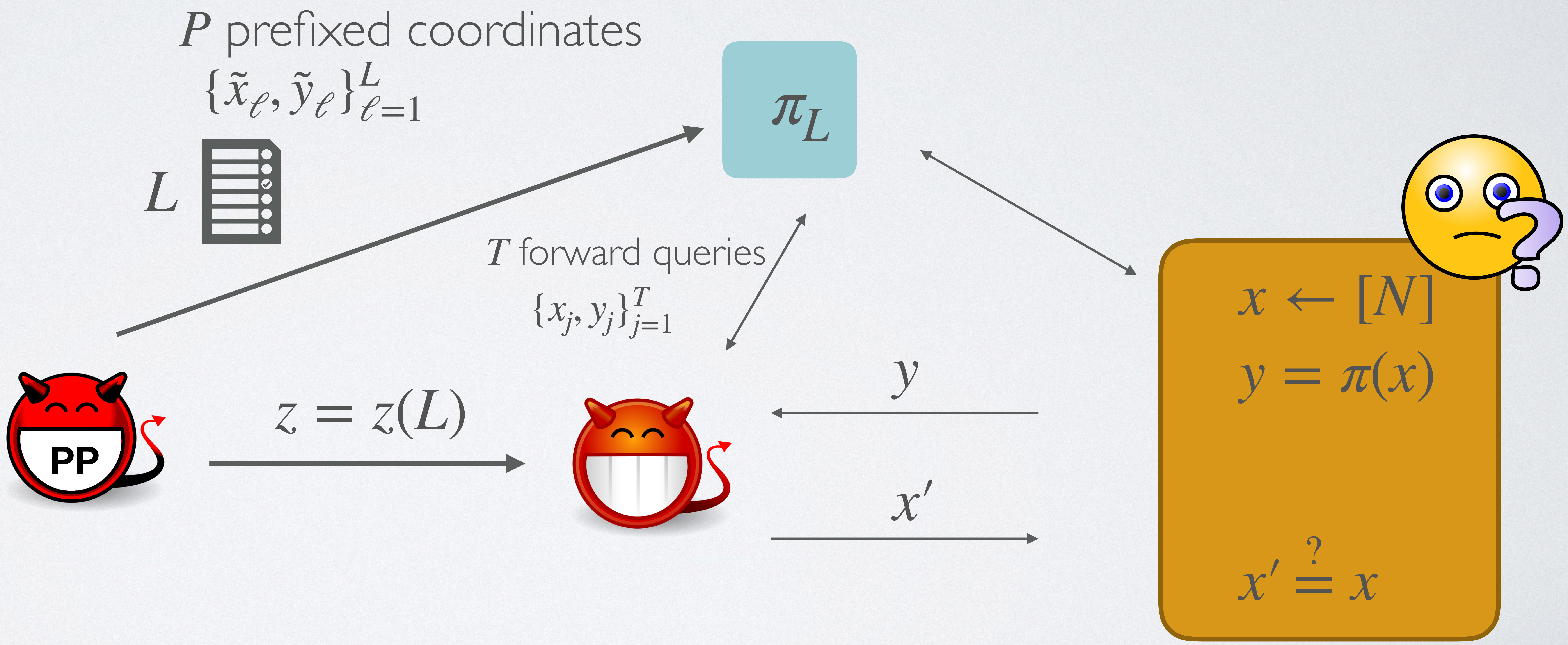
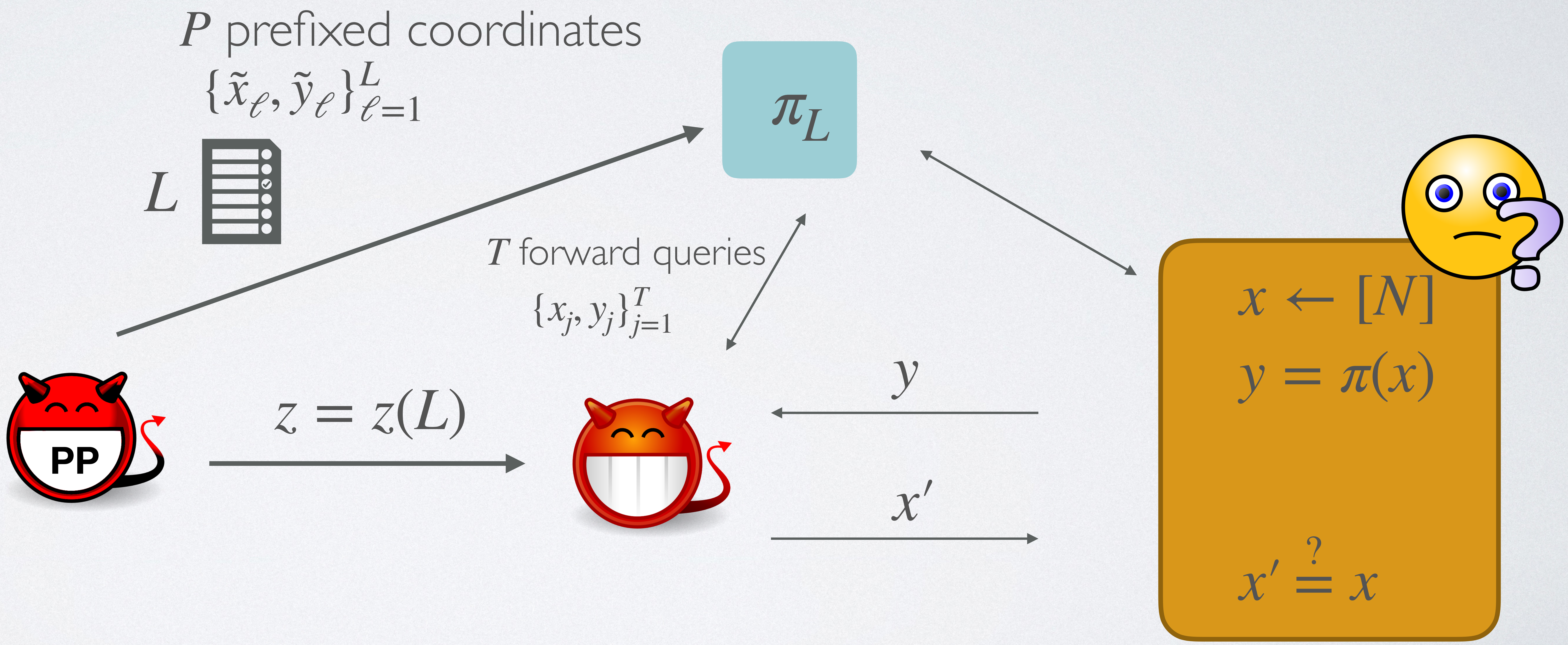


OWP in BF Model

Prefixed random permutation $\pi_L : [N] \to [N]$

Event BAD:



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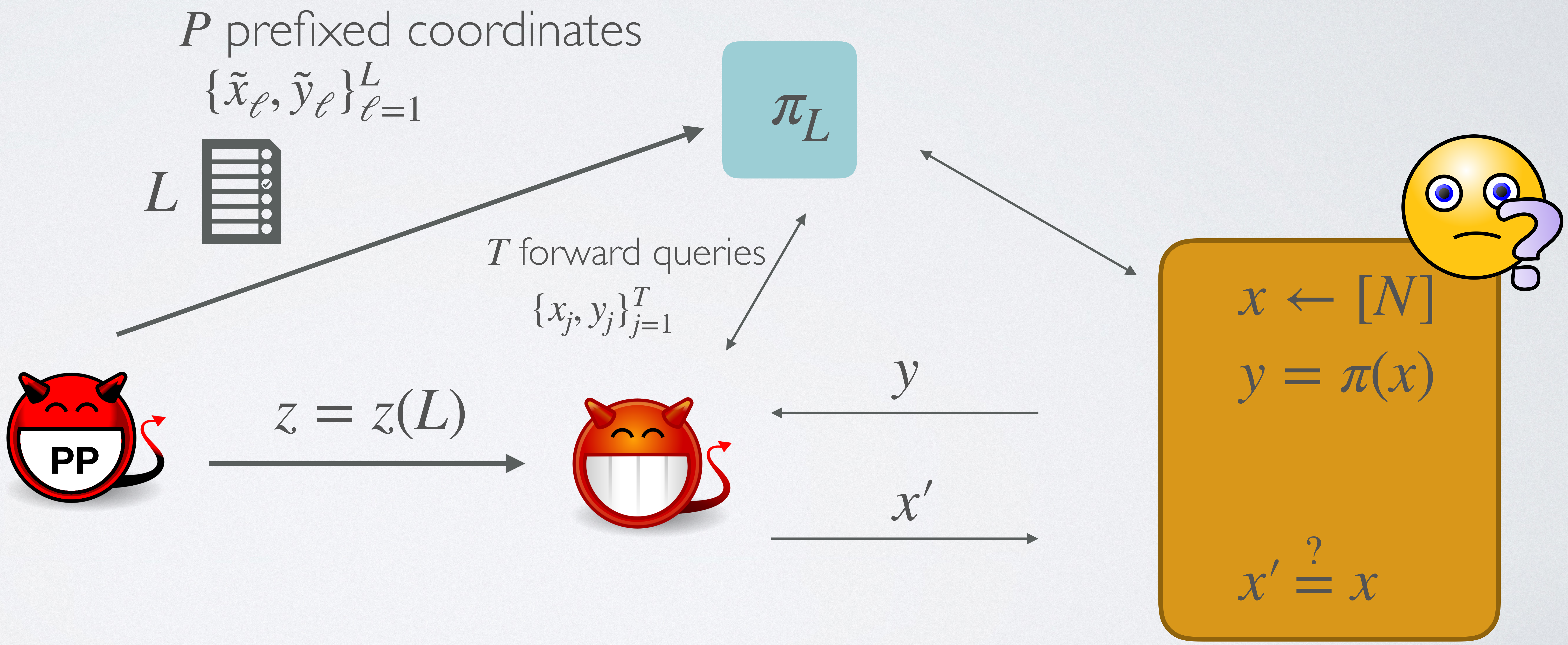
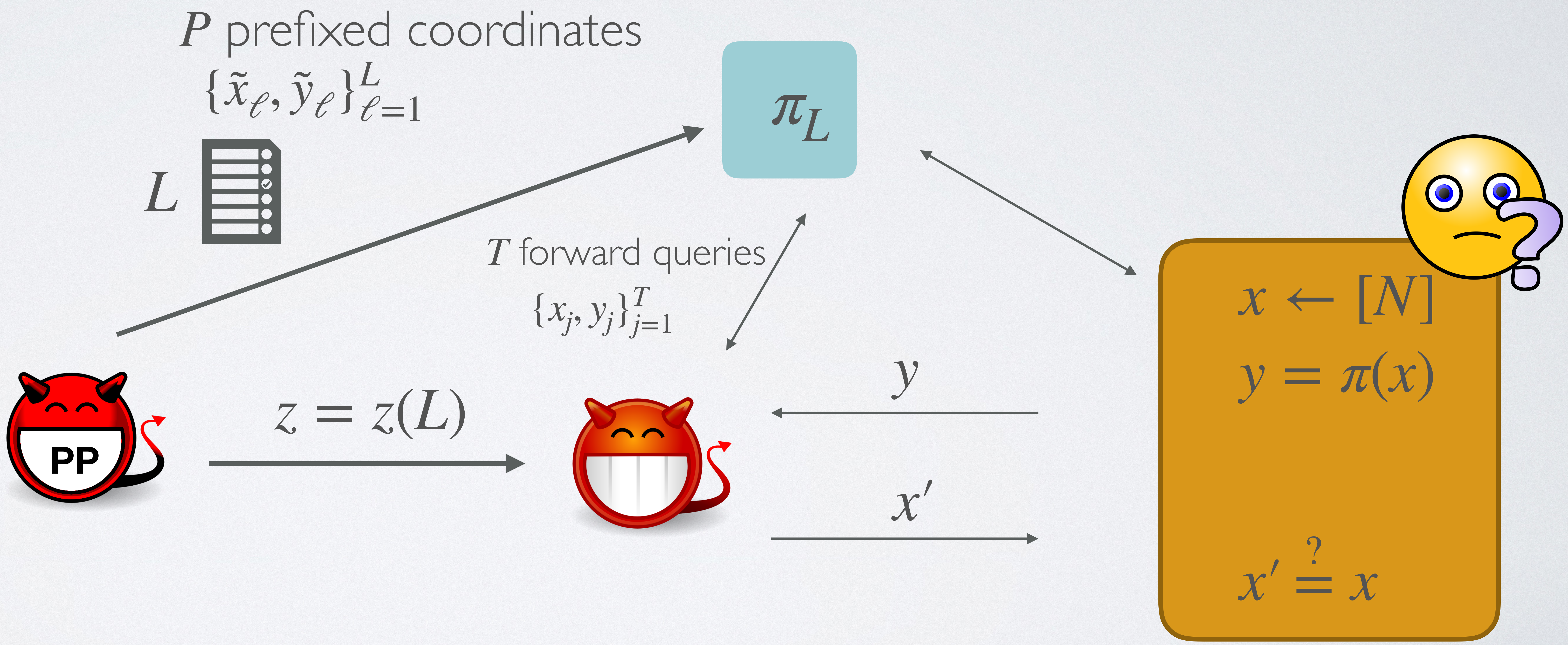


Event **BAD**:

$\exists j : \quad x_j = x$

OWP in BF Model

Prefixed random permutation $\pi_L : [N] \to [N]$



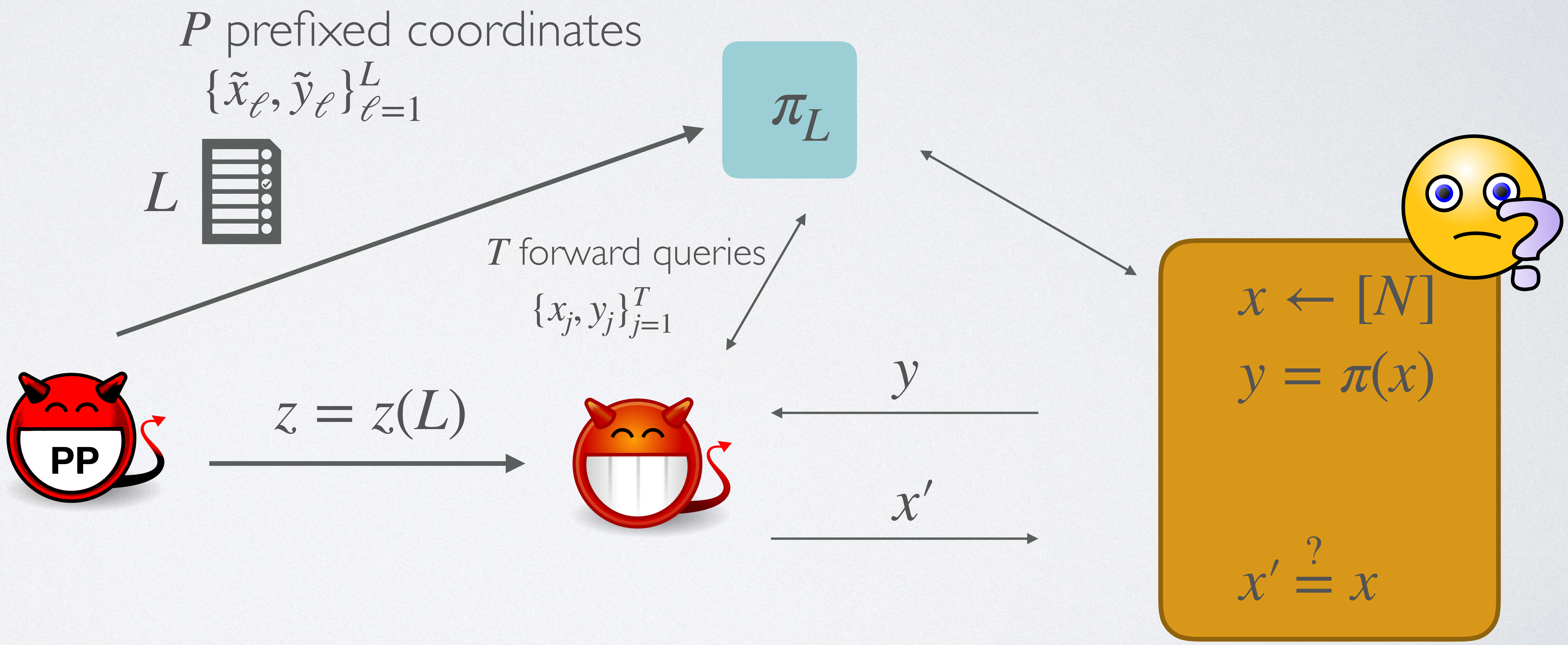
Event **BAD**:

$\exists j : \quad x_j = x$

$\exists \ell : \quad \tilde{x}_\ell = x$

OWP in BF Model

Prefixed random permutation $\pi_L : [N] \to [N]$



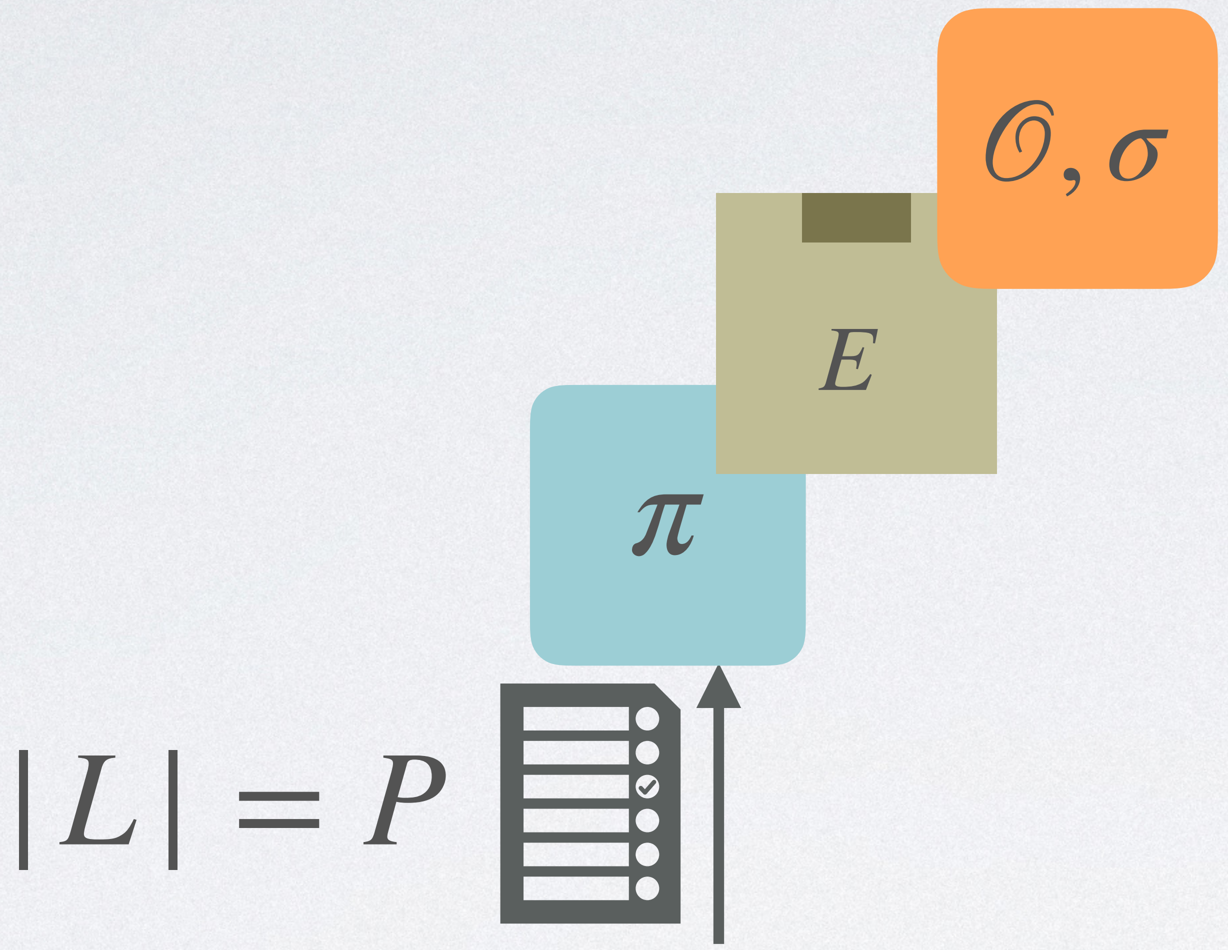
Event BAD:

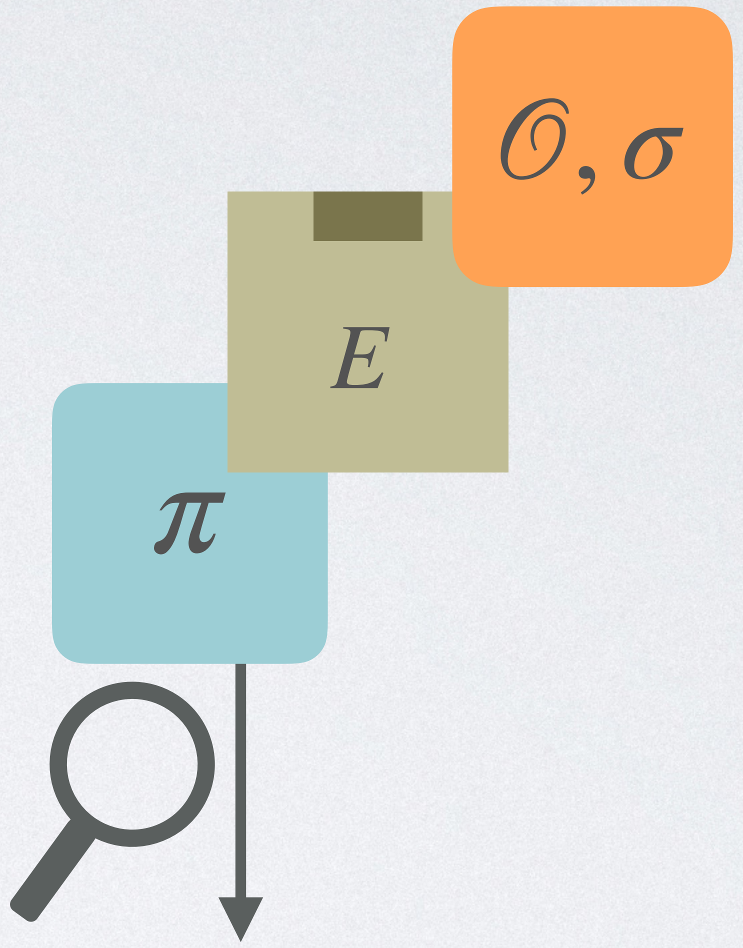
$\exists j : \quad x_j = x$

$\exists \ell : \quad \tilde{x}_\ell = x$

$$\text{P[BAD]} \leq \frac{T}{N} + \frac{P}{N}$$

Bit-Fixing to Auxiliary Input





Theorem: (S, T, ε)-secure $\Longrightarrow$ (S, T, ε')-secure

where $\varepsilon' \leq 2\varepsilon$
and $P \approx ST$

For **unpredictability** applications

OWP in BF Model

Bound in BF-GGM: $\frac{T}{N} + \frac{P}{N} \longrightarrow P \approx ST$

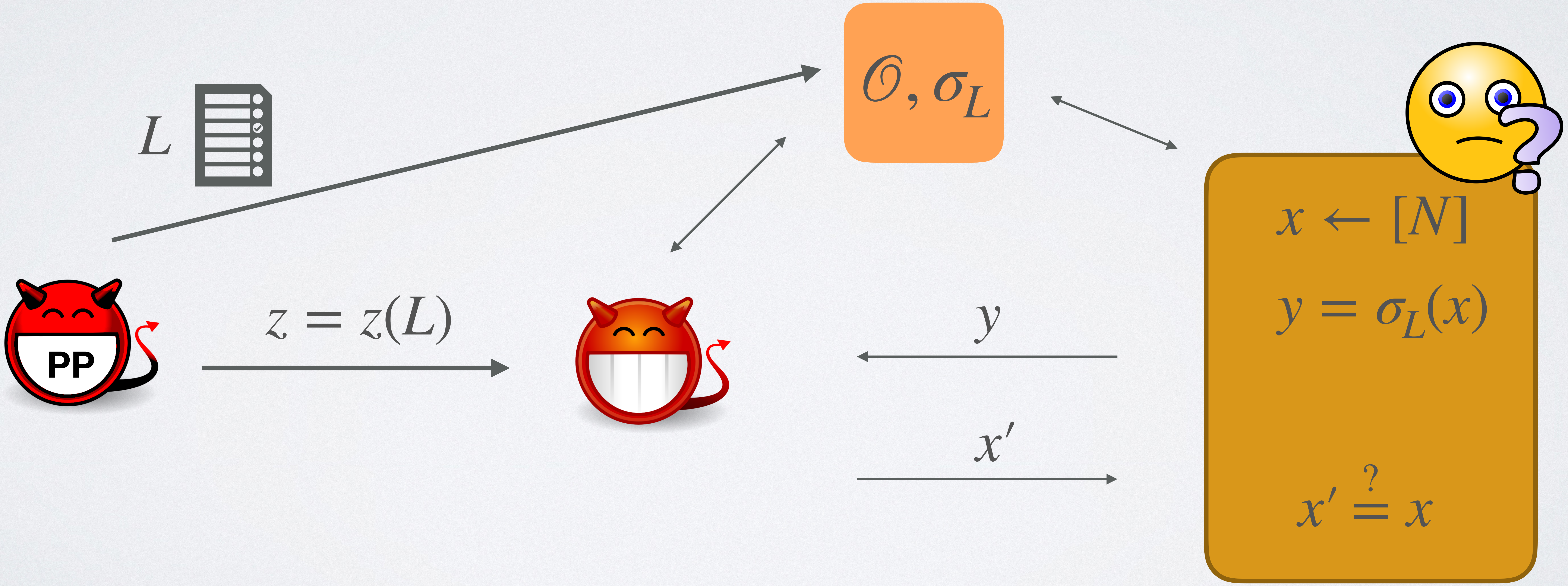
Bound in AI-GGM: $\frac{T}{N} + \frac{ST}{N}$

Bound in AI-GGM (compression proof): $\frac{ST}{N}$

De, Trevisan, Tulsiani '10

Discrete Logarithms in BF Model

Random injection $\sigma : [N] \to [M]$



Discrete Logarithms in BF Model

Prefixed random injection $\sigma_L : [N] \to [M]$

P prefixed coordinates

$\{\tilde{x}_\ell, \tilde{y}_\ell\}_{\ell=1}^{L}$

By making queries to $\mathcal{O}$:

$\mathcal{A}$ "generates" degree-1 polynomials in X

Event BAD: two polynomials collide at $X = x$

Discrete Logarithms in BF Model

Prefixed random injection $\sigma_L : [N] \to [M]$

P prefixed coordinates

$\{\tilde{x}_\ell, \tilde{y}_\ell\}_{\ell=1}^{L}$

By making queries to $\mathcal{O}$:

$\mathcal{A}$ "generates" degree-1 polynomials in X

Event BAD: two polynomials collide at $X = x$

$\exists i, j$: some polynomial evaluates to $\tilde{x}_j$ at $X = x$

Discrete Logarithms in BF Model

Prefixed random injection $\sigma_L : [N] \to [M]$

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By making queries to $\mathcal{O}$:

$\mathcal{A}$ "generates" degree-1 polynomials in X

Event BAD: two polynomials collide at $X = x$

$\exists i, j$: some polynomial evaluates to $\tilde{x}_j$ at $X = x$

$$\text{P[BAD]} \leq \frac{T^2}{N} + \frac{PT}{N}$$

Discrete Logarithms in BF Model

Bound in BF-GGM: $\frac{T^2}{N} + \frac{PT}{N} \longrightarrow P \approx ST$

Bound in AI-GGM: $\frac{T^2}{N} + \frac{ST^2}{N}$

Discrete Logarithms in BF Model

Bound in BF-GGM: $\frac{T^2}{N} + \frac{PT}{N} \longrightarrow P \approx ST$

Bound in AI-GGM: $\frac{T^2}{N} + \frac{ST^2}{N}$

Bound in AI-GGM (compression proof): $\frac{ST^2}{N}$

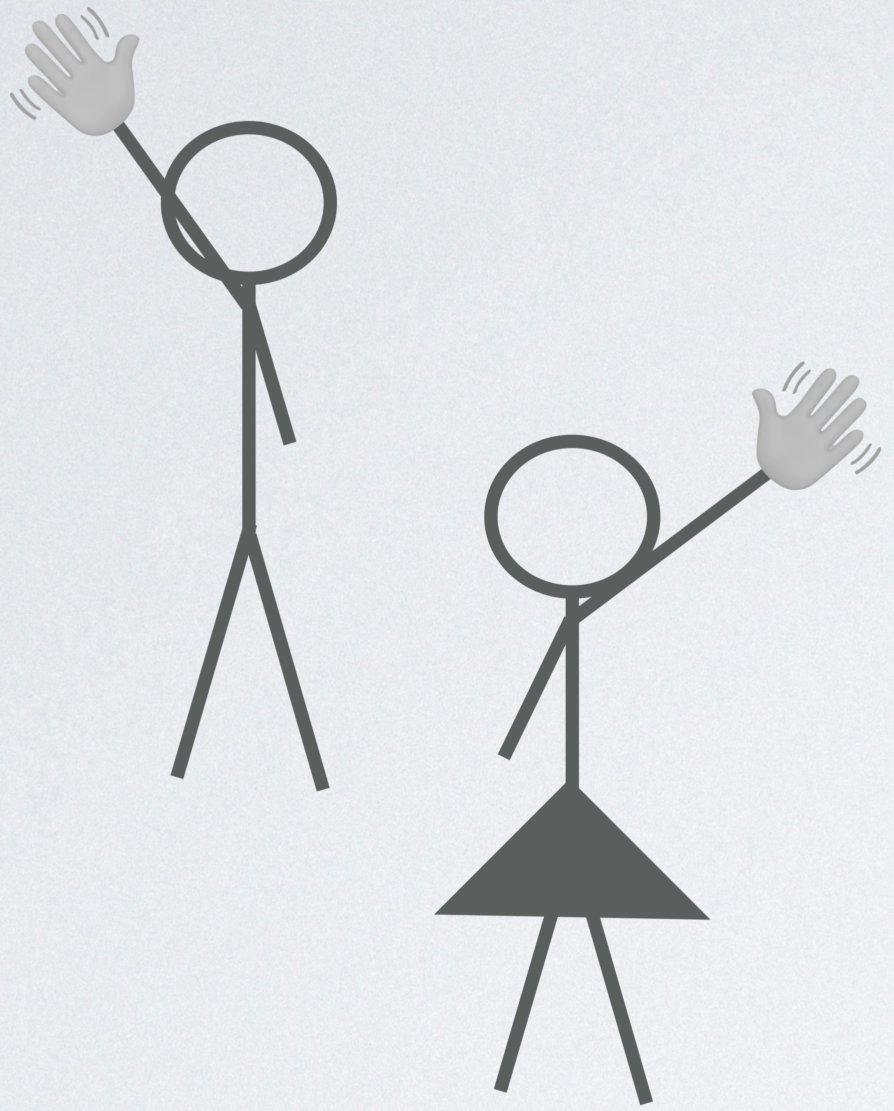
Corrigan-Gibbs, Kogan '18

Summary of Bounds

- **Basic:** OWP, Even-Mansour, ideal cipher as block cipher, Davies-Meyer as a PRF, CRHF based on Davies-Meyer
- **Symmetric:** Merkle-Damgard with Davies-Meyer and sponges (as CRHF, PRF, MAC), …
- **Generic group model:** DL, CDH, DDH, OM-DL, KEA, …
- **Computational:** Full-domain permutation encryption

Some Future Work

- Close gaps:
  - EM: bound: $\sqrt{\frac{ST^2}{N}}$ attack: $\frac{ST^2}{N}$
  - DDH: bound: $\sqrt{\frac{ST^2}{N}}$ attack: $\frac{ST^2}{N}$, $\sqrt{\frac{S}{N}}$
- Tight bounds for other primitives (e.g., KAC)



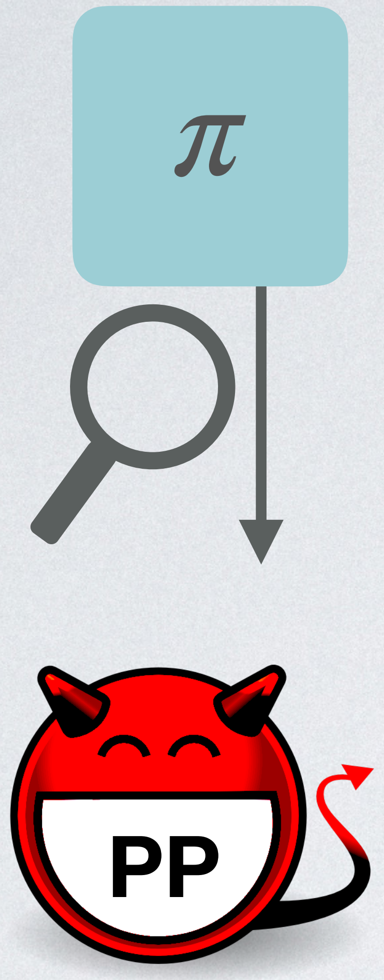
Thank you!

eprint.iacr.org/2018/226

Proof of Presampling

Göös, Lovett, Meka '16

C, Dodis, Guo, Steinberger '18



Proof of Presampling

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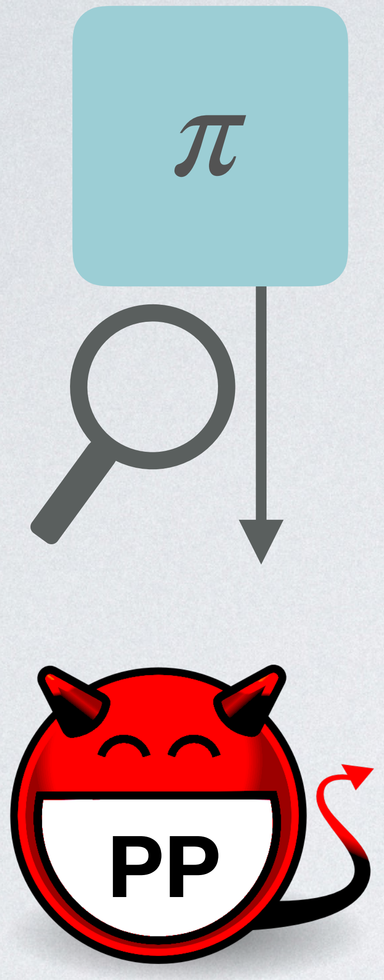

Before leakage: $\mathrm{H}_\infty(\pi) = \log N!$

Proof of Presampling

Göös, Lovett, Meka '16

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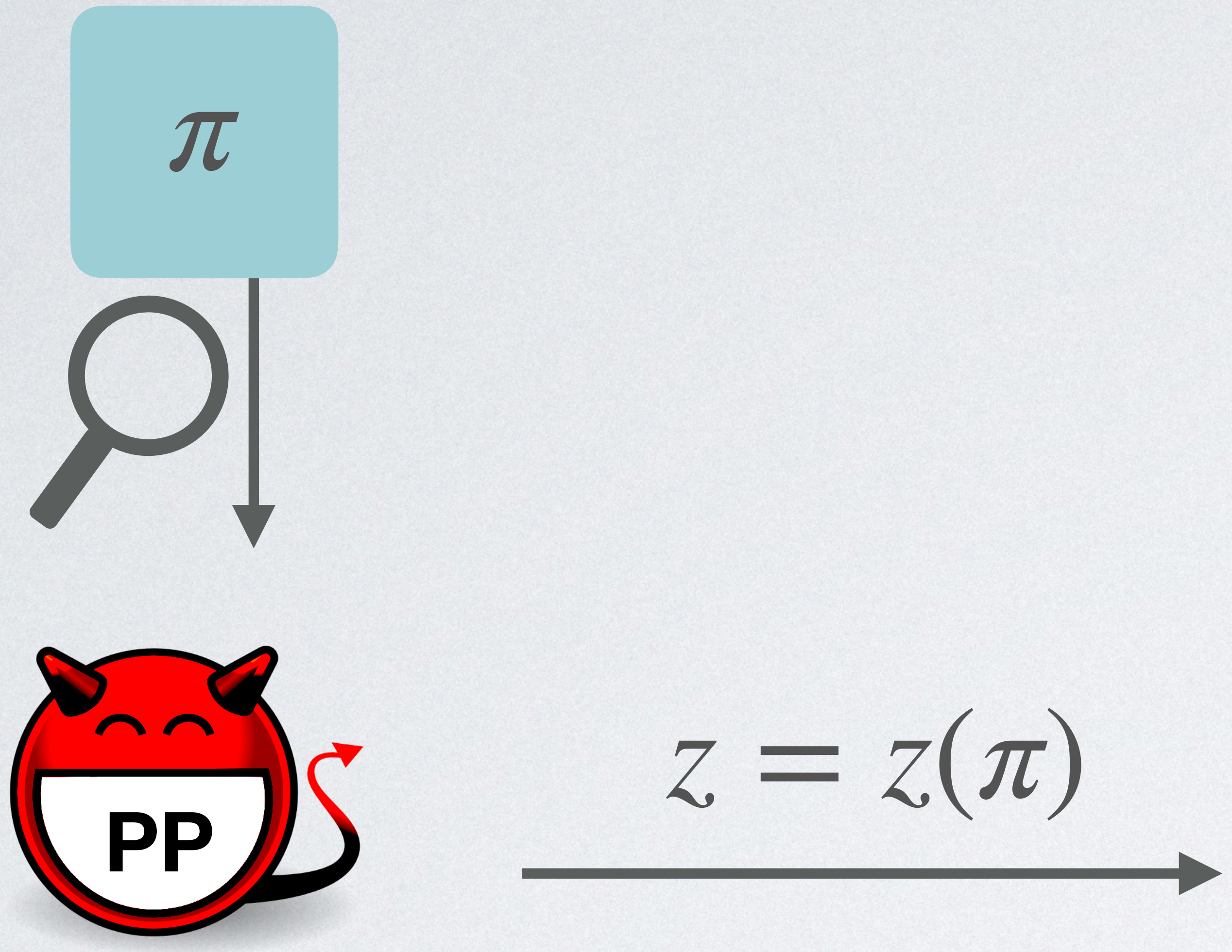

Before leakage: $\mathrm{H}_\infty(\pi) = \log N!$

After leakage: $\mathrm{H}_\infty(\pi \mid z) = \log N! - S$

Proof of Presampling

Göös, Lovett, Meka ‘16

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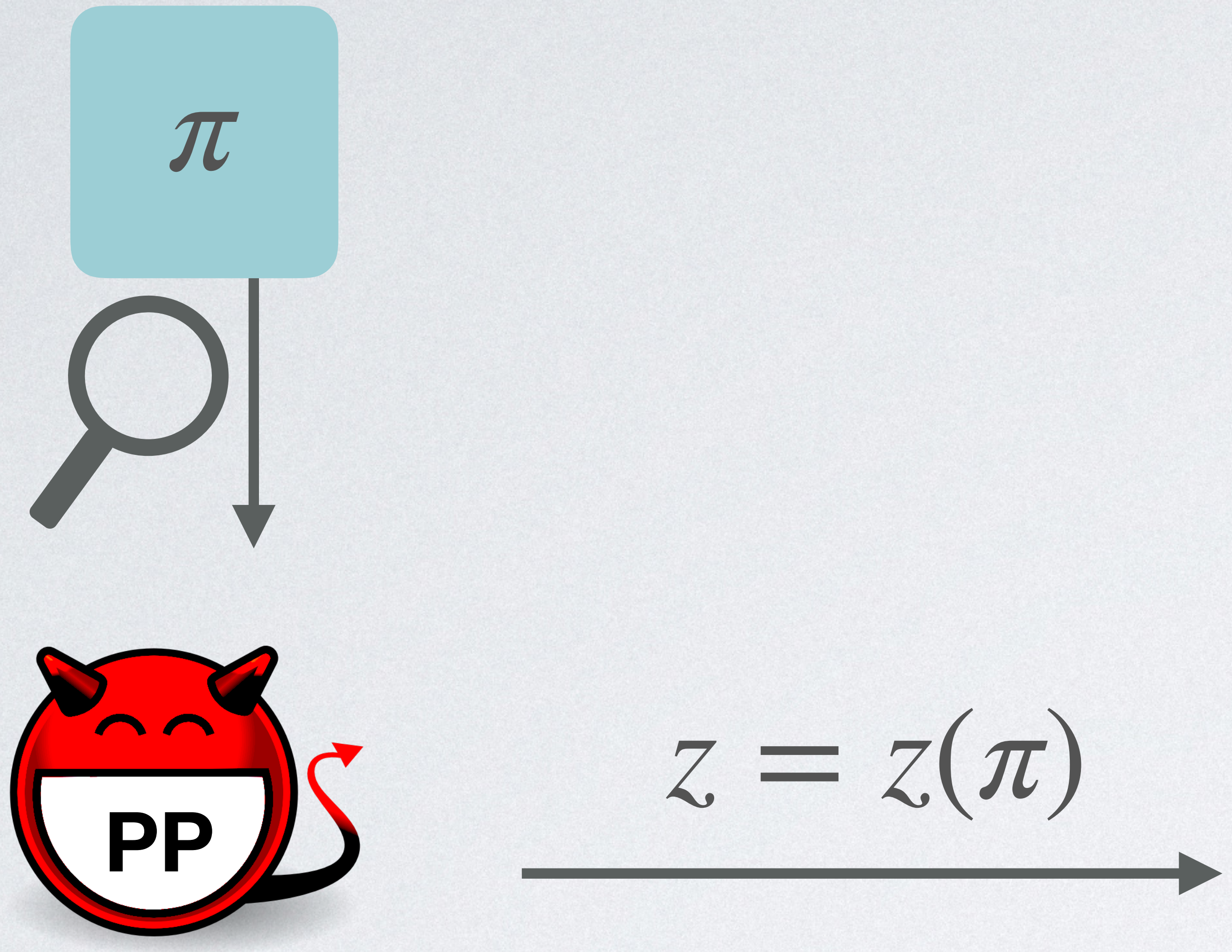


Before leakage: $\mathrm{H}_\infty(\pi) = \log N!$

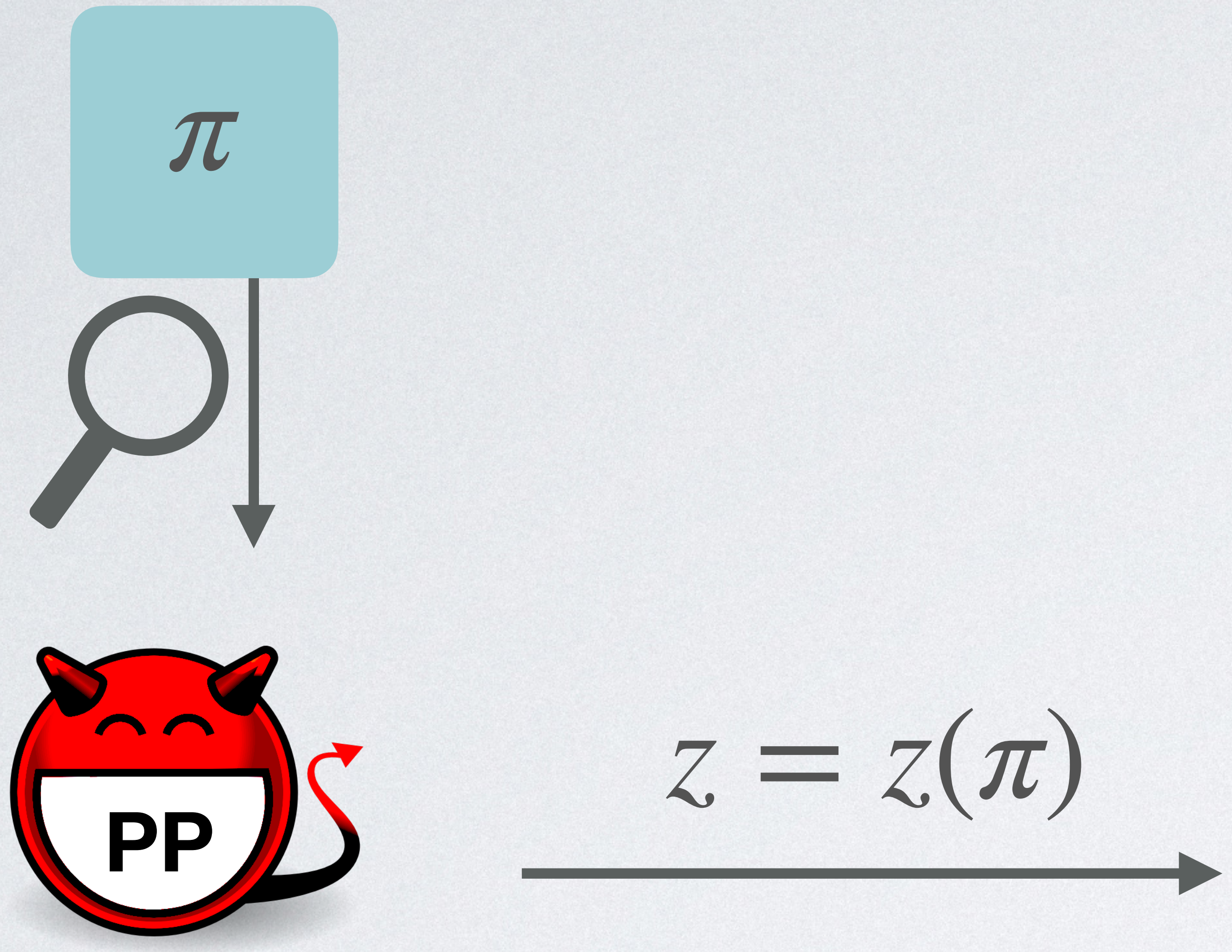
After leakage: $\mathrm{H}_\infty(\pi \,|\, z) = \log N! - S$

1. Decompose $\pi' := \pi \,|\, z$ into dense sources

Göös, Lovett, Meka ‘16

C, Dodis, Guo, Steinberger ‘18

Proof of Presampling



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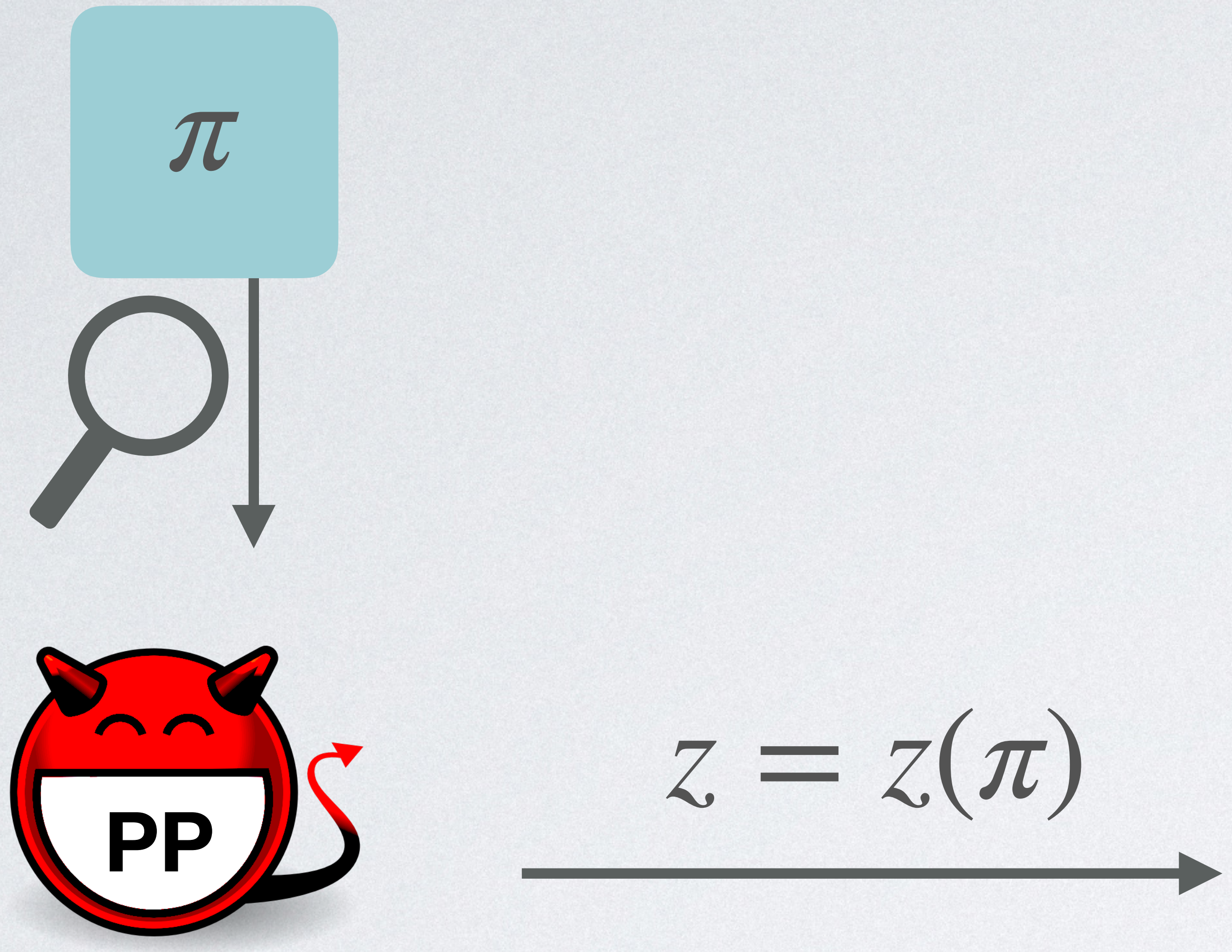
1. Decompose $\pi' := \pi \,|\, z$ into dense sources

   (a) Fixed on P coordinates $L \subseteq [N]$

Göös, Lovett, Meka '16

C, Dodis, Guo, Steinberger '18

Proof of Presampling



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After leakage: $\mathrm{H}_\infty(\pi \,|\, z) = \log N! - S$

1. Decompose $\pi' := \pi \,|\, z$ into dense sources

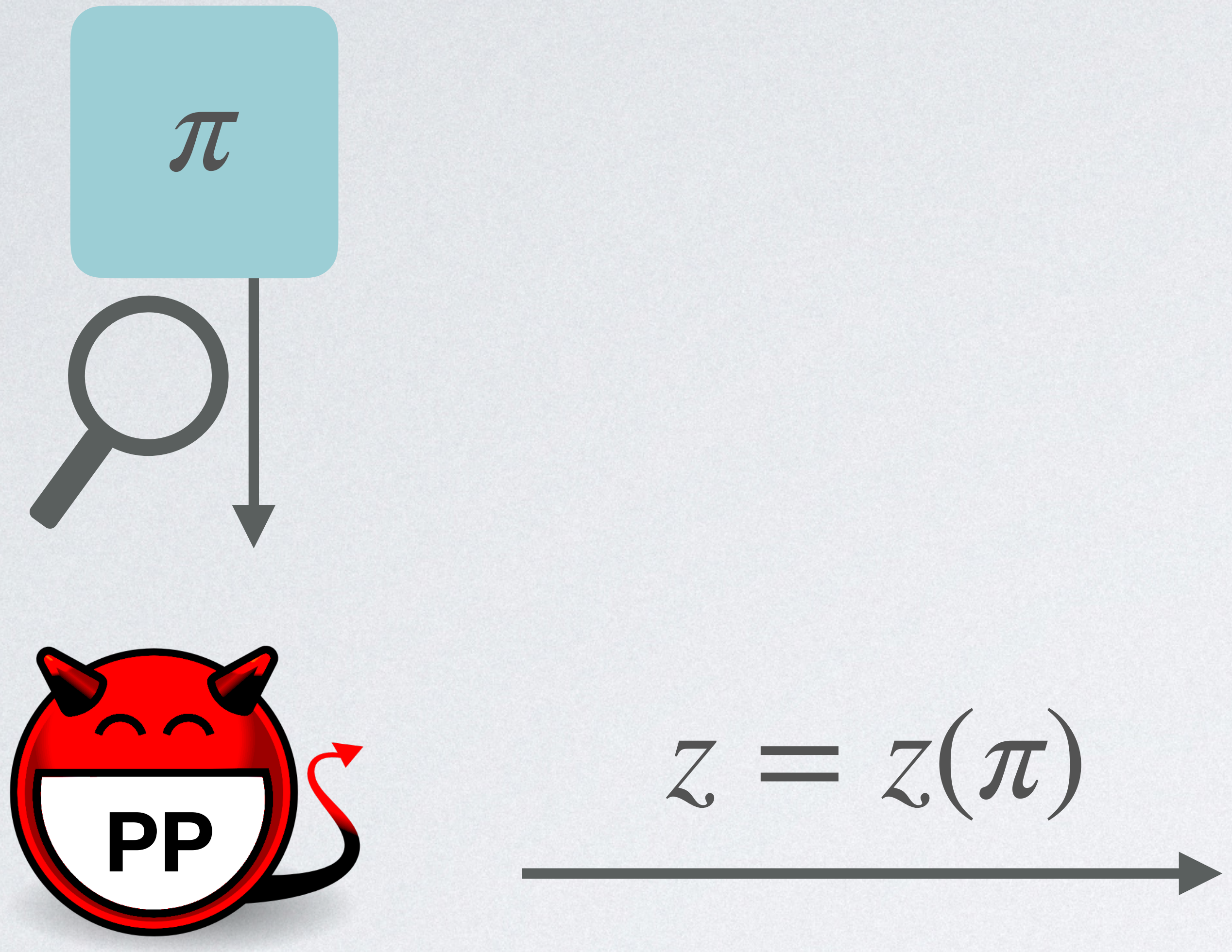
(a) Fixed on P coordinates $L \subseteq [N]$

(b) $\forall\ Q \subseteq [N] \backslash L$:

Proof of Presampling

Göös, Lovett, Meka '16

C, Dodis, Guo, Steinberger '18



Before leakage: $\mathrm{H}_\infty(\pi) = \log N!$

After leakage: $\mathrm{H}_\infty(\pi \mid z) = \log N! - S$

1. Decompose $\pi' := \pi \mid z$ into dense sources

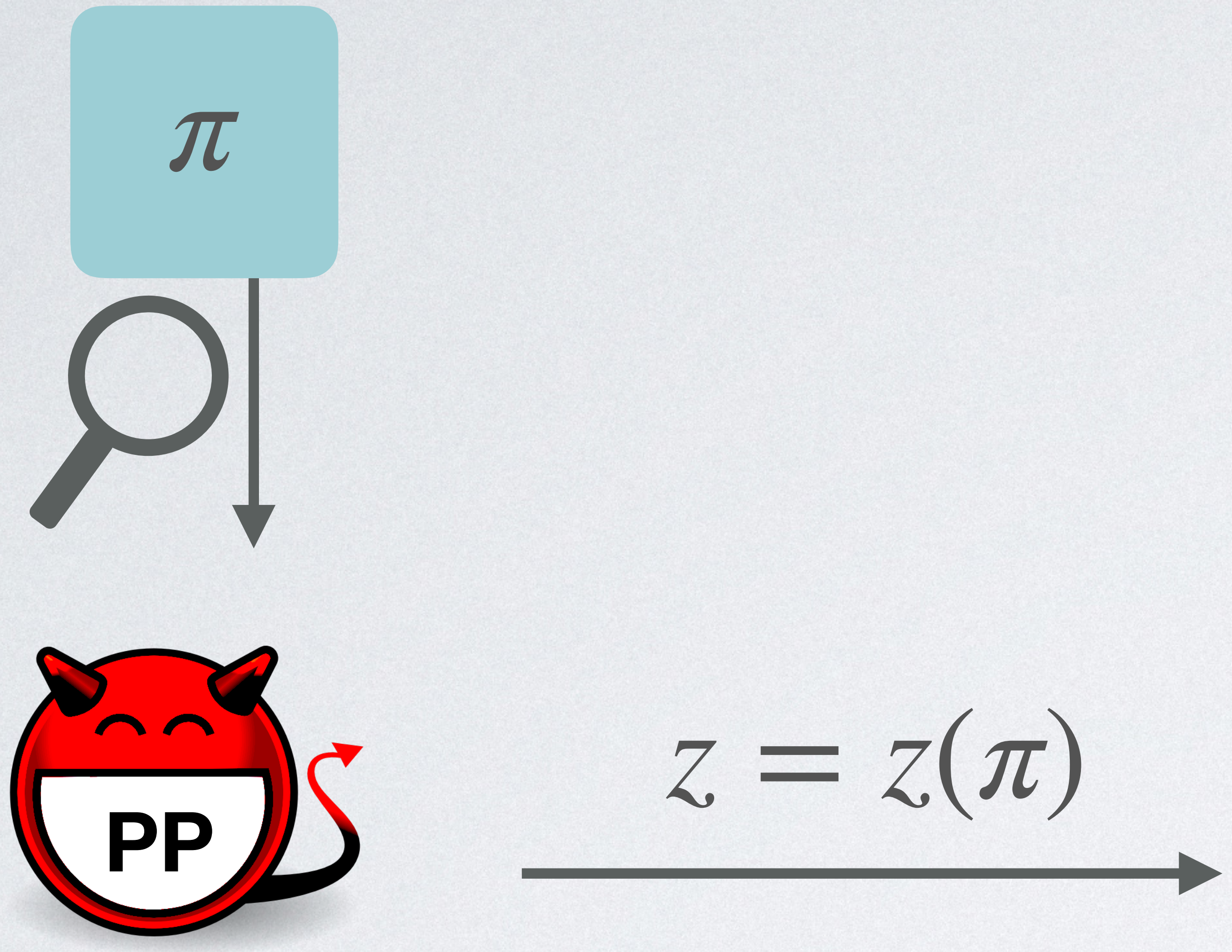
   (a) Fixed on P coordinates $L \subseteq [N]$

   (b) $\forall\ Q \subseteq [N] \setminus L : \quad \mathrm{H}_\infty(\pi'_Q)$

Proof of Presampling

Göös, Lovett, Meka '16

C, Dodis, Guo, Steinberger '18



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1. Decompose $\pi' := \pi \,|\, z$ into dense sources

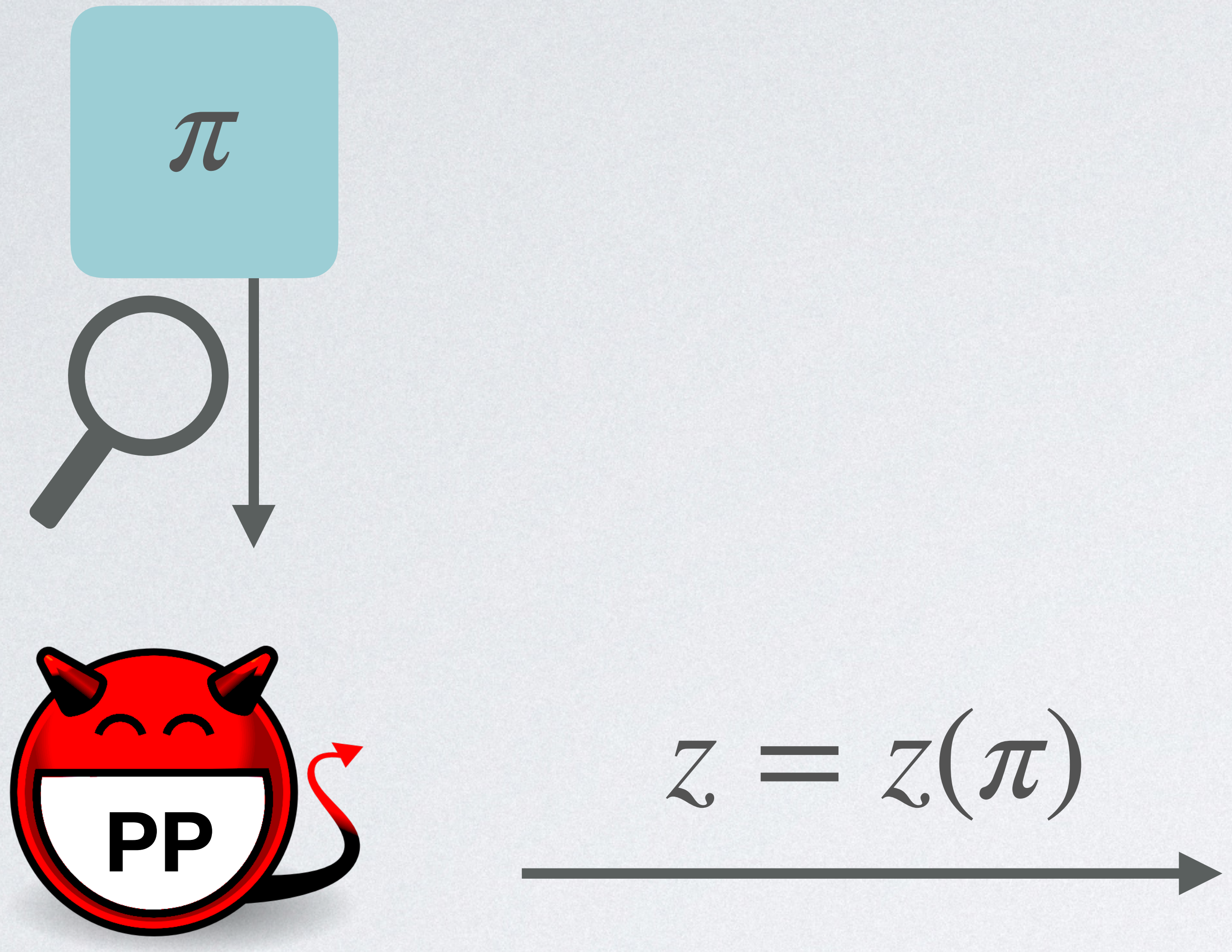
   (a) Fixed on P coordinates $L \subseteq [N]$

   (b) $\forall\ Q \subseteq [N] \backslash L : \quad \mathrm{H}_\infty(\pi'_Q) \ \geq\ (1-\delta) \cdot$

Proof of Presampling

Göös, Lovett, Meka '16

C, Dodis, Guo, Steinberger '18



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1. Decompose $\pi' := \pi \,|\, z$ into dense sources

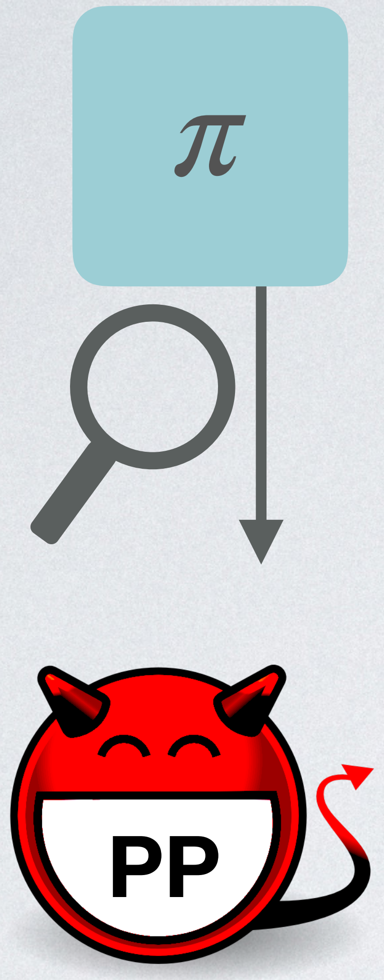
(a) Fixed on P coordinates $L \subseteq [N]$

(b) $\forall\, Q \subseteq [N] \backslash L: \quad \mathrm{H}_\infty(\pi'_Q) \;\geq\; (1-\delta) \cdot \log \dfrac{(N-P)!}{(N-P-|Q|)!}$

Göös, Lovett, Meka '16

C, Dodis, Guo, Steinberger '18

Proof of Presampling



$z = z(\pi)$

Before leakage: $H_\infty(\pi) = \log N!$

After leakage: $H_\infty(\pi \mid z) = \log N! - S$

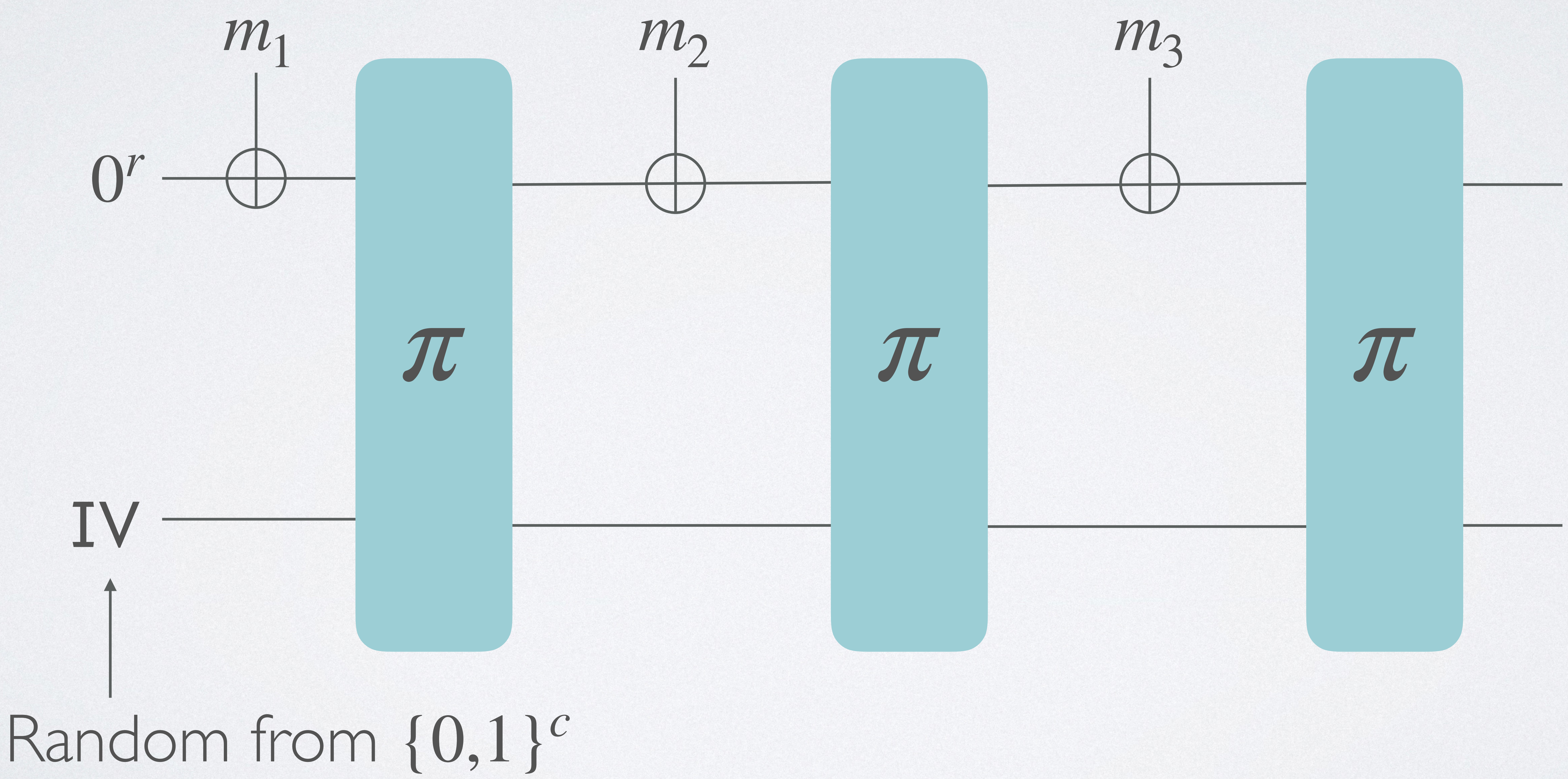
1. Decompose $\pi' := \pi \mid z$ into dense sources

   (a) Fixed on P coordinates $L \subseteq [N]$

   (b) $\forall\, Q \subseteq [N] \backslash L: \quad H_\infty(\pi'_Q) \;\geq\; (1-\delta) \cdot \log \dfrac{(N-P)!}{(N-P-|Q|)!}$

2. Show dense is indistinguishable from uniform

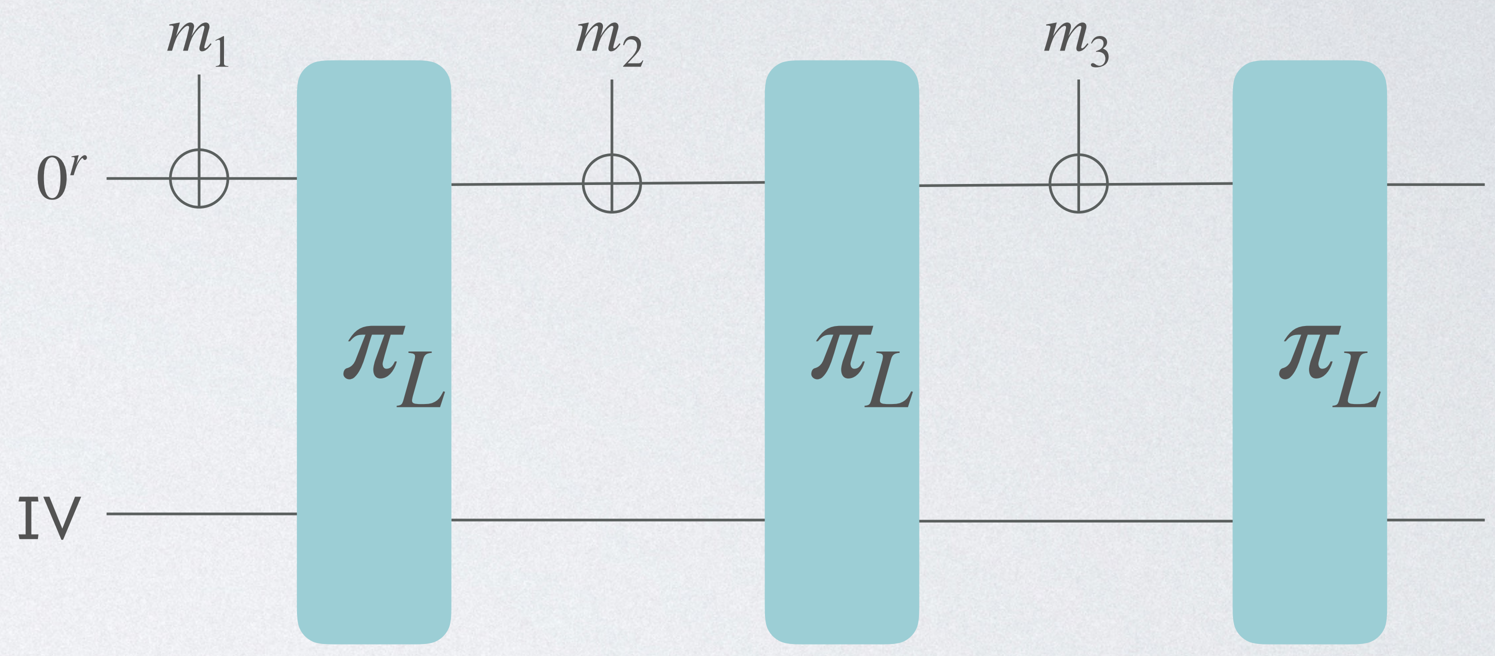
SHA-3 Collision Resistance in the AI-RPM



SHA-3 Collision Resistance in the BF-RPM

P prefixed coordinates

$\{\tilde{x}_\ell, \tilde{y}_\ell\}_{\ell=1}^{L}$



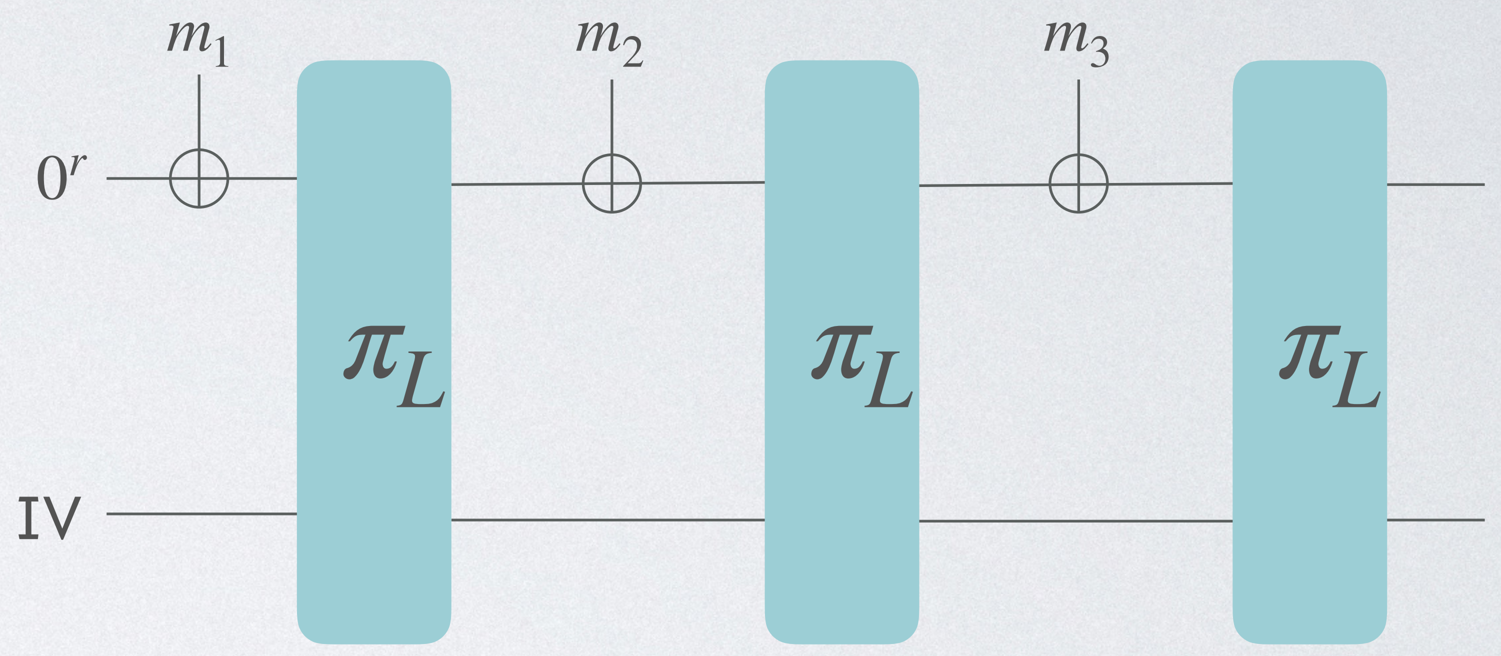
SHA-3 Collision Resistance in the BF-RPM

T queries

$\{x_j, y_j\}_{j=1}^{T}$

P prefixed coordinates

$\{\tilde{x}_\ell, \tilde{y}_\ell\}_{\ell=1}^{L}$



SHA-3 Collision Resistance in the BF-RPM

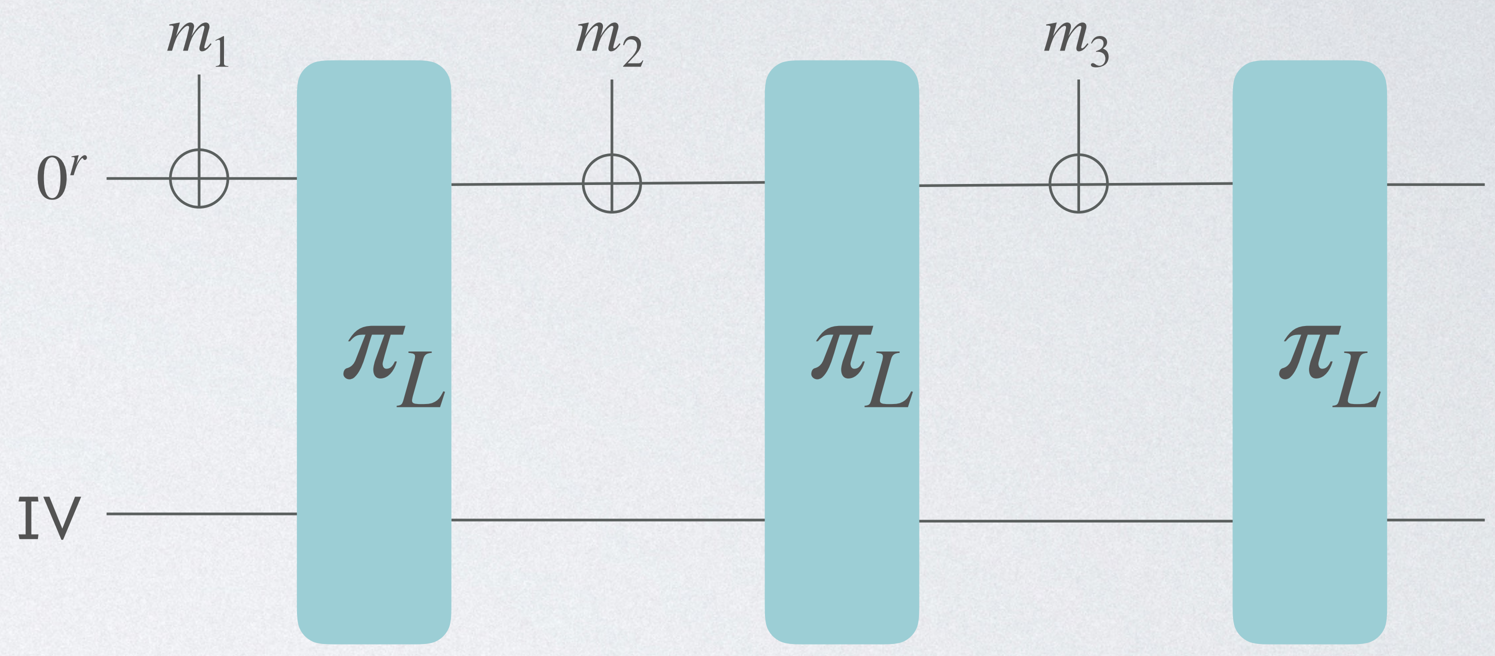
T queries

$\{x_j, y_j\}_{j=1}^{T}$

P prefixed coordinates

$\{\tilde{x}_\ell, \tilde{y}_\ell\}_{\ell=1}^{L}$

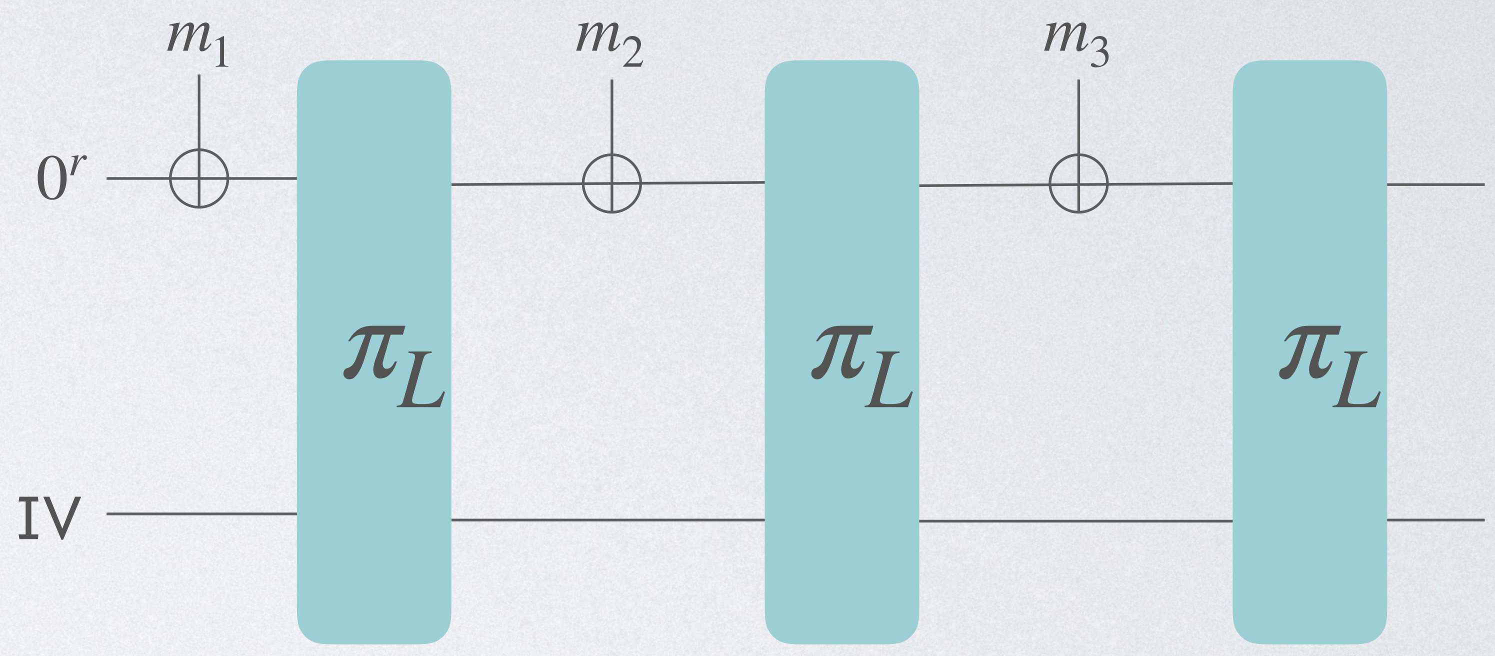
Queries: graph starting at $(0^r, IV)$



SHA-3 Collision Resistance in the BF-RPM

T queries $\{x_j, y_j\}_{j=1}^{T}$

P prefixed coordinates $\{\tilde{x}_\ell, \tilde{y}_\ell\}_{\ell=1}^{L}$



Queries: graph starting at $(0^r, IV)$

Event **BAD**: capacity part not fresh

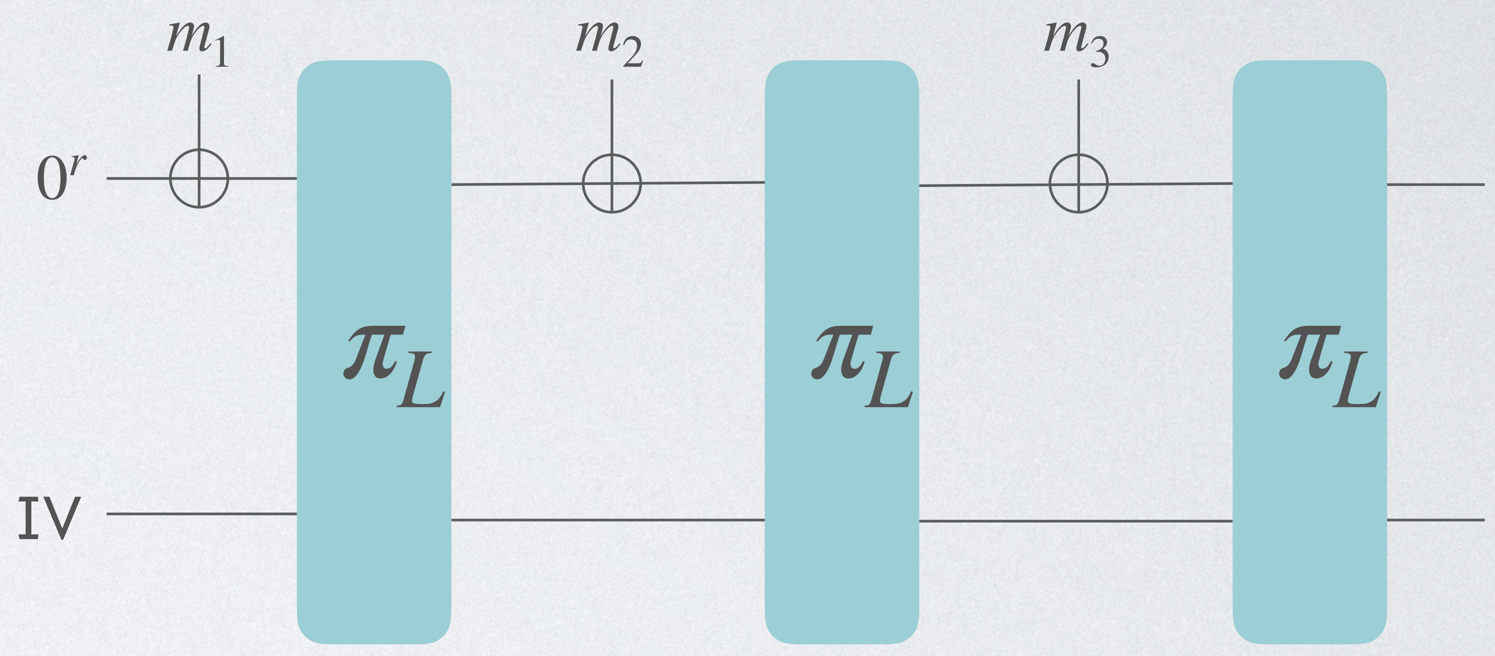
SHA-3 Collision Resistance in the BF-RPM

T queries

$\{x_j, y_j\}_{j=1}^T$

P prefixed coordinates

$\{\tilde{x}_\ell, \tilde{y}_\ell\}_{\ell=1}^L$



Queries: graph starting at $(0^r, IV)$

Event BAD: capacity part not fresh

$$\text{P[BAD]} \leq \frac{T^2}{2^c} + \frac{TP}{2^c}$$

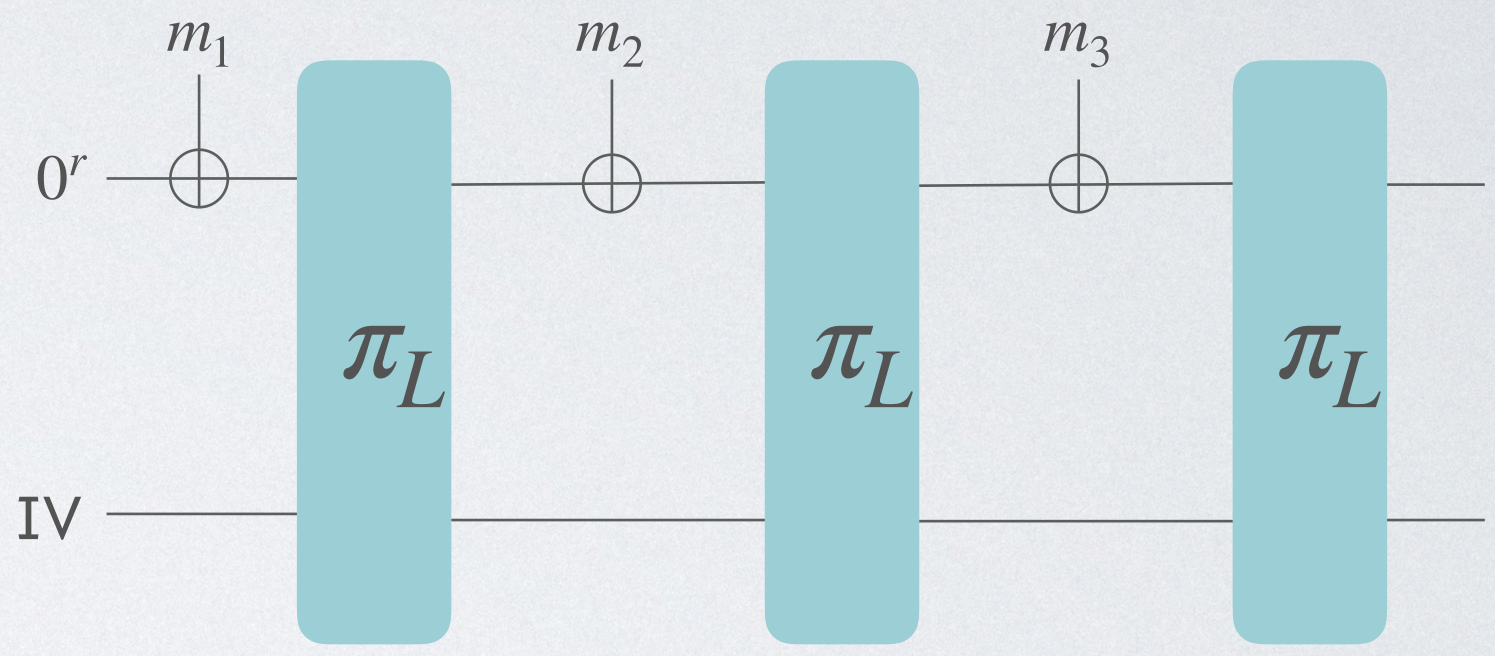
SHA-3 Collision Resistance in the BF-RPM

T queries

$\{x_j, y_j\}_{j=1}^{T}$

P prefixed coordinates

$\{\tilde{x}_\ell, \tilde{y}_\ell\}_{\ell=1}^{L}$



Queries: graph starting at $(0^r, IV)$

Event **BAD**: capacity part not fresh

$$\text{P[BAD]} \leq \frac{T^2}{2^c} + \frac{TP}{2^c}$$

$$\text{P[COLL | BAD]} \leq \frac{T^2}{2^r}$$

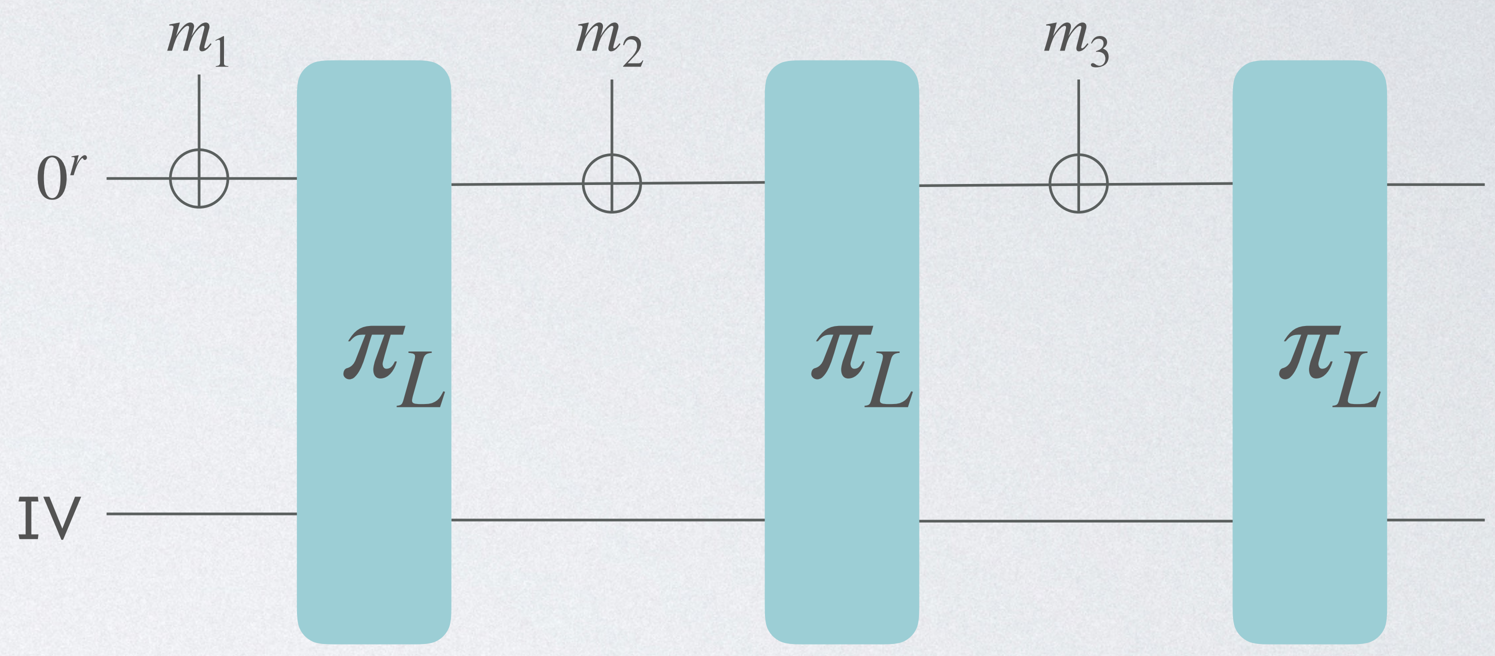
SHA-3 Collision Resistance in the BF-RPM

T queries

$\{x_j, y_j\}_{j=1}^{T}$

P prefixed coordinates

$\{\tilde{x}_\ell, \tilde{y}_\ell\}_{\ell=1}^{L}$



Queries: graph starting at $(0^r, IV)$

Event BAD: capacity part not fresh

$$\text{P[BAD]} \leq \frac{T^2}{2^c} + \frac{TP}{2^c}$$

$$\text{P[COLL | BAD]} \leq \frac{T^2}{2^r}$$

$$P \approx ST \longrightarrow$$

AI-ROM: $\dfrac{T^2}{2^r} + \dfrac{ST^2}{2^c}$