### On Tightly Secure Non-Interactive Key Exchange









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### Non-Interactive Key Exchange (NIKE)



 $\begin{array}{ll} (\mathsf{pk}_1,\mathsf{sk}_1) \leftarrow \texttt{KeyGen} & (\mathsf{pk}_2,\mathsf{sk}_2) \leftarrow \texttt{KeyGen} \\ \\ \mathcal{K}_{21} = \texttt{SharedKey}(\mathsf{pk}_2,\mathsf{sk}_1) & = & \mathcal{K}_{12} = \texttt{SharedKey}(\mathsf{pk}_1,\mathsf{sk}_2) \end{array}$ 

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#### Why do we care?

- $\blacktriangleright$  Theory: closer relation between  ${\cal P}$  and  ${\cal S}$
- **Practice:** smaller keys  $\Rightarrow$  more efficient instantiations



(Simplified) Security model



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(Simplified) Security model  $\mathsf{pk}_1, \cdots, \mathsf{pk}_n$ 0.00.0100 -

### (Simplified) Security of NIKE w/ extractions



$$\mathsf{Advantage}^{\mathsf{nike}}_\mathcal{A} := |\operatorname{\mathsf{Pr}}[b^\star = b] - 1/2|$$

### Recap: DH Key Exchange - Security w/ extractions

**Idea:**  $i^{\star}, j^{\star} \leftarrow_{R} \{1, \dots, n\}$ , embed DDH-challenge in  $\mathsf{pk}_{i^{\star}}, \mathsf{pk}_{i^{\star}}$ 

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**[BJLS16]:** This loss is inherent!

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- + Generic transformation with tight instantiation:
  - ► NIKE with passive security ~ NIKE with active security

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- rules out tight simple black-box reductions

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 $\forall (\mathsf{pk}_1,\mathsf{sk}_1), (\mathsf{pk}_2,\mathsf{sk}_2) \colon \mathtt{SharedKey}(\mathsf{pk}_2,\mathsf{sk}_1) = \mathtt{SharedKey}(\mathsf{pk}_1,\mathsf{sk}_2)$ 

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### Recap: Subset membership problem (SMP)

X set,  $L \subseteq X$  NP-language

Subset membership assumption for (X, L):

 $\{x \mid x \leftarrow_R L\} \qquad \approx_c \qquad \{x \mid x \leftarrow_R X \setminus L\}$ 

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# Recap: Hash proof system [CS98]

HPS = (Gen, PubEval, PrivEval) is HPS for language L if:

$$\left. \begin{array}{l} \operatorname{PubEval}(hpk, x, w) \\ \operatorname{PrivEval}(hsk, x) \end{array} \right\} \text{ return the same key } K \text{ for all } x \in L \text{ with witness } w \end{array} \right\}$$

**Universality:**  $\forall x \notin L$ ,  $(hpk, hsk) \leftarrow Gen$ :

 $(hpk, x, \texttt{PrivEval}(hsk, x)) \equiv (hpk, x, \texttt{random})$ 

# Our NIKE

Variation of the PAKE of [KOY01; GL03]

HPS = (Gen, PubEval, PrivEval) for L, SMP for  $L \subseteq X$  hard



 $x_1 \leftarrow L$  with witness  $w_1$ 

 $(hpk_2, hsk_2) \leftarrow \texttt{Gen}$  $K_{21} = \texttt{PubEval}(hpk_2, x_1, w_1) = K_{12} = \texttt{PrivEval}(hsk_2, x_1)$ 

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#### Note:

- hsk not unique
- can switch x to  $X \setminus L$

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 $\rightsquigarrow$  security loss of only *n* 

Reduction knows sk<sub>i</sub>

 $i \neq i^{\star}$ 

Reduction doesn't know sk<sub>i</sub>

 $i = i^{\star}$ 

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#### Instantiation:

- generic instantiation from standard components
- optimized tightly secure instantiation for our NIKE

Reference	pk	sec. model	sec. loss	assumption	uses
[DH76]	$1 imes \mathbb{G}$	passive	n <sup>2</sup>	DDH	-
Ours	$3 imes \mathbb{G}$	passive	п	DDH	-
[CKS08]	$2 imes \mathbb{G}$	active*	2	CDH	ROM
[FHKP13]	$1 imes \mathbb{Z}_{N}$	active	n <sup>2</sup>	factoring	ROM
[FHKP13]	$2  imes \mathbb{G} + 1  imes \mathbb{Z}_p$	active	$n^2$	DBDH	pairing
Ours	$12  imes \mathbb{G}$	active	п	DLIN	pairing

\*w/o extractions

#### Modular constructions

#### New lower bound:

- > applies to all schemes where invalid public keys have no secret keys
- yields a loss of  $\Omega(n)$  for all simple black-box reductions

#### Generic transformation from passive to active secure NIKE Thank you!!

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