On The Complexity of Compressing Obfuscation

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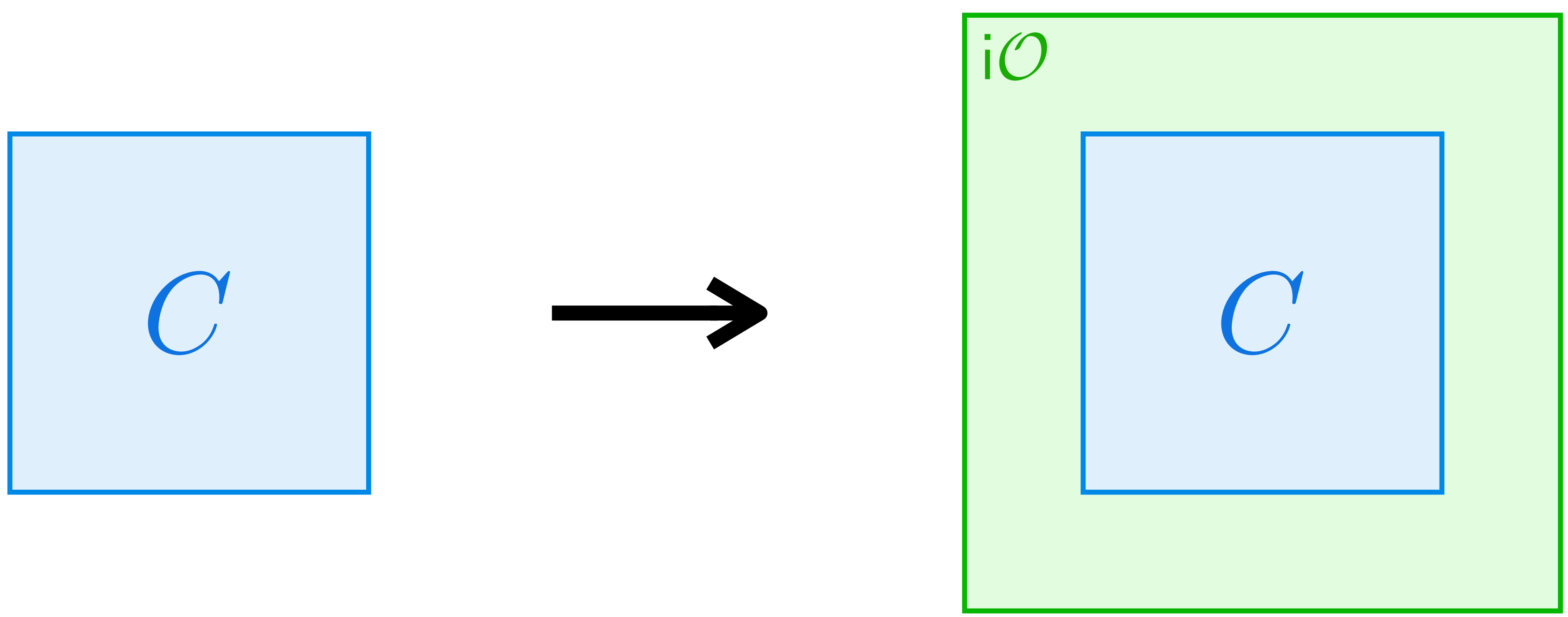
Indistinguishability Obfuscation (iO)

An obfuscator is a *compiler* which

- preserves functionality
- obfuscated circuit is "unintelligible"



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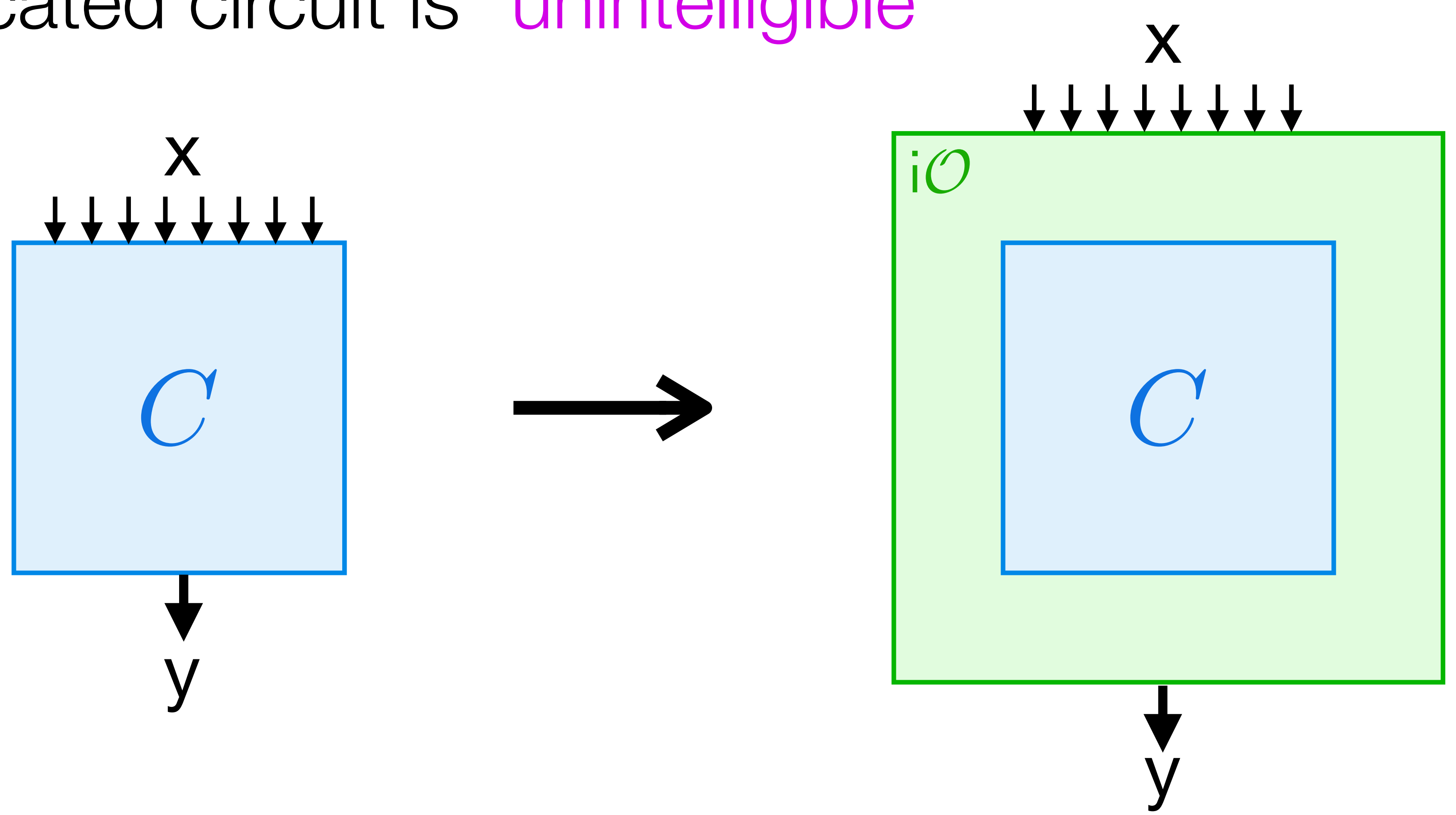

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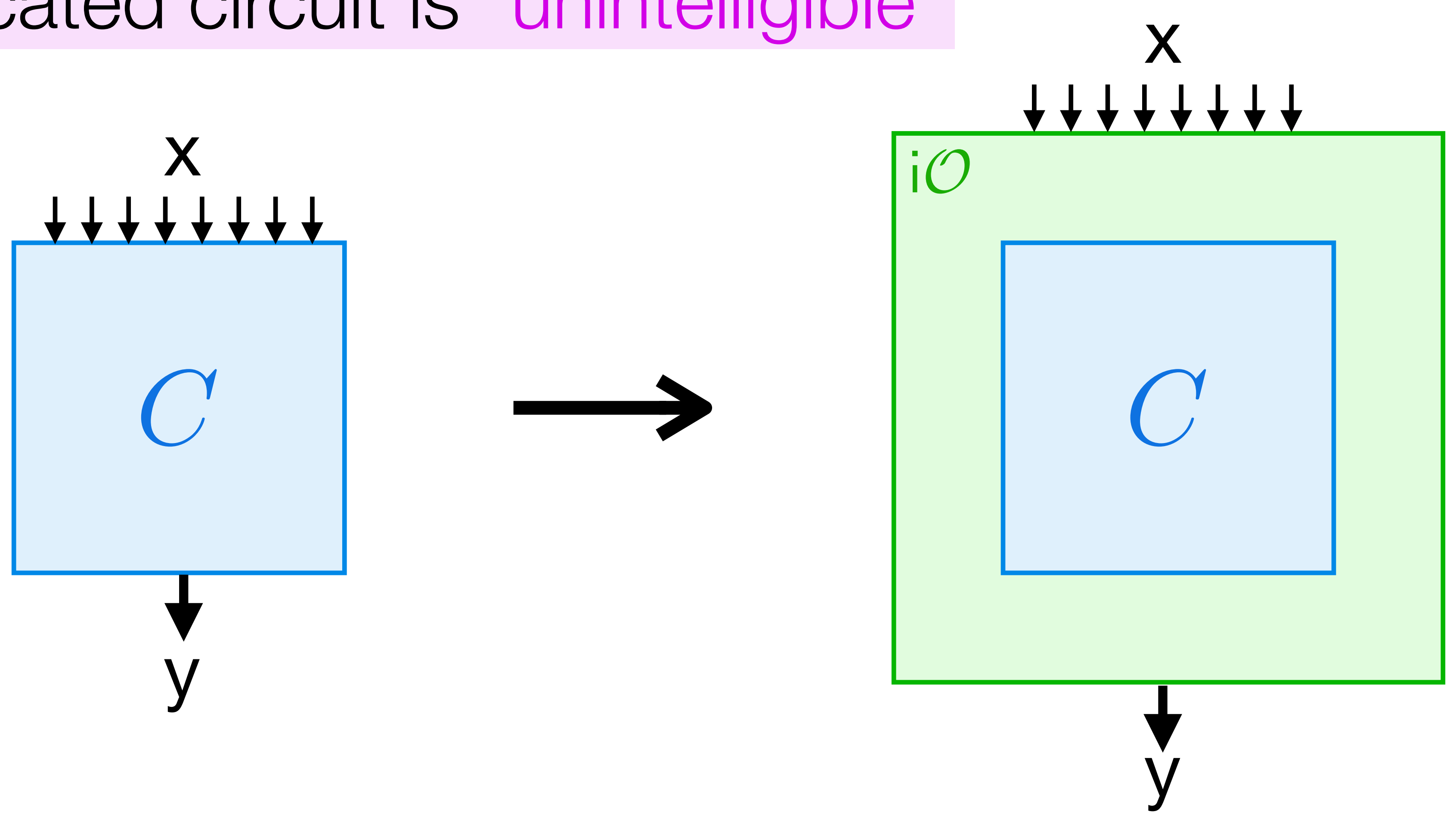

If C_0 and C_1 compute the same function and $|C_0|=|C_1|$, then $iO(C_0)$ and $iO(C_1)$ are hard to distinguish

Power of iO

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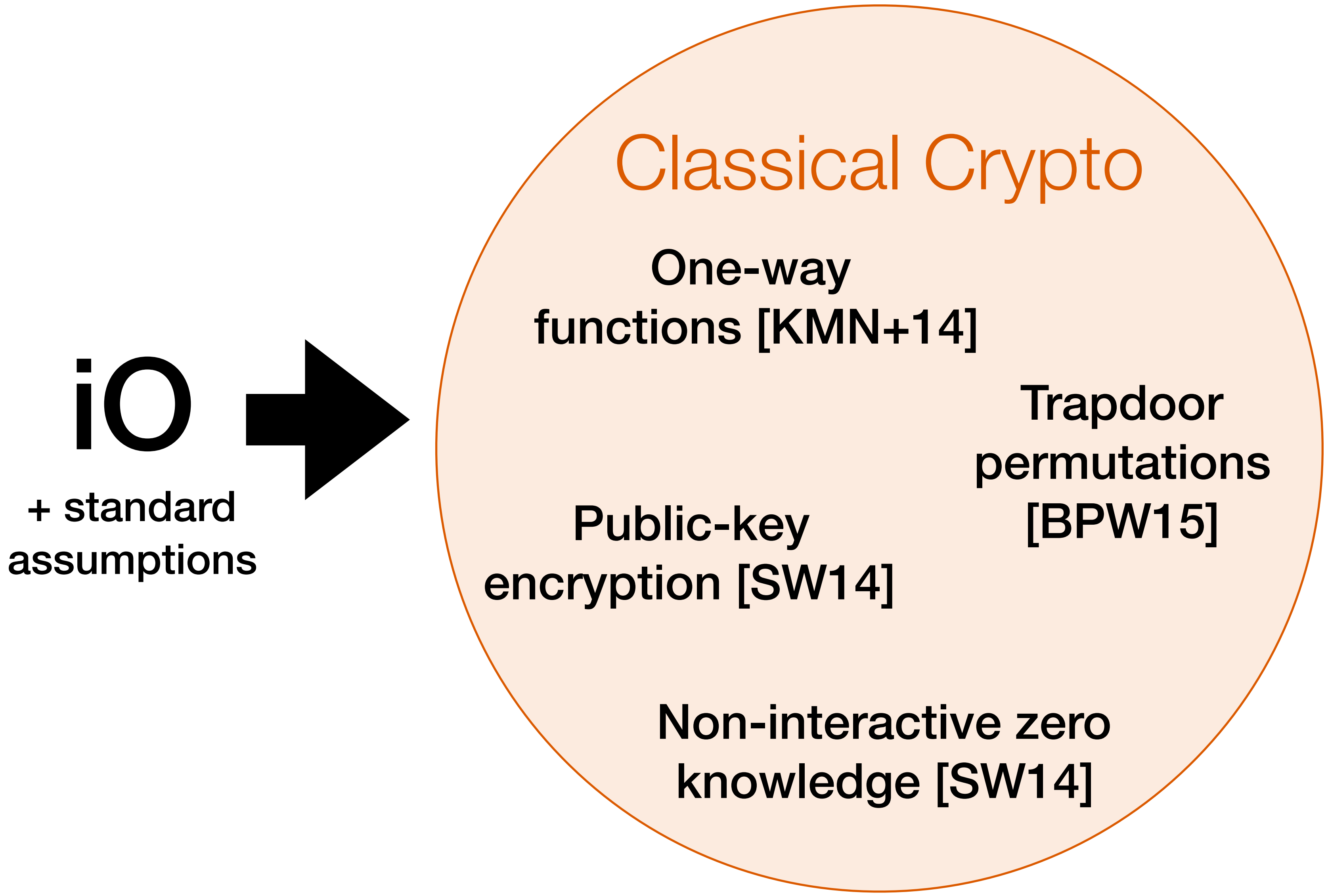
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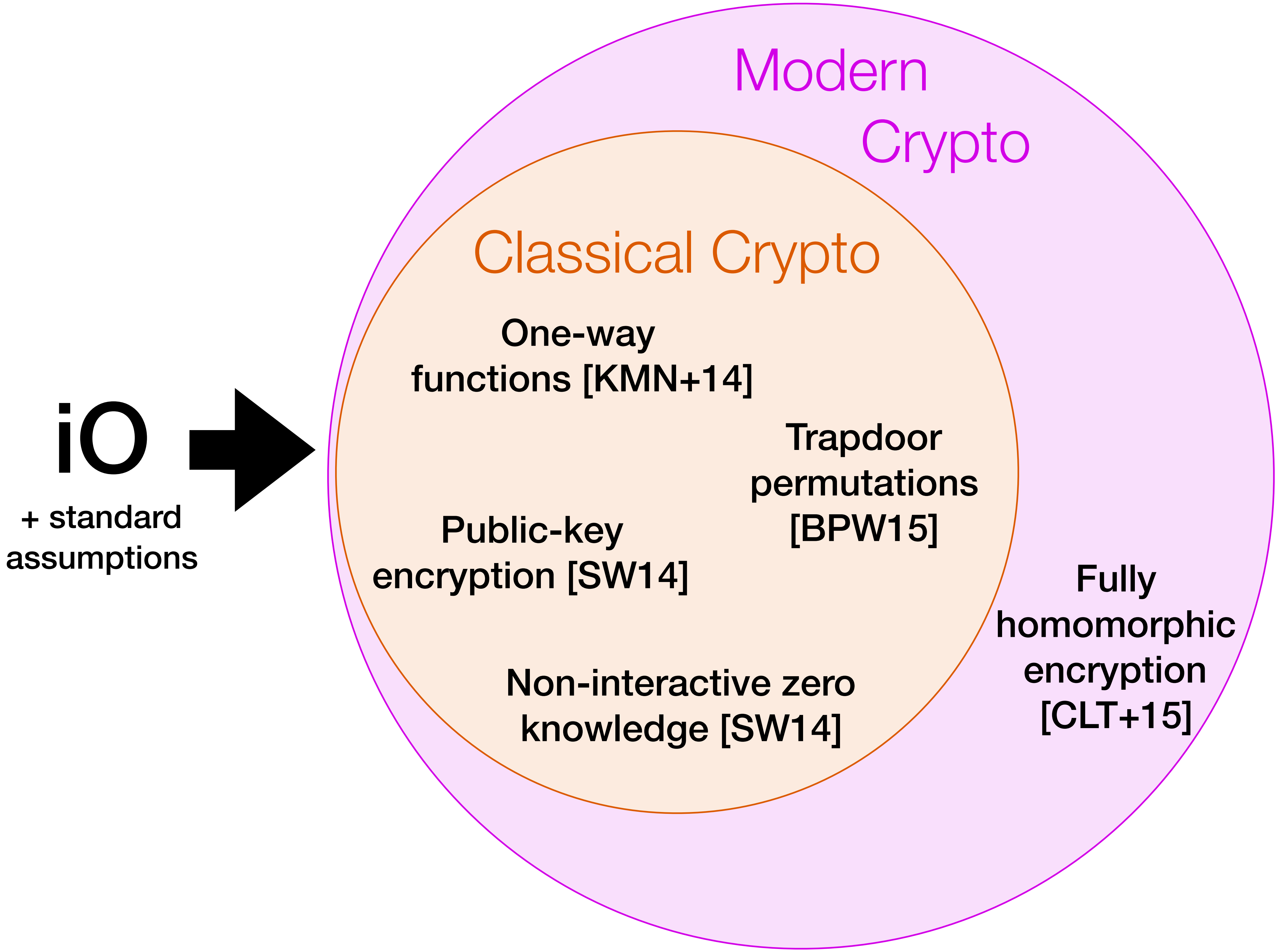
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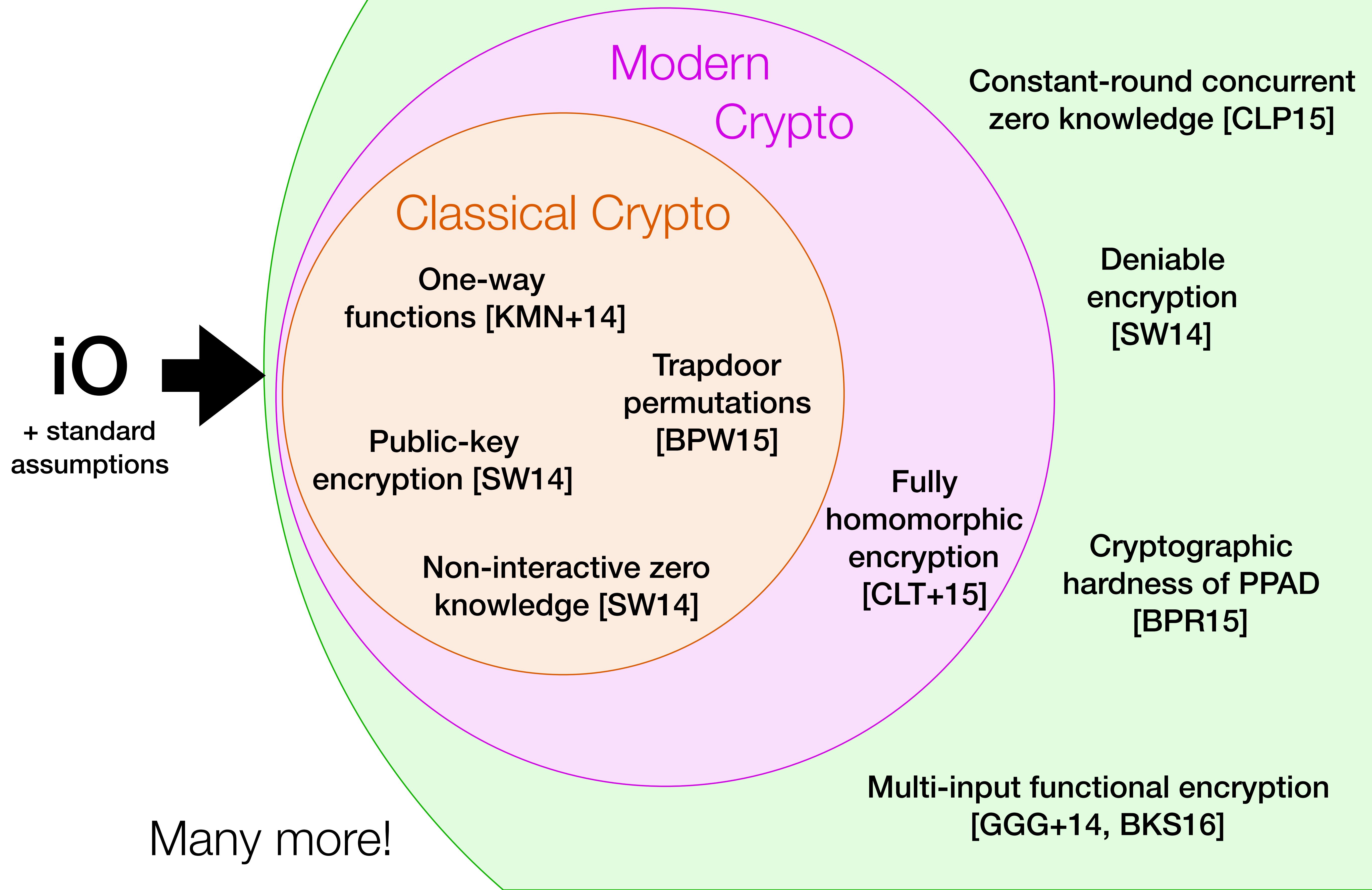
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Constant-round concurrent zero knowledge [CLP15]
Deniable encryption [SW14]
Cryptographic hardness of PPAD [BPR15]
Multi-input functional encryption [GGG+14, BKS16]
Many more!

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Reduce iO to seemingly weaker **building blocks**

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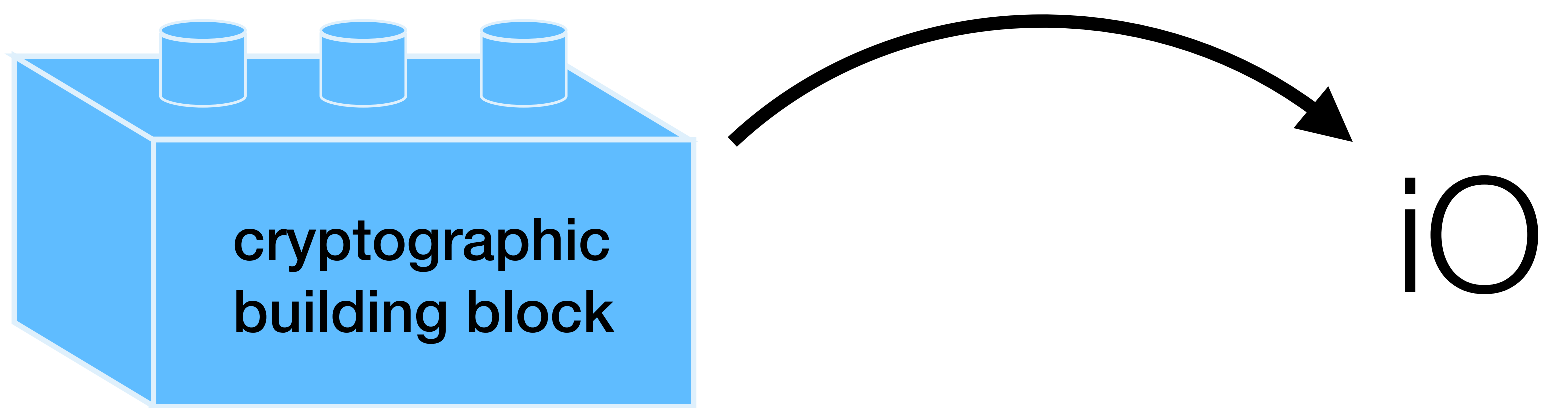

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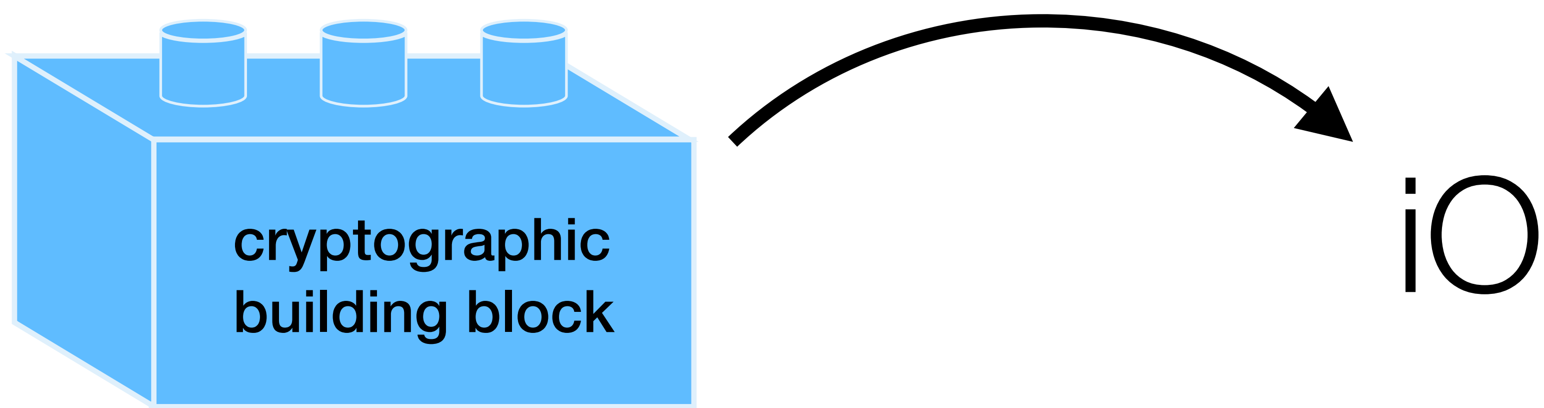

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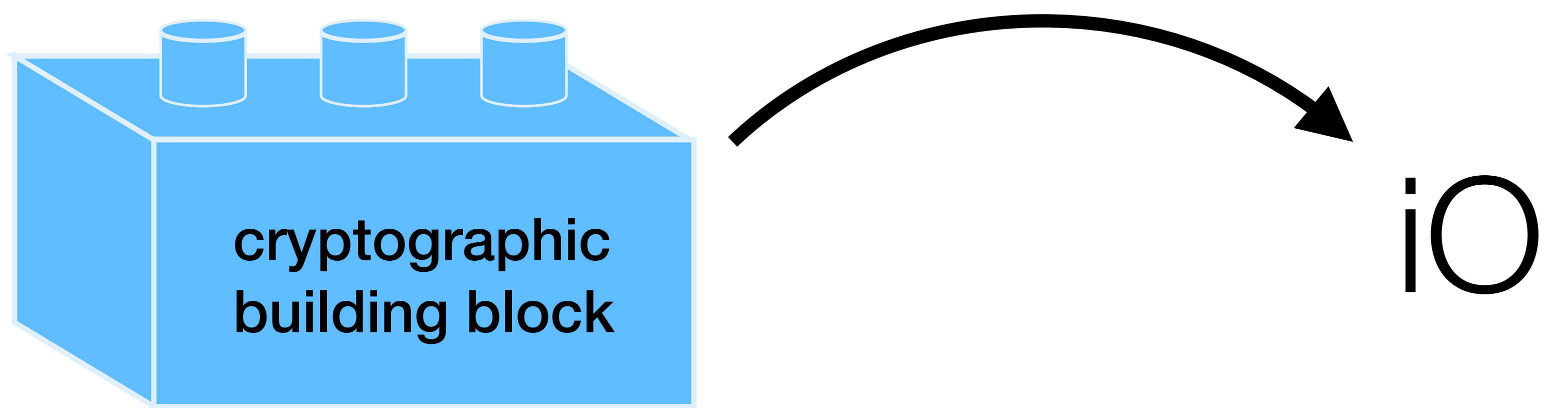

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[ADGM17,BBKK17,BWZ14,CGH17,CHLRS15,GHMS14,LV17,MSZ16]

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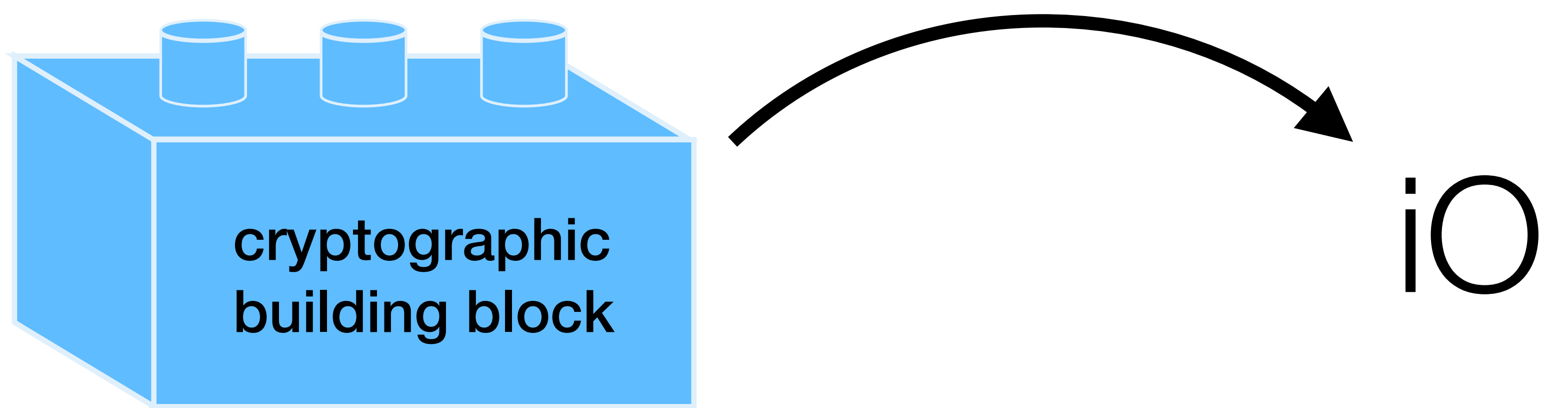

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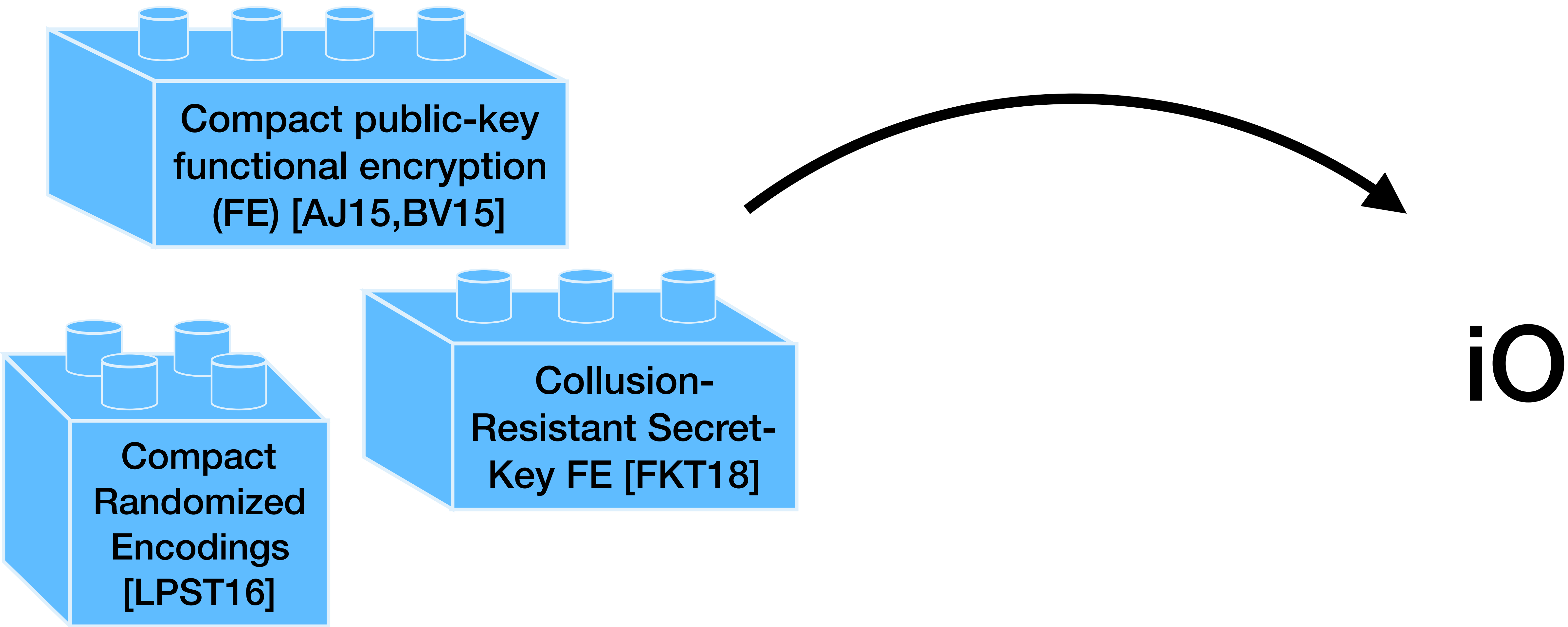

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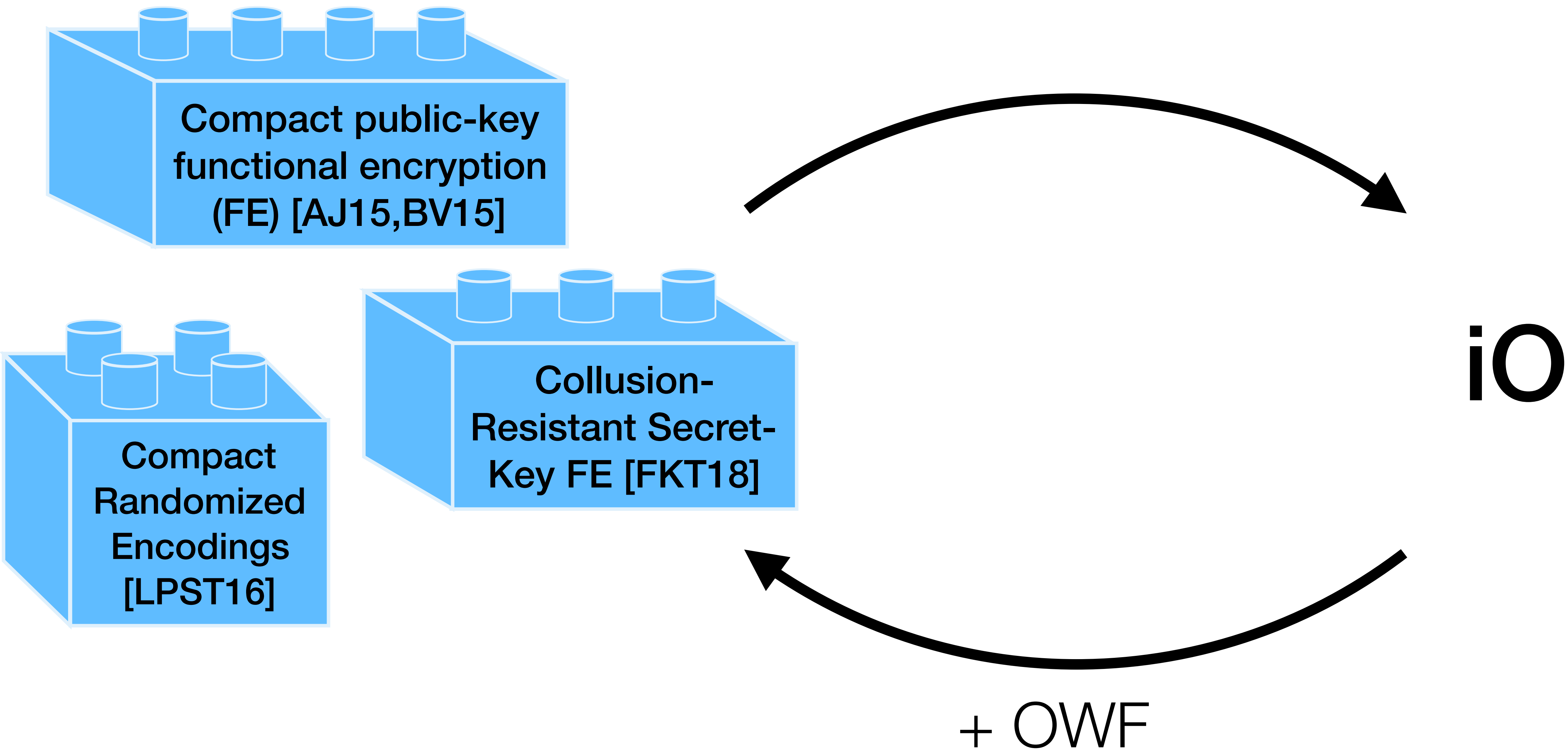

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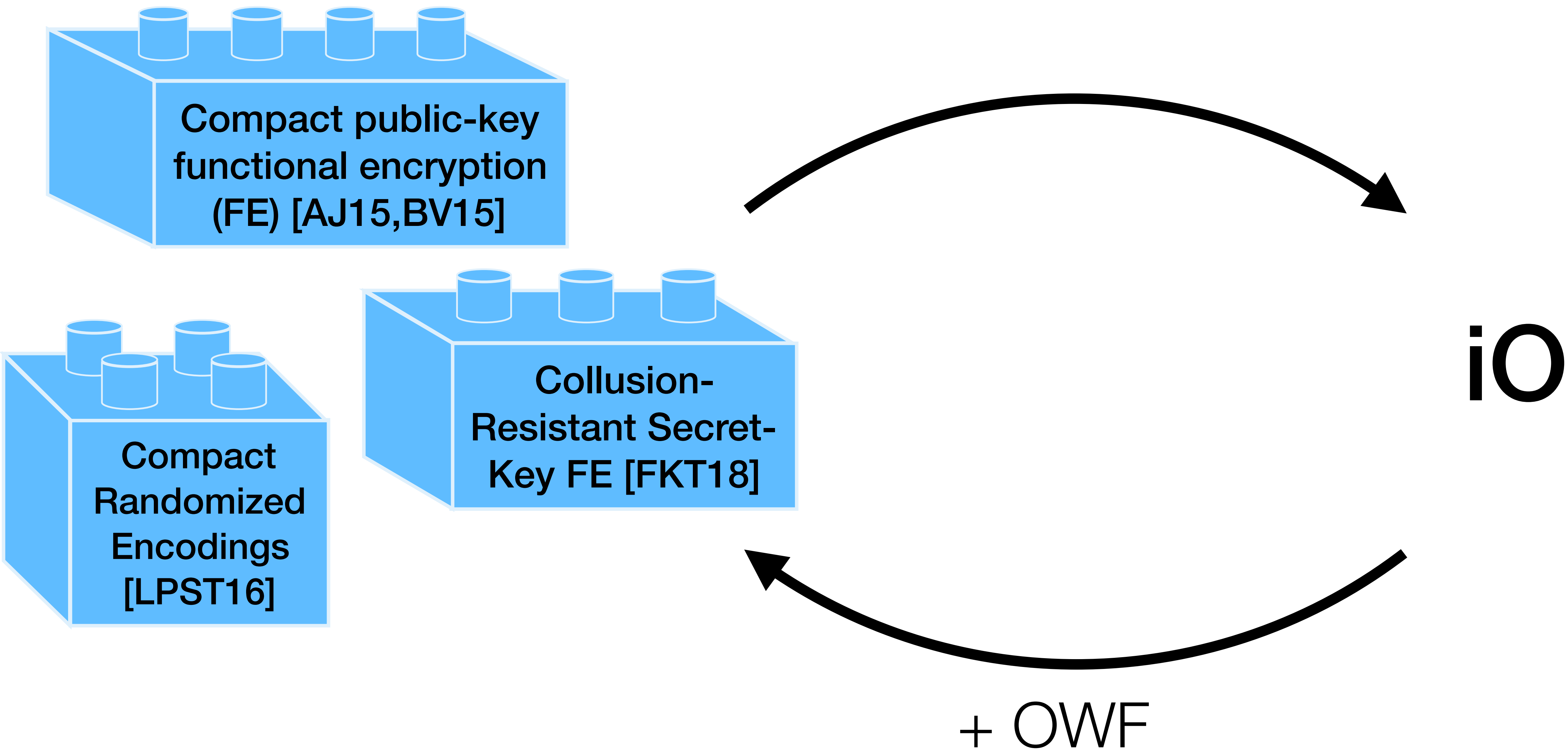

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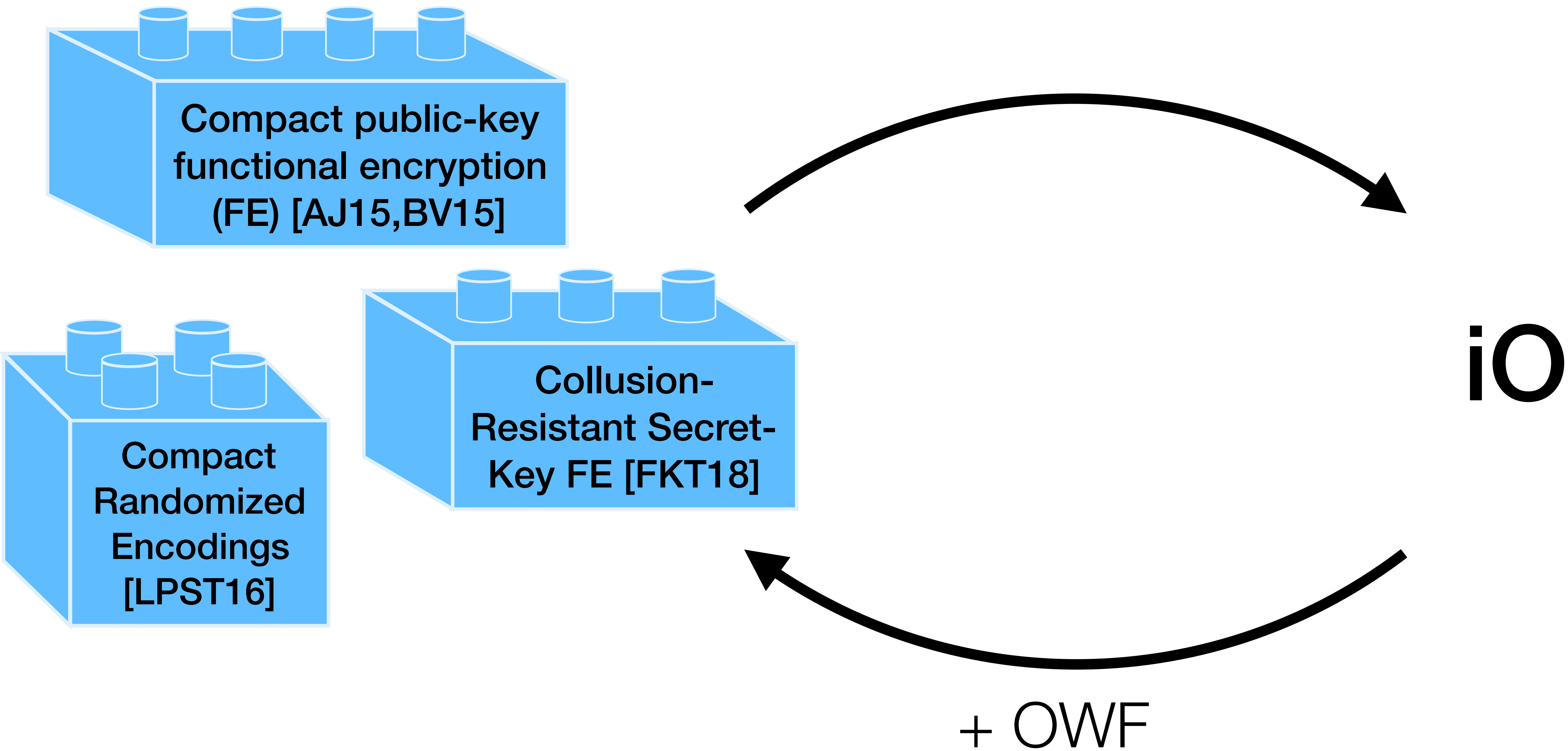

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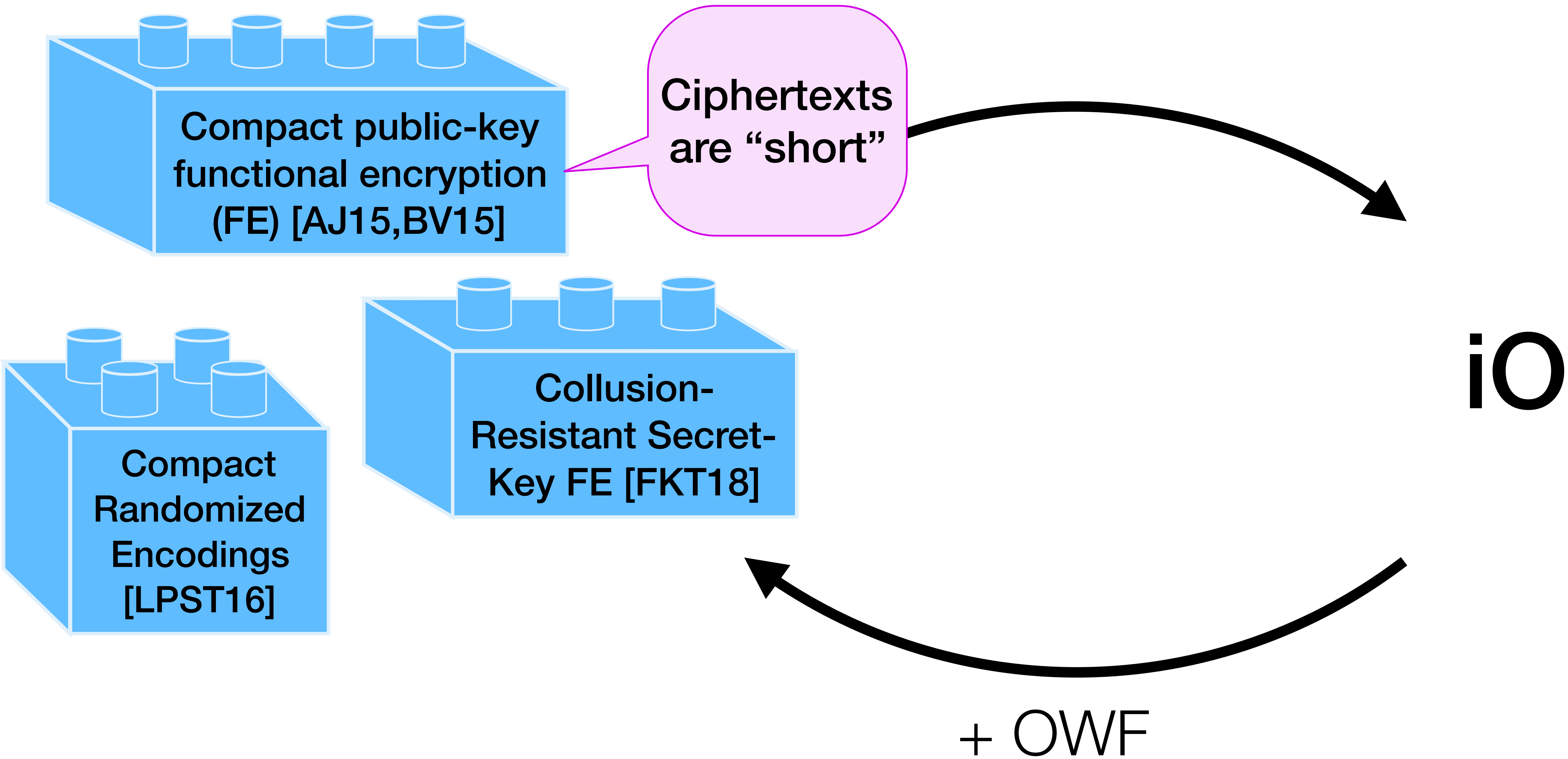

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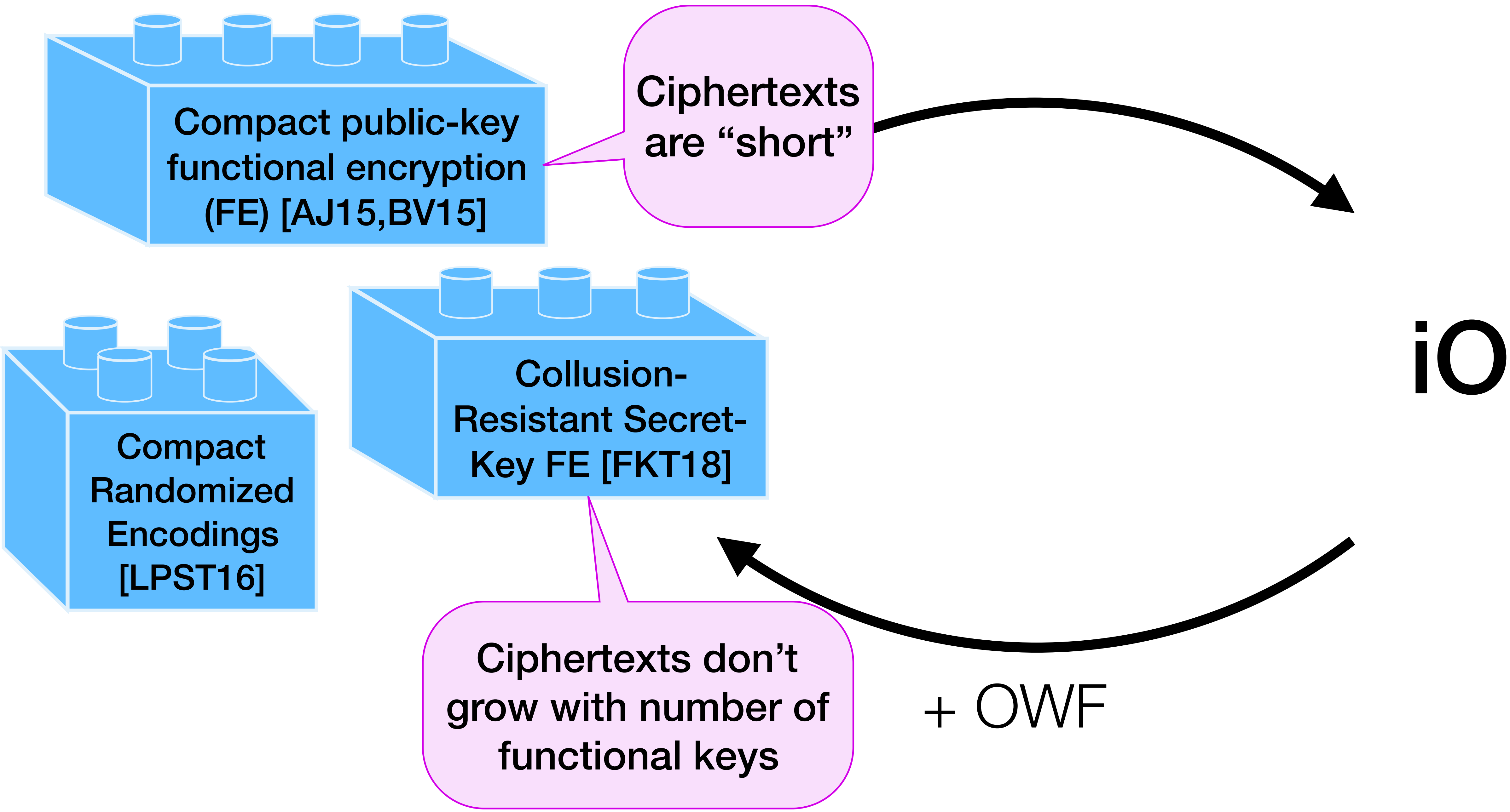

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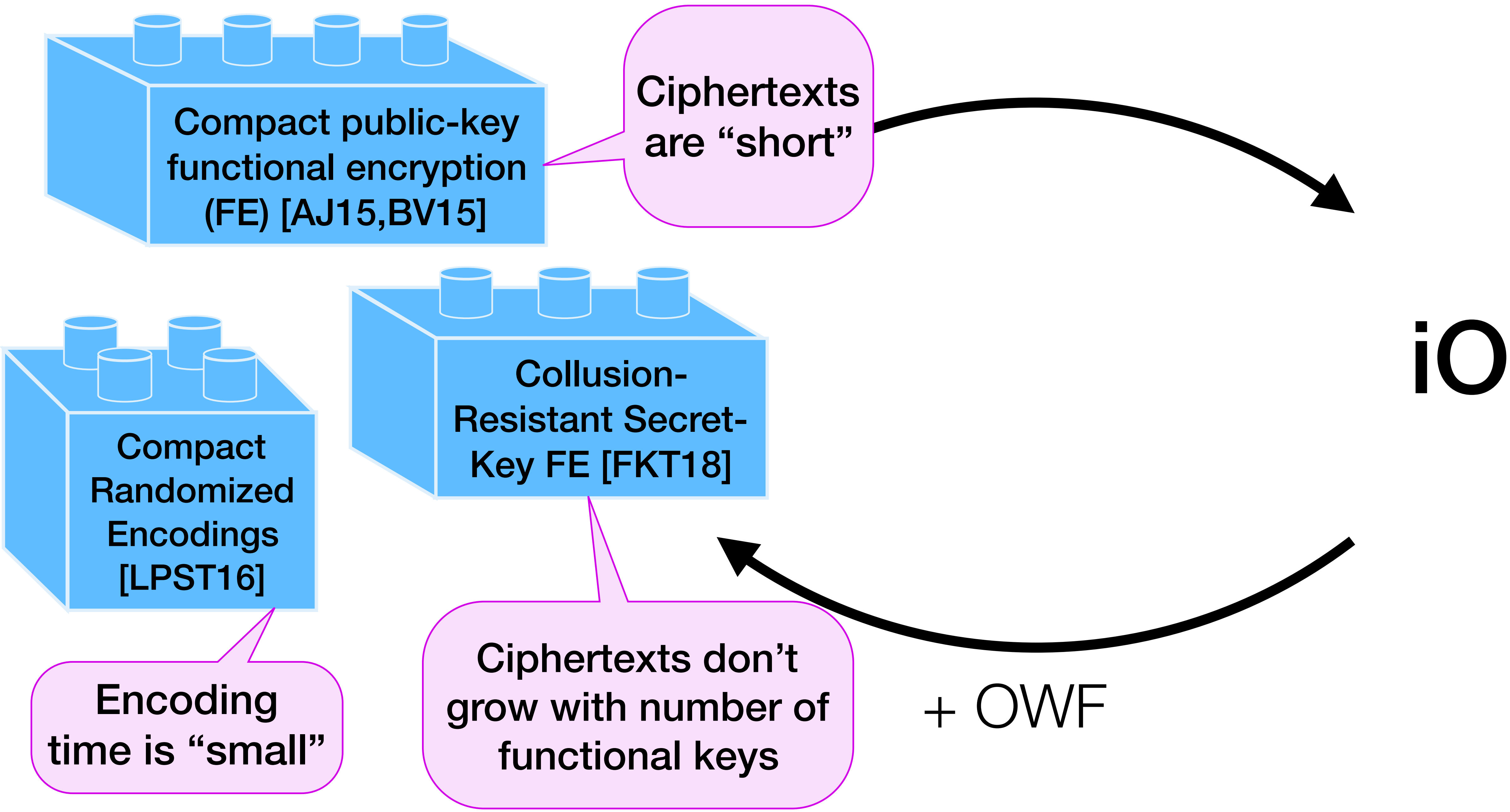

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A (t,ℓ)-compressing obfuscator has:

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Size of the obfuscation is ℓ(s,n)

This talk: circuits C
- Size s
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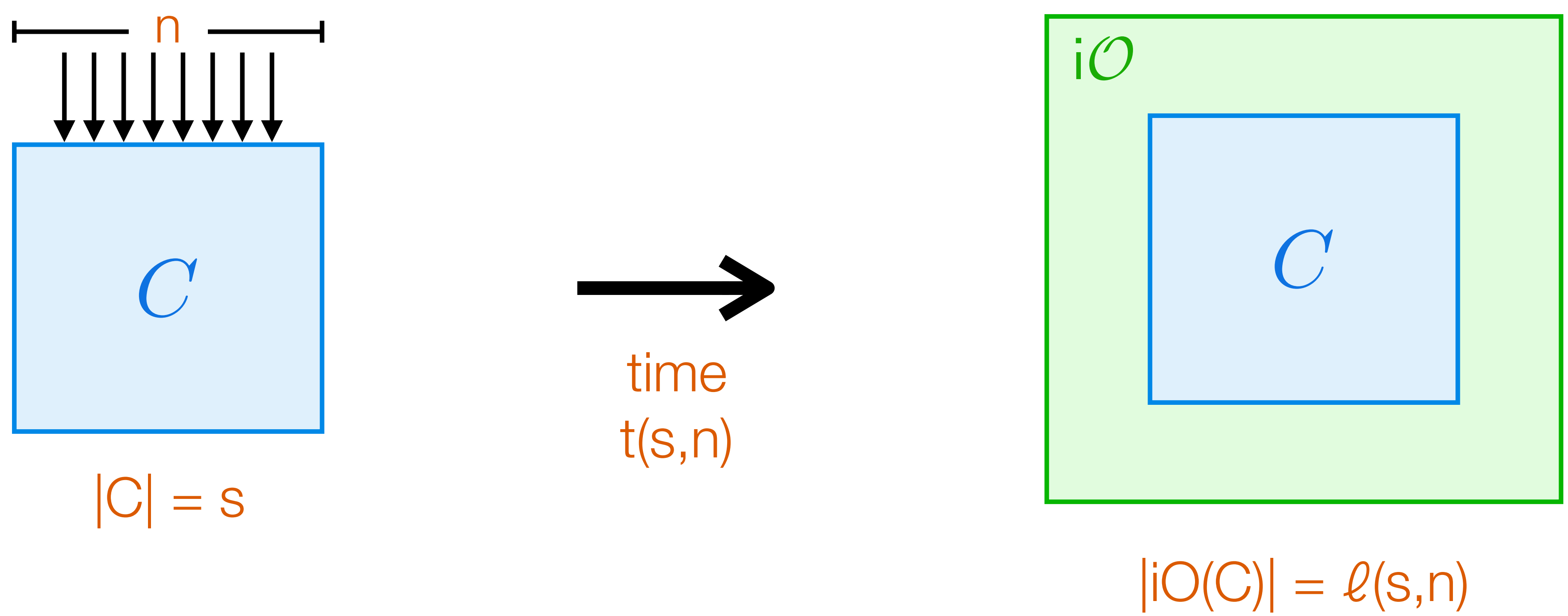

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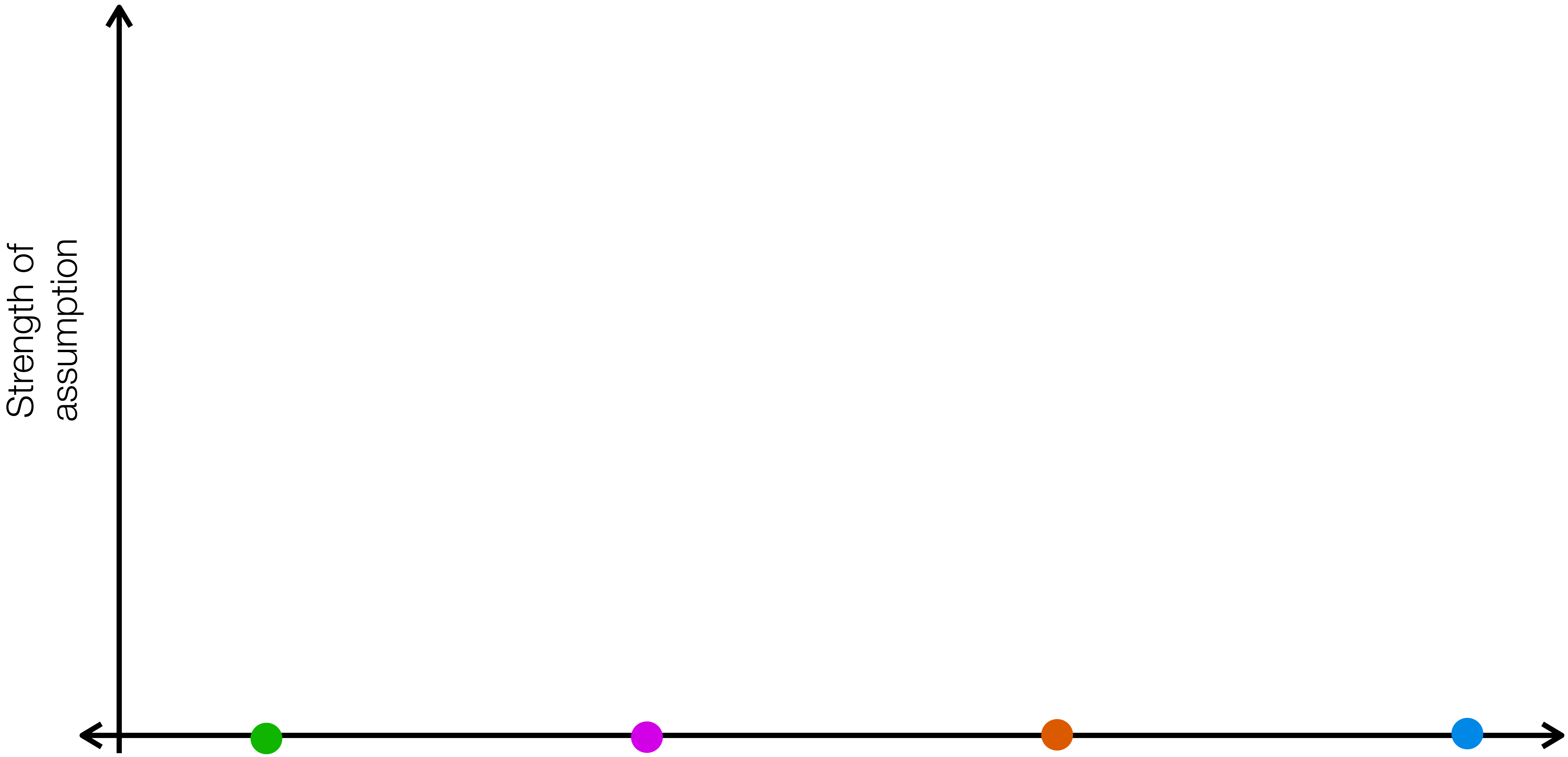

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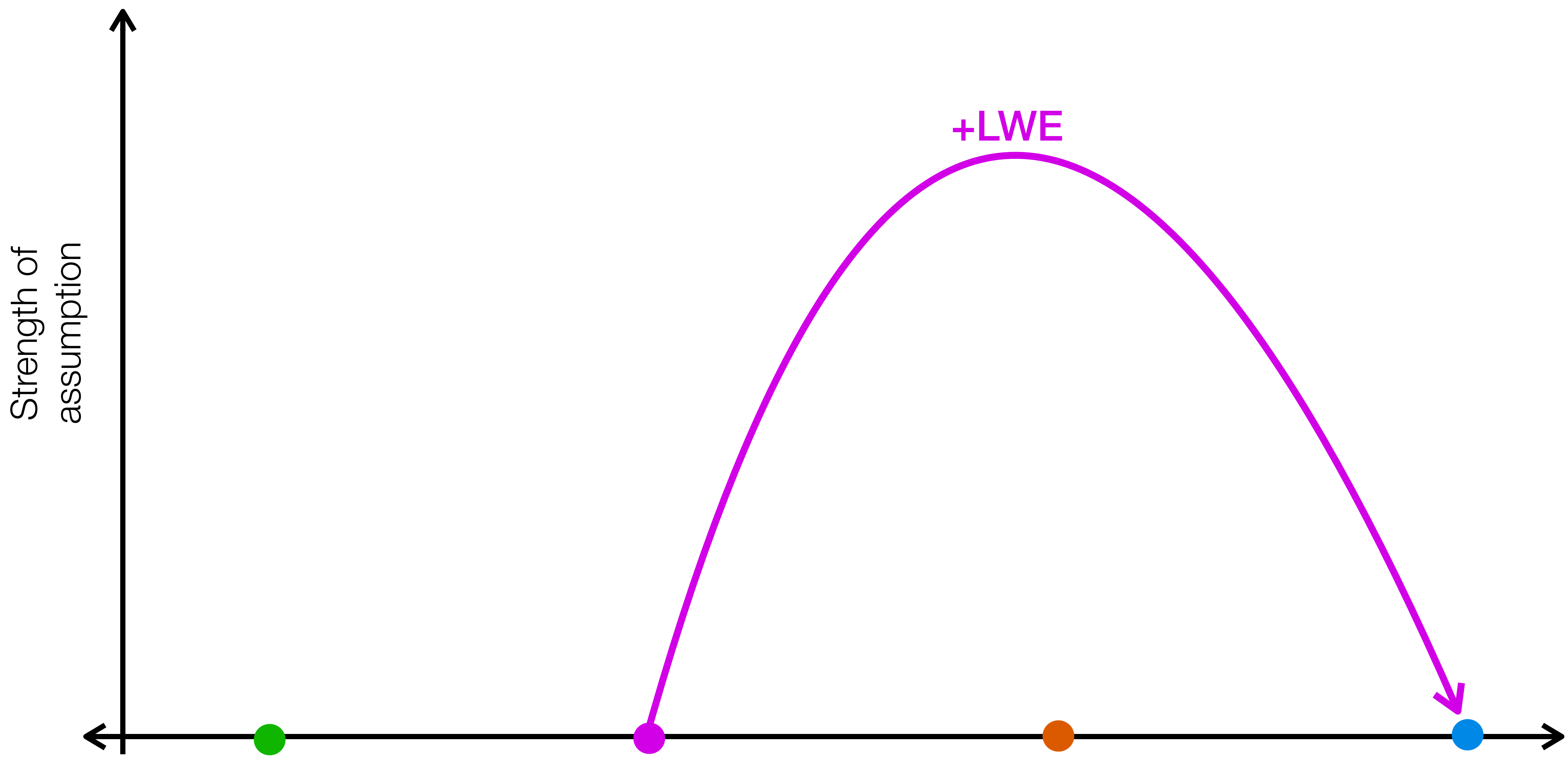

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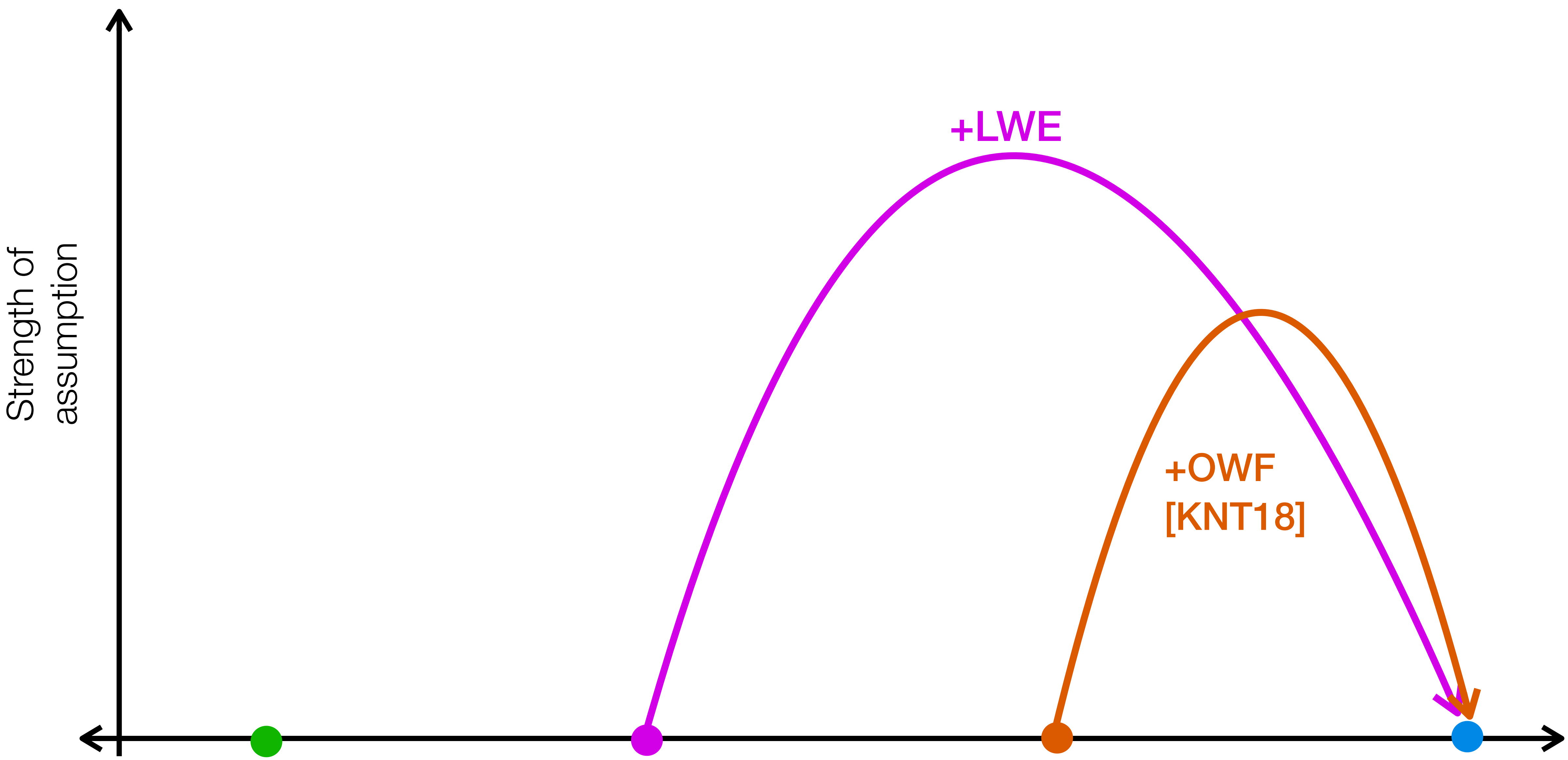

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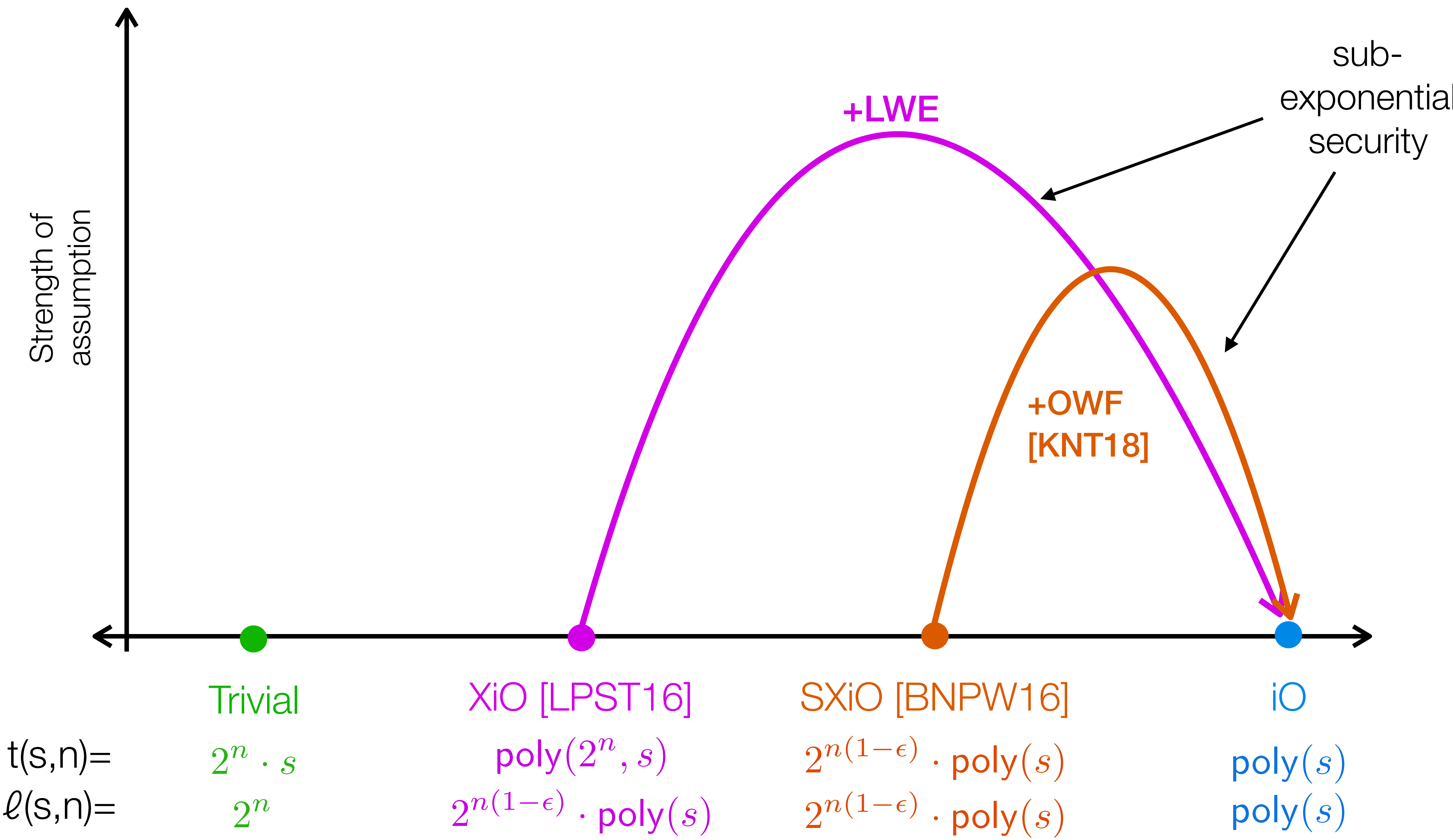

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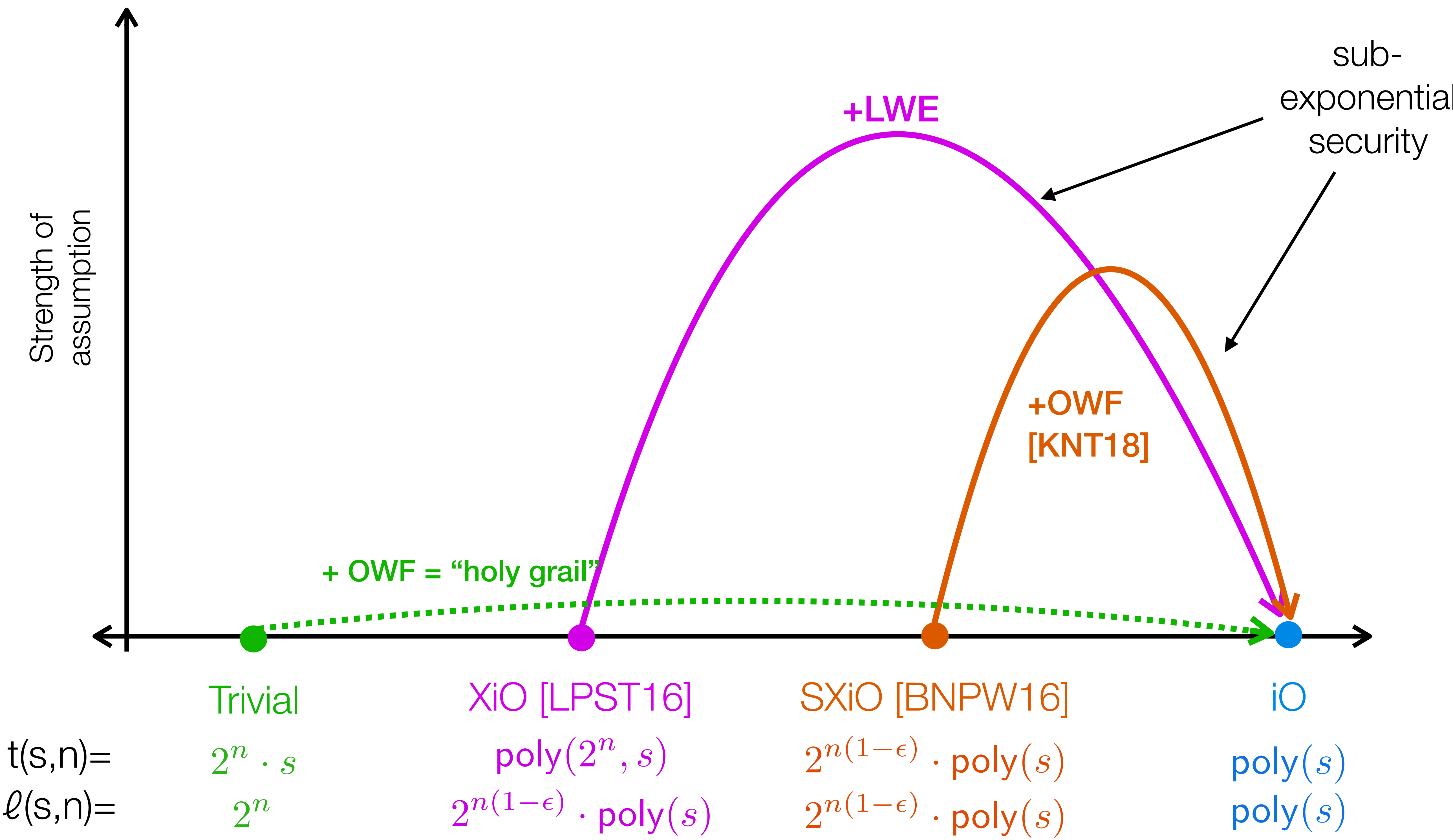

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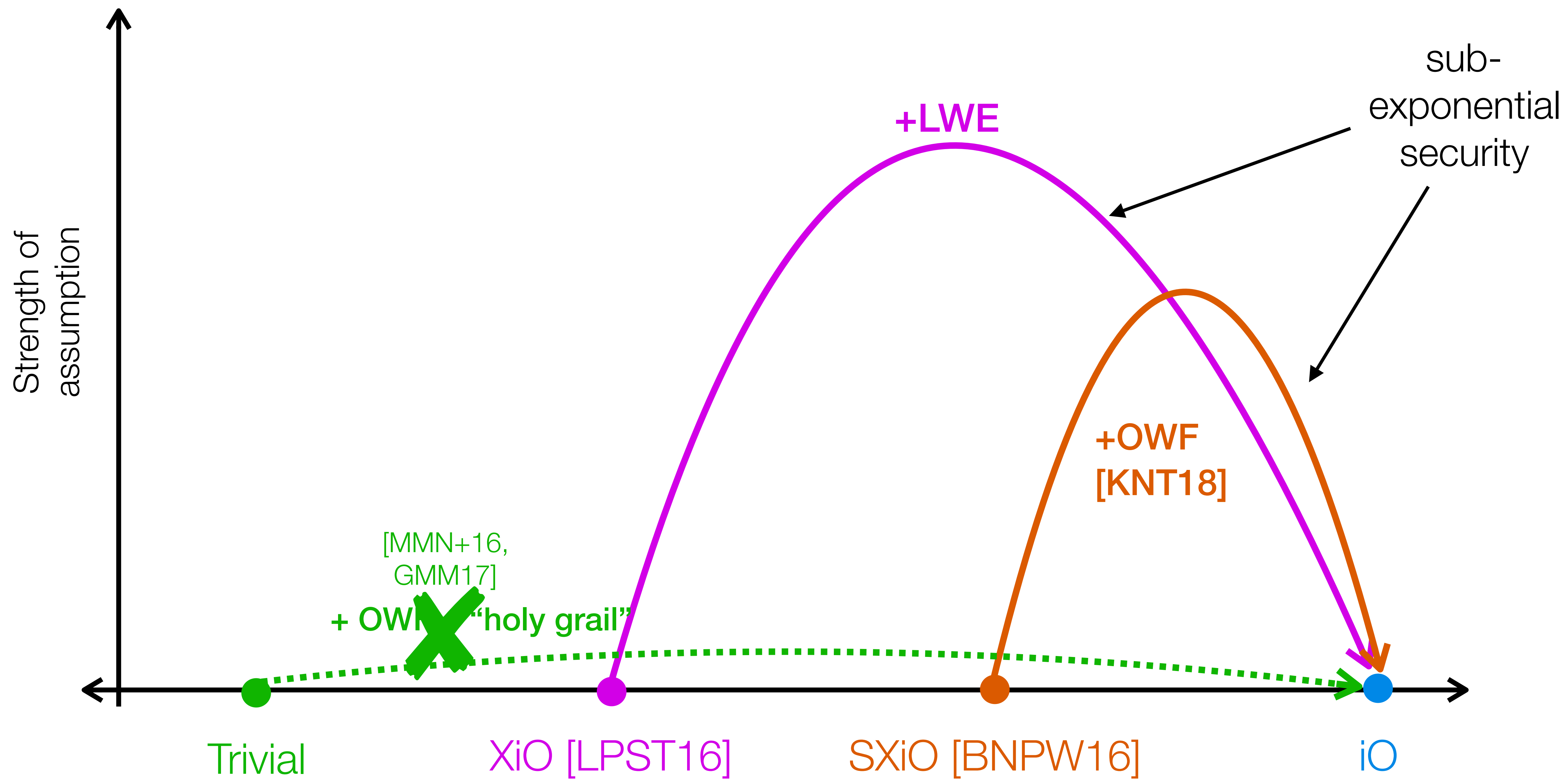

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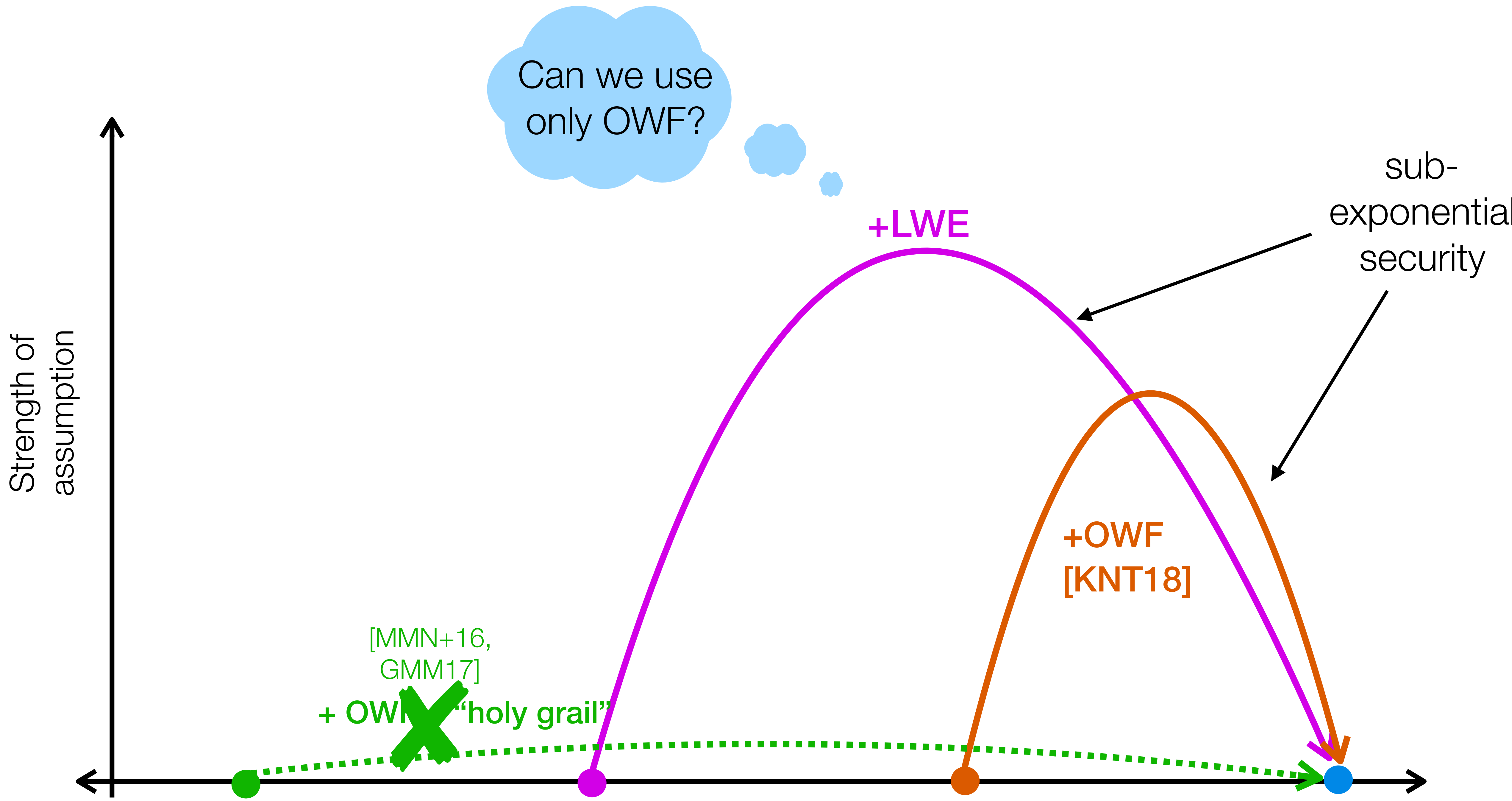

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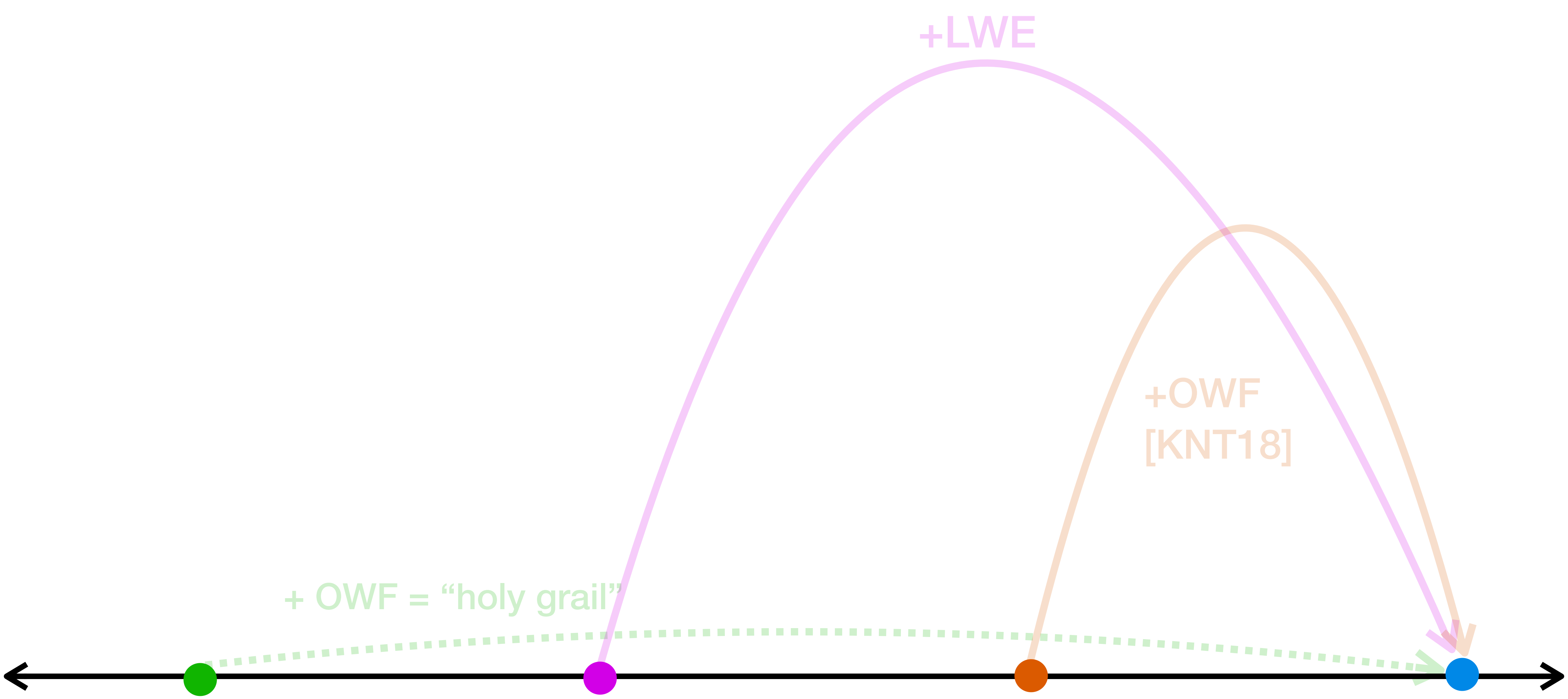

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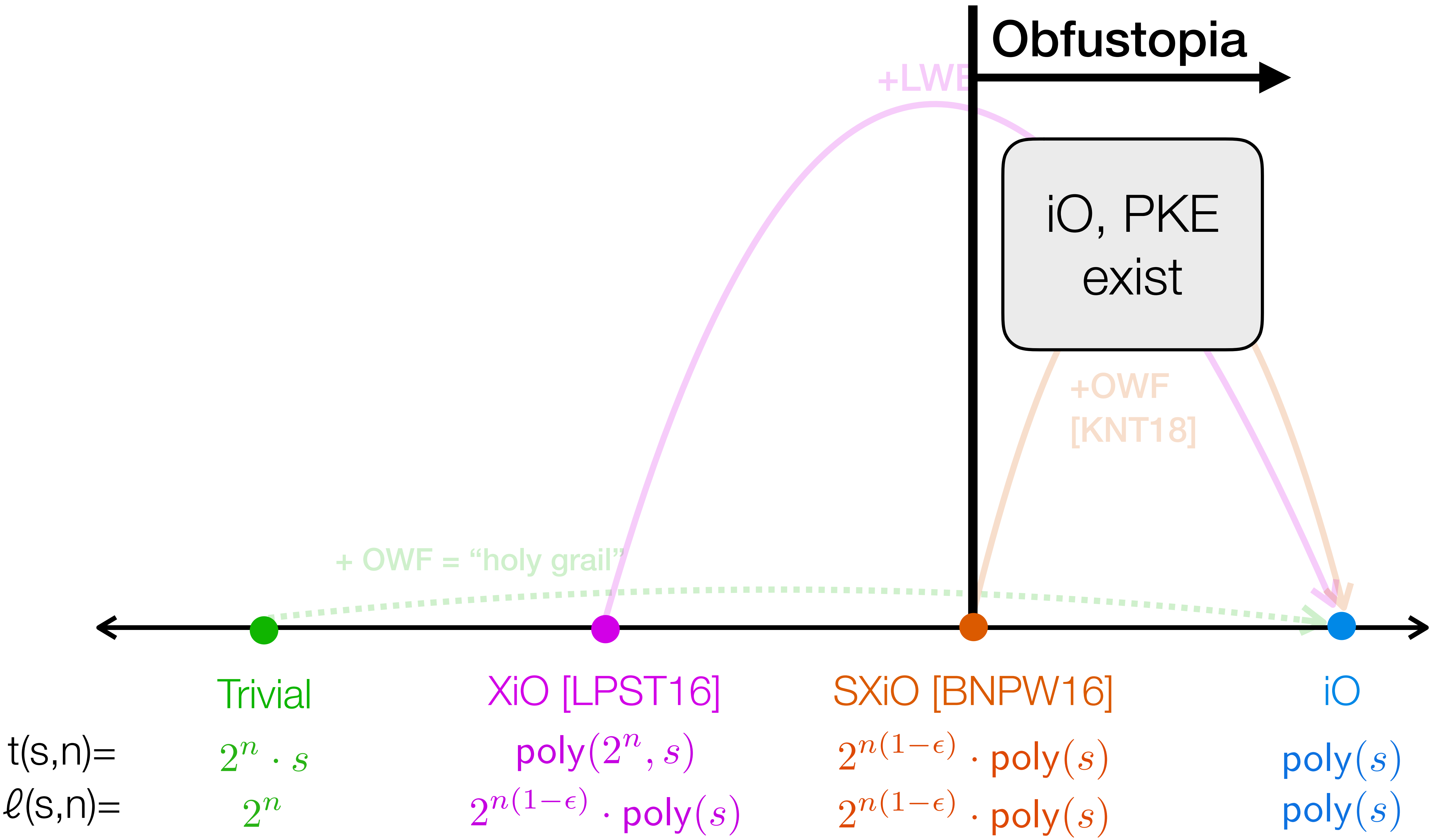

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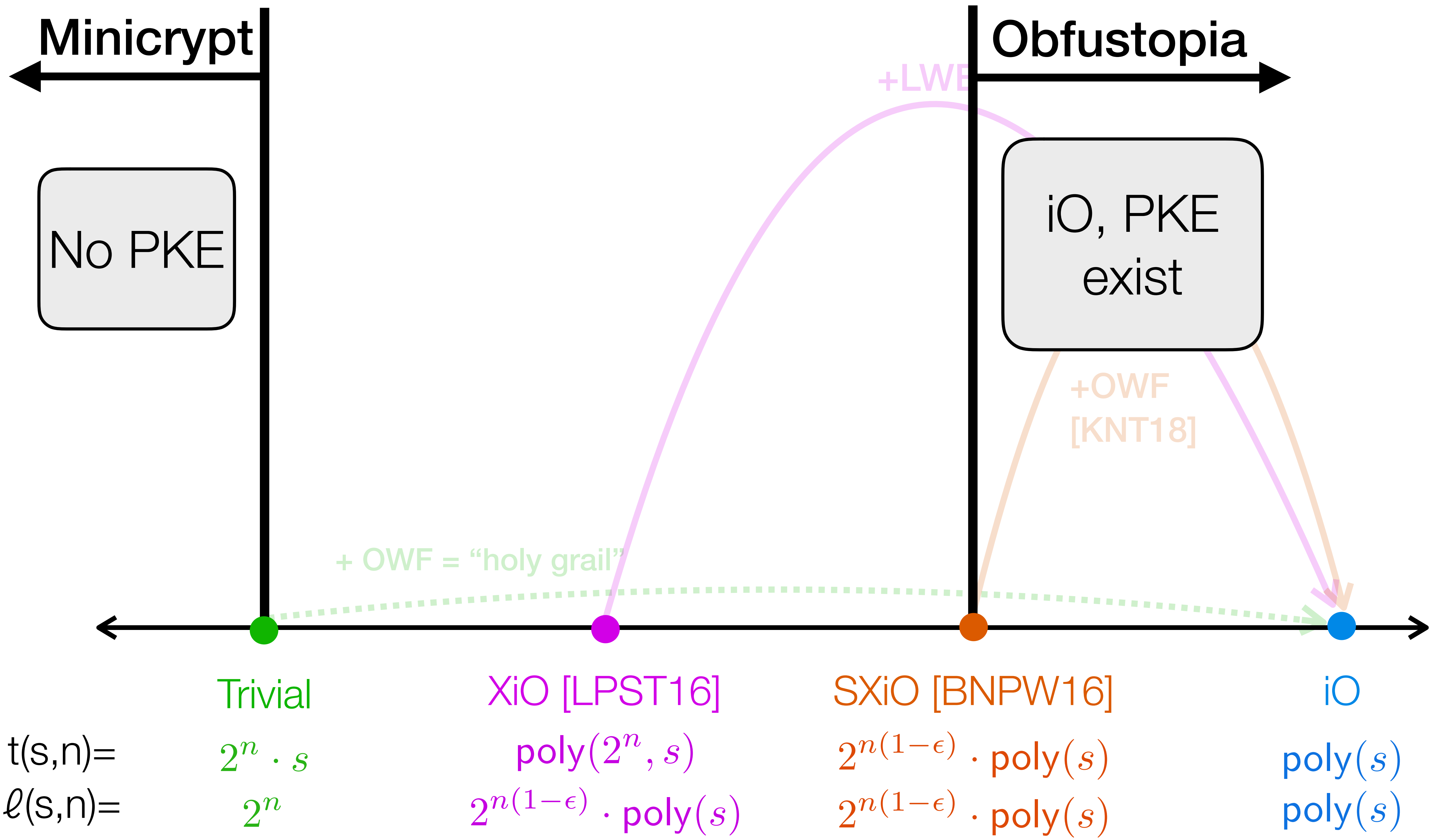

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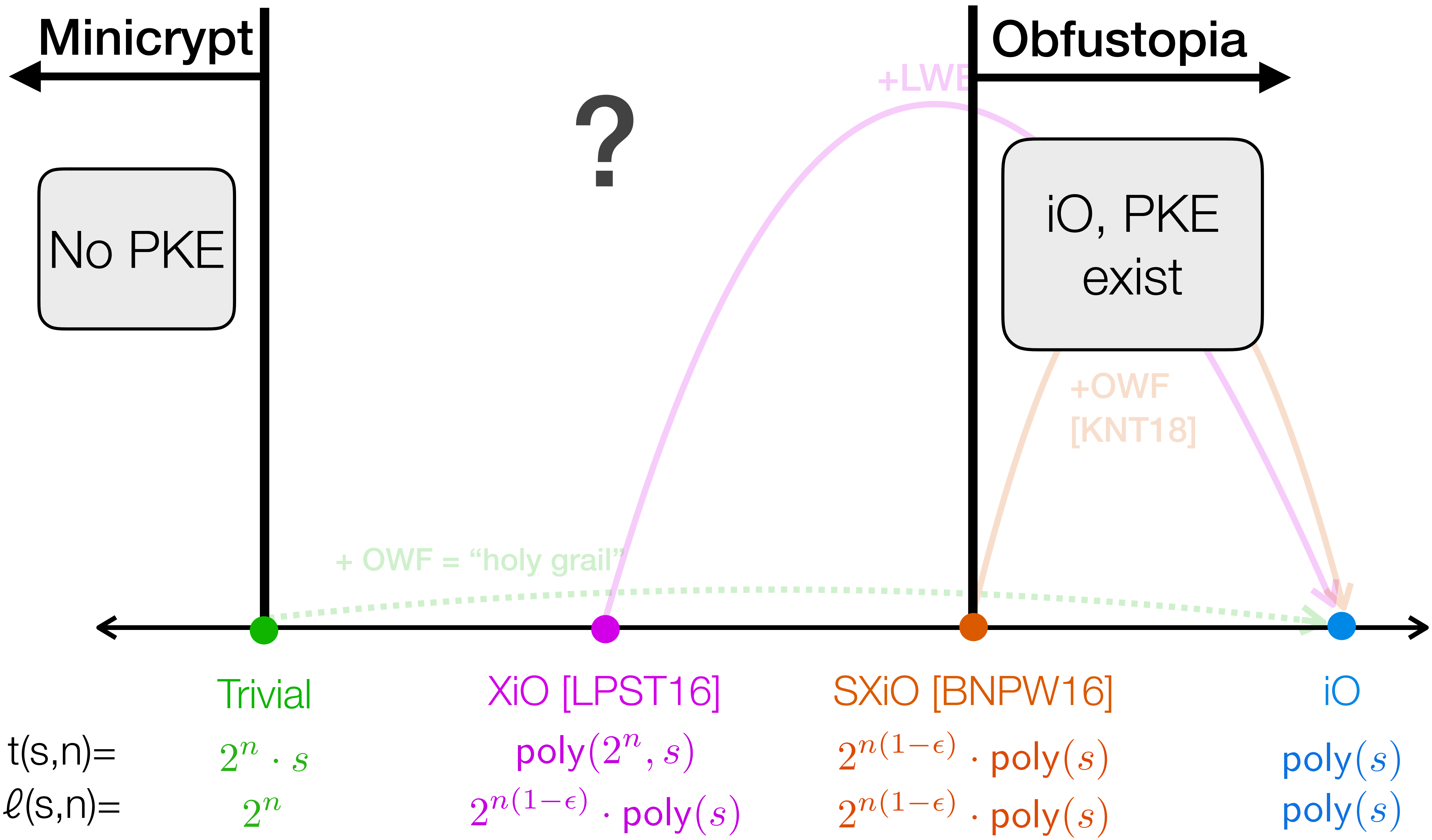

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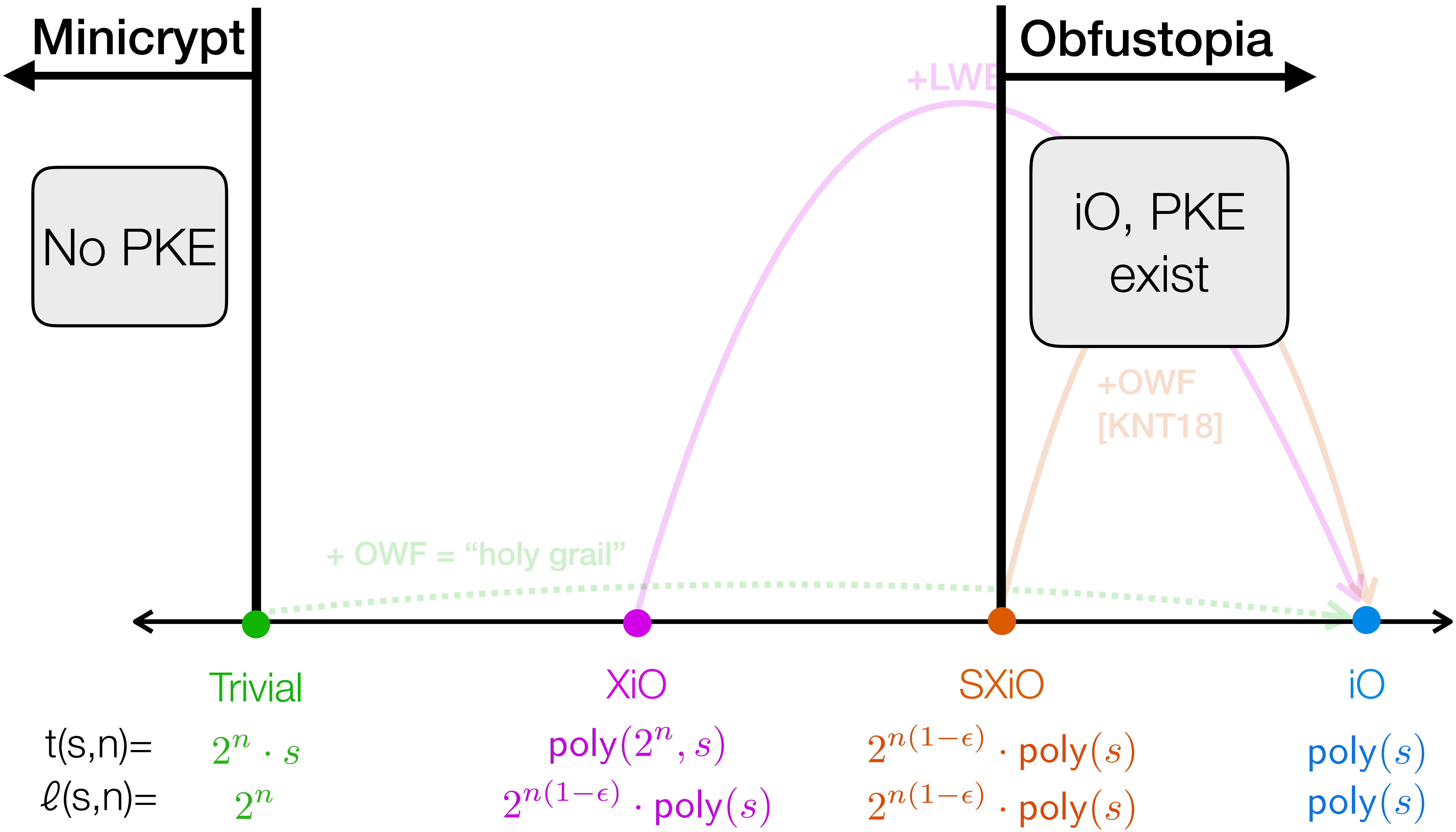

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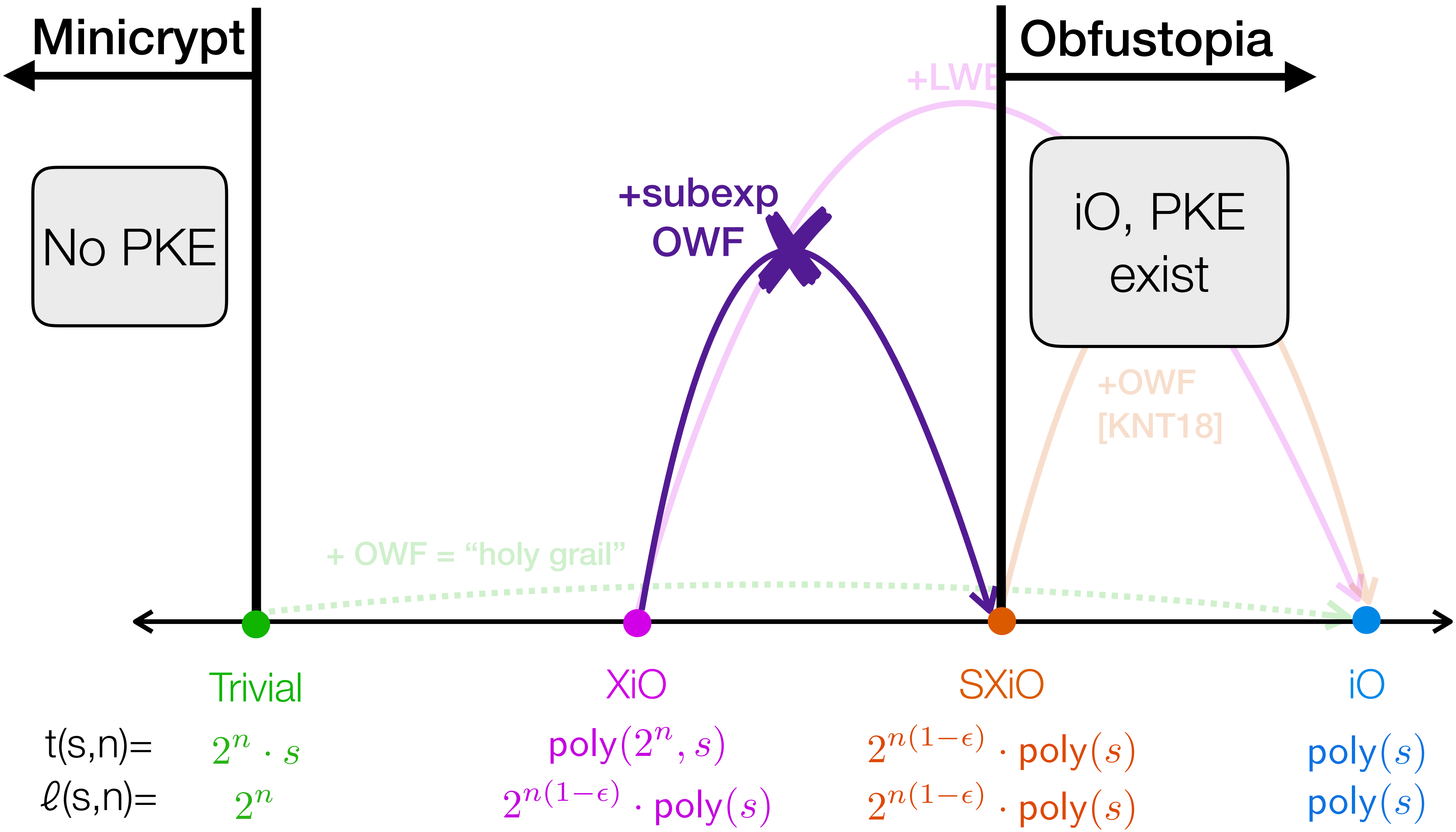

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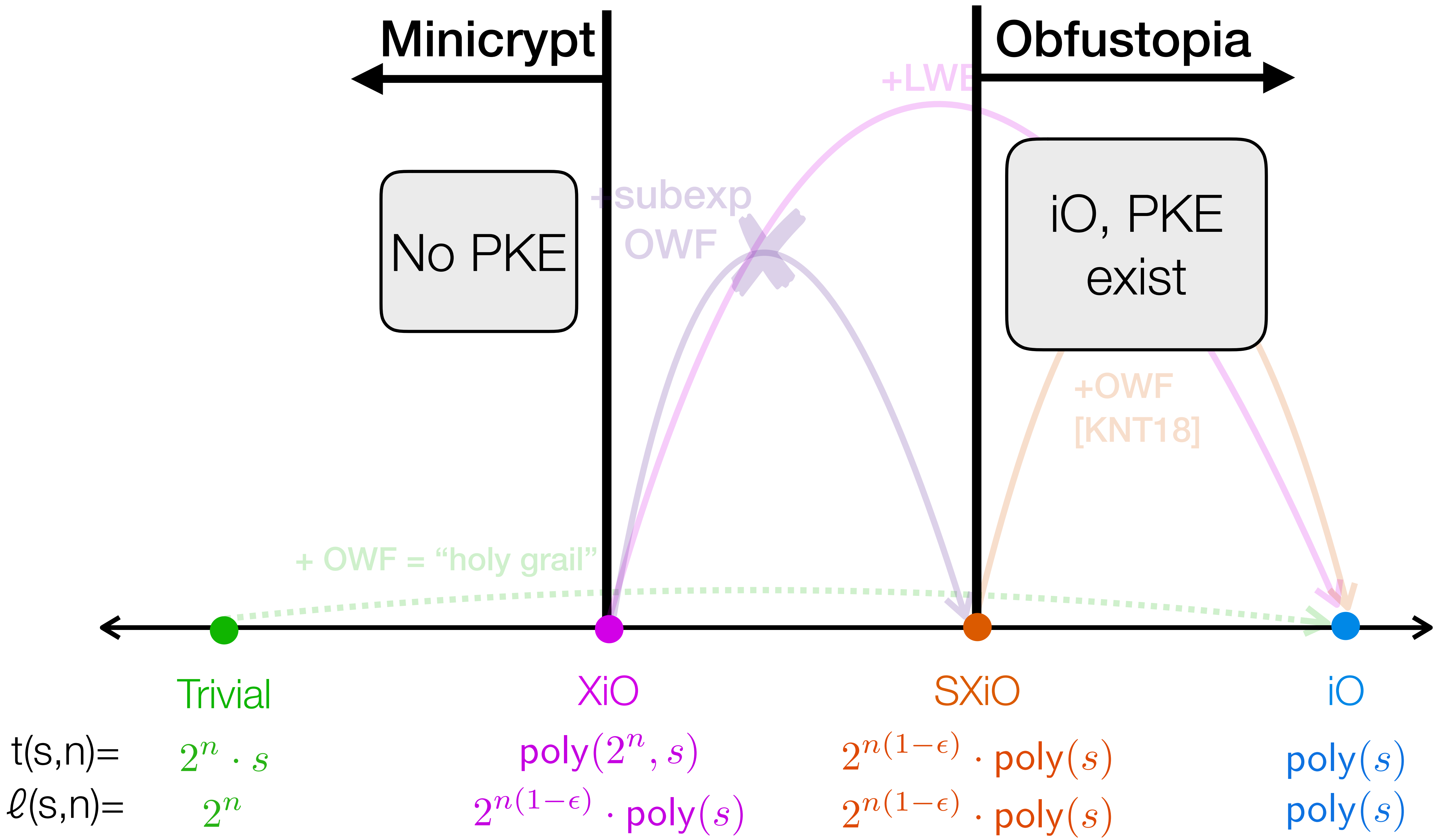

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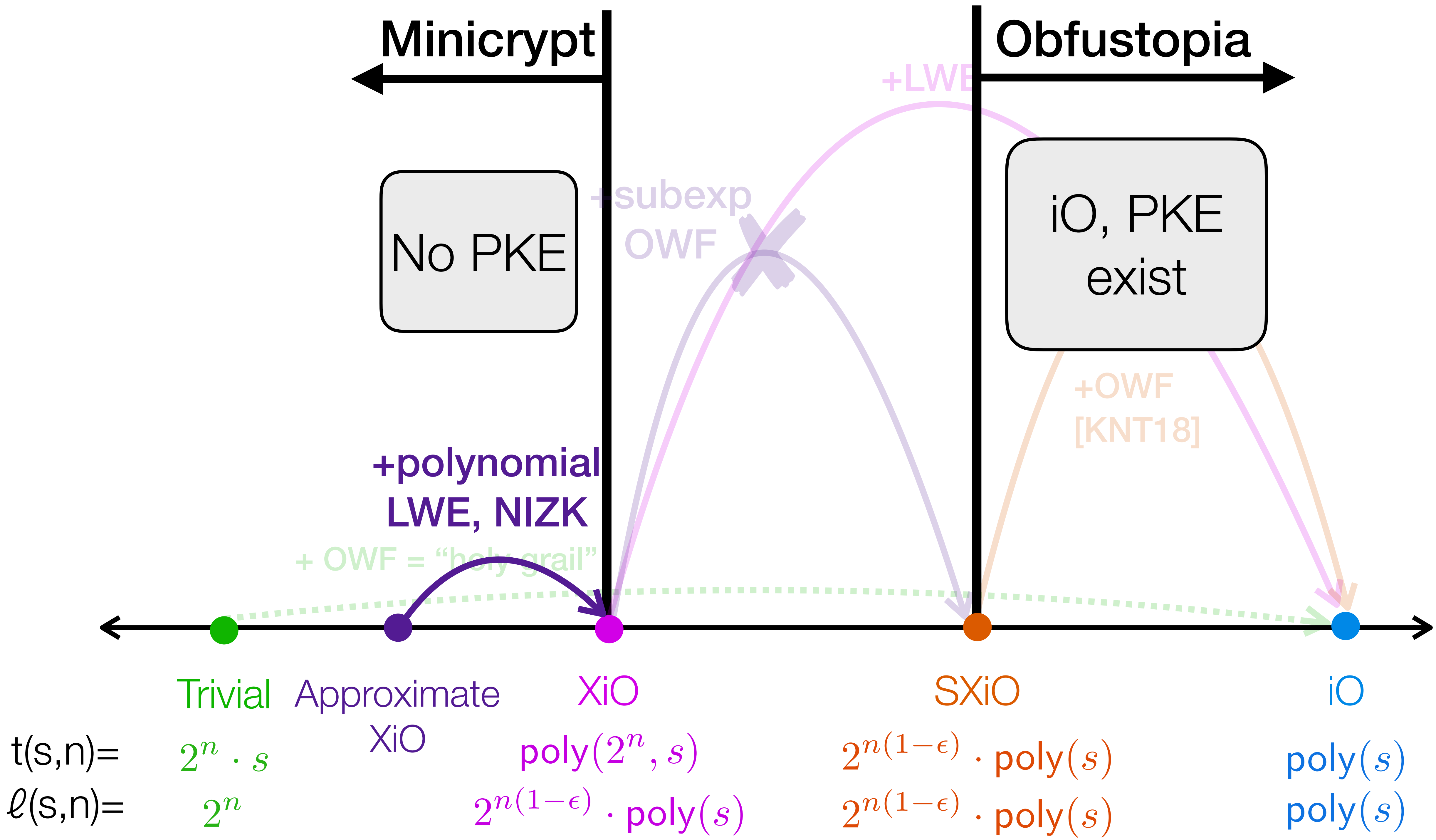

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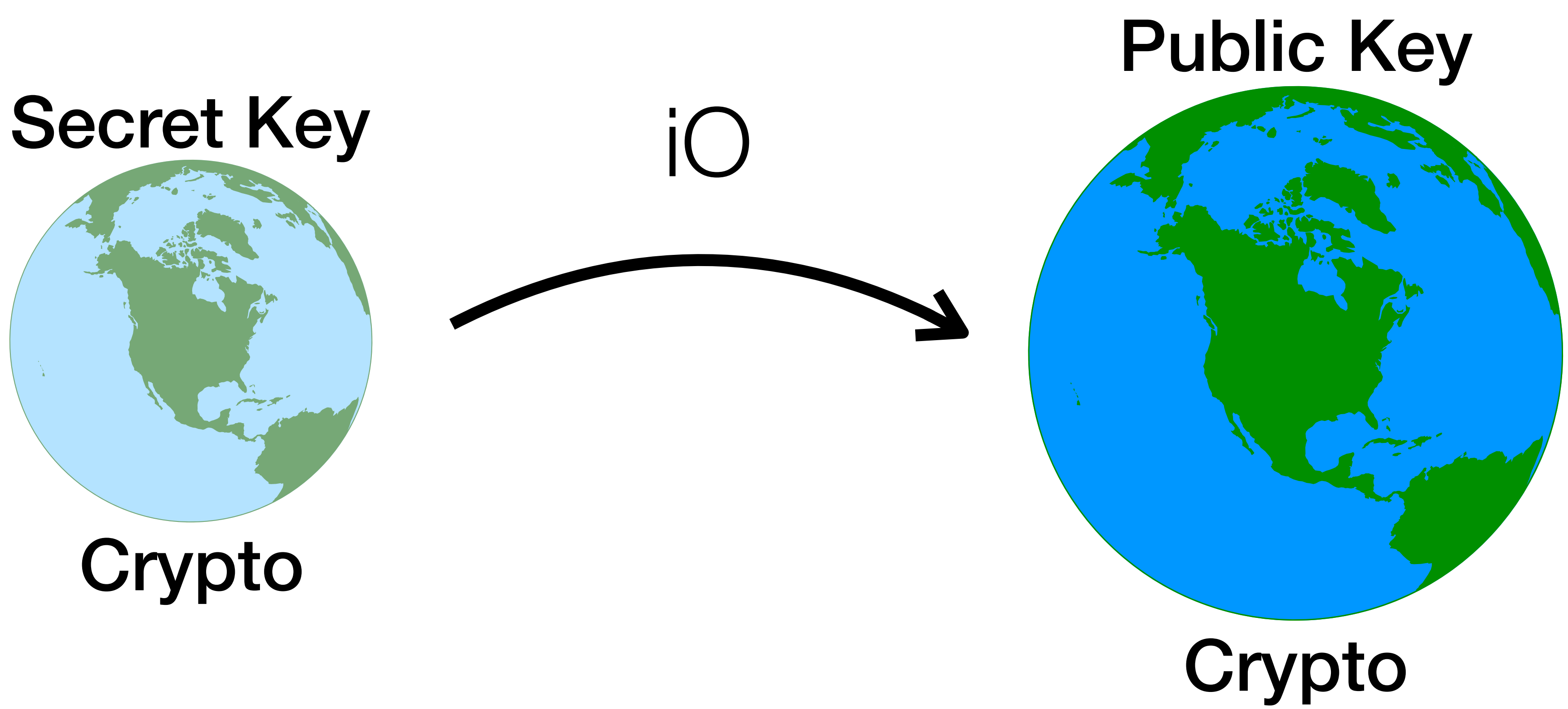

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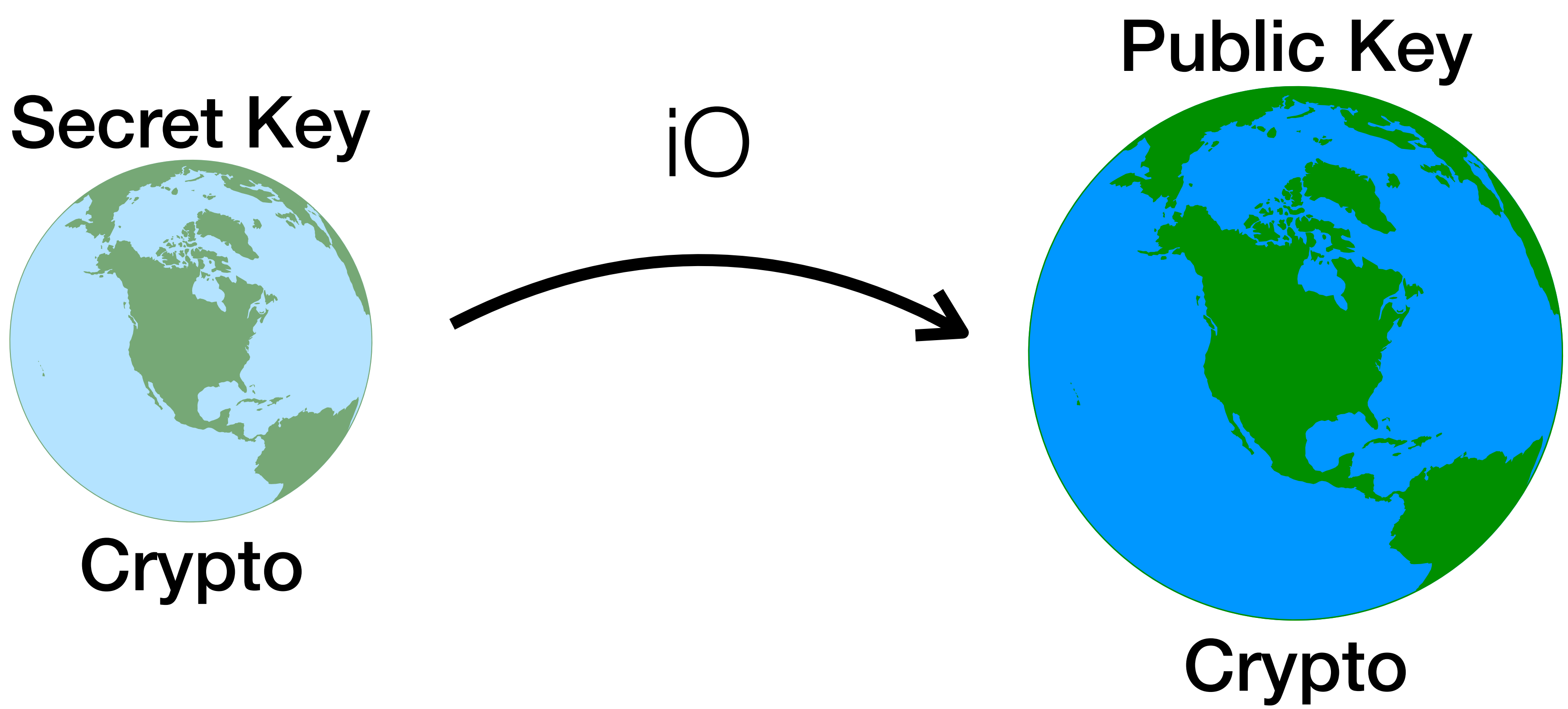
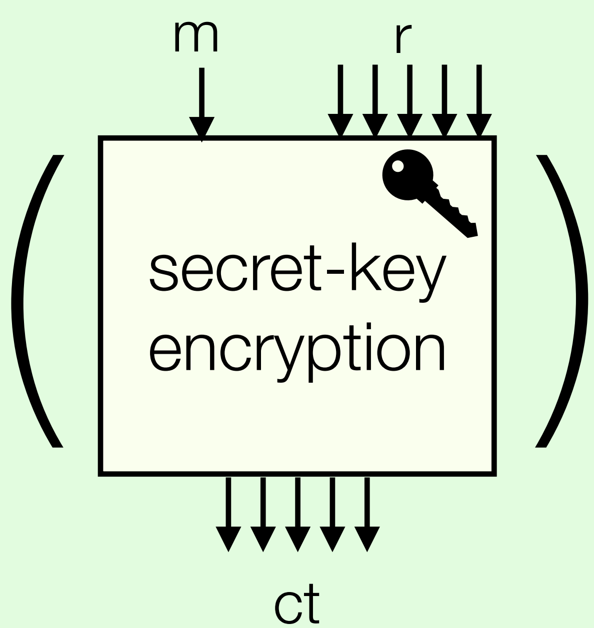


Theorem: **XiO** + **OWF** $\not\Rightarrow$ **PKE** in a black-box way

Intuition: PKE from iO + OWF

[SW14]

pk ← iO.Obf



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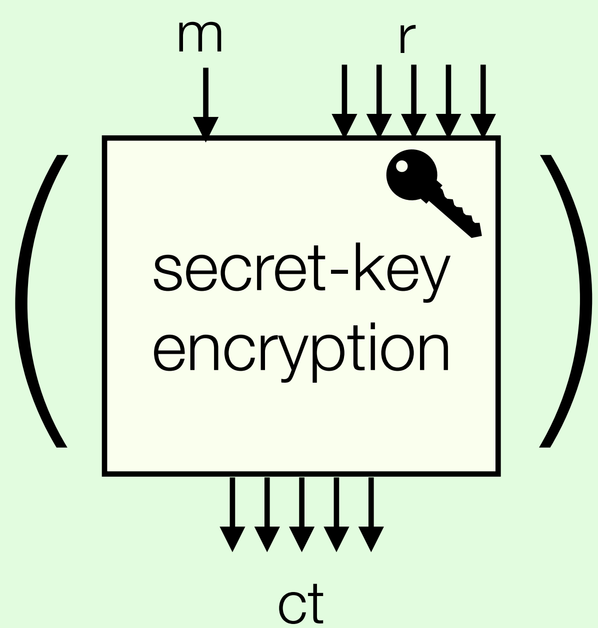

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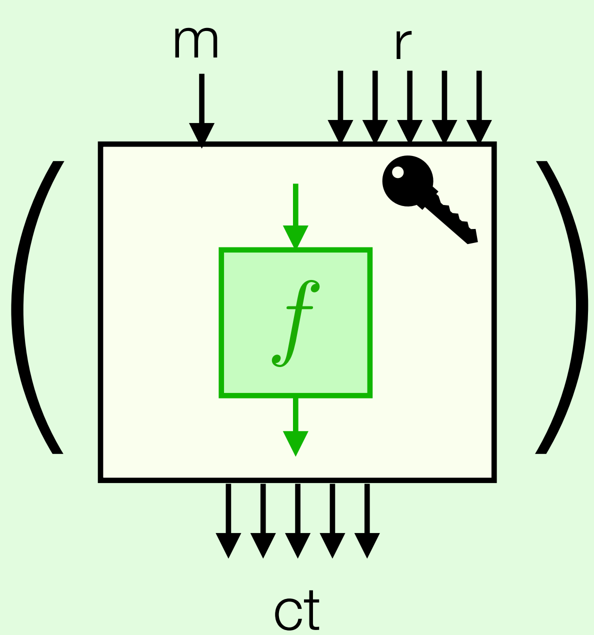
pk ← iO.Obf 

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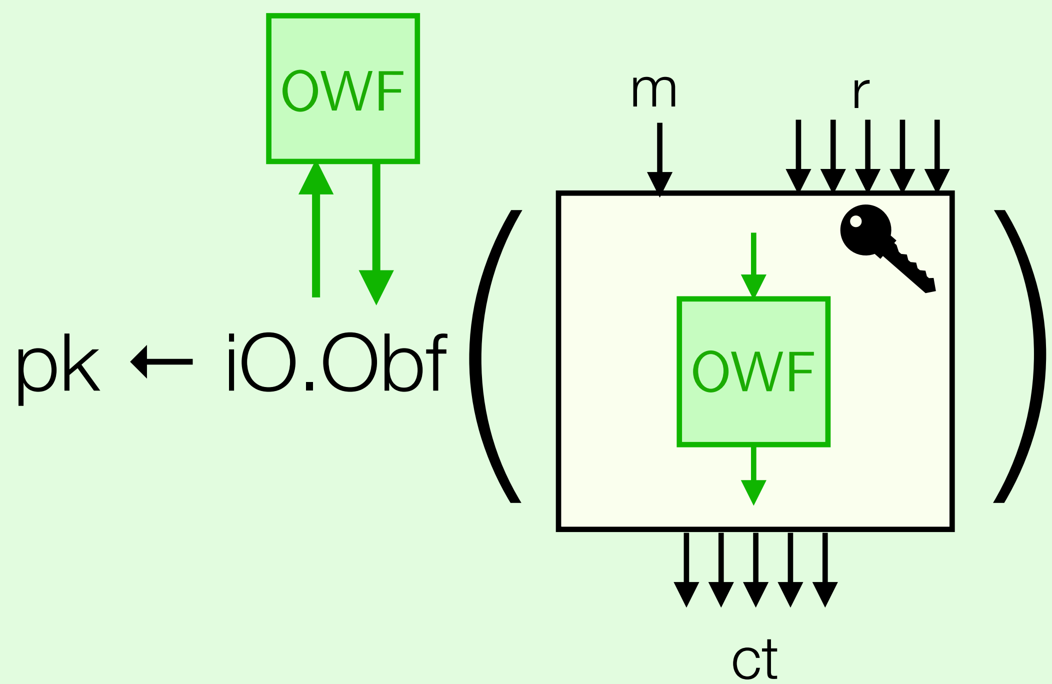

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- First used for circuits with OWF gates [BKSY11, AS16]

  Problem: Separation overcome by new constructions (e.g., PKE from SXiO + OWFs [BNPW16])

- Extended to circuits with iO and OWF gates [GMM17]

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We consider XiO for *oracle-aided* circuits

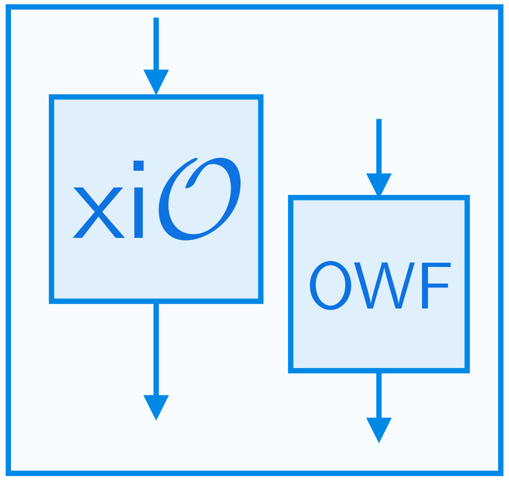
- First used for circuits with OWF gates [BKSY11, AS16]

  Problem: Separation overcome by new constructions (e.g., PKE from SXiO + OWFs [BNPW16])

- Extended to circuits with iO and OWF gates [GMM17]

Our result — extended model

Captures known techniques for iO, e.g., **"self-feeding"** techniques



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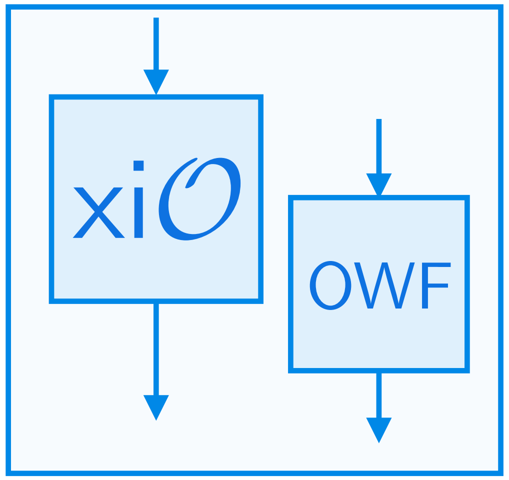

Non-black-box extension of Impagliazzo-Rudich separation [IR89]

Our Results and Outline

1. Power of compressing obfuscation

XiO + one-way functions $\not\Rightarrow$ public-key encryption in a black-box way

2. Existence with statistical security

- ✅ Constructions for “powerful” class of circuits (e.g., AC^0)
- ❎ Unlikely to exist with stronger compression

3. Existence under computational assumptions

Approximately-correct (S)XiO + polynomial LWE + NIZK $\Rightarrow$ correct (S)XiO

Statistically Secure Compressing Obfuscation

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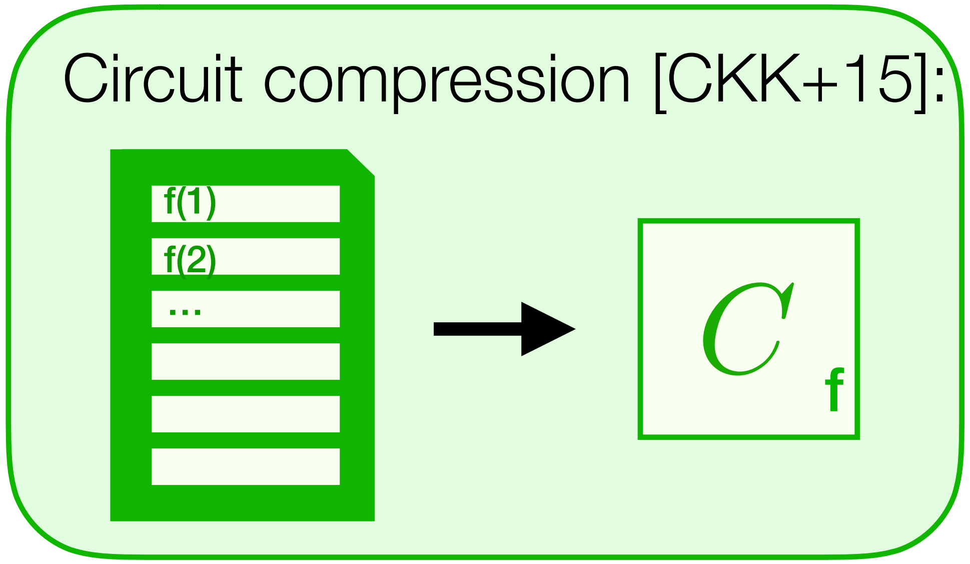

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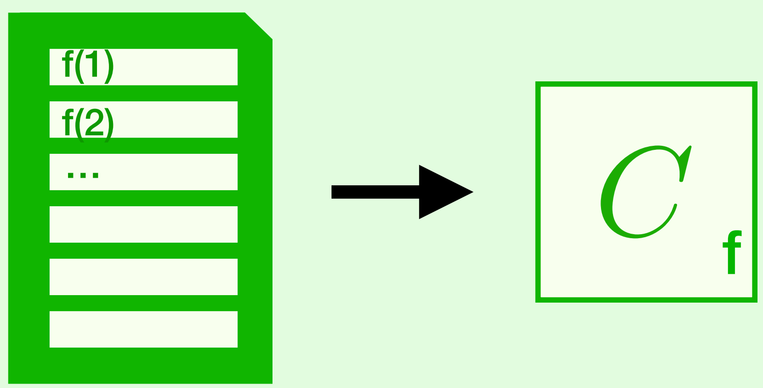
Circuit compression [CKK+15]:



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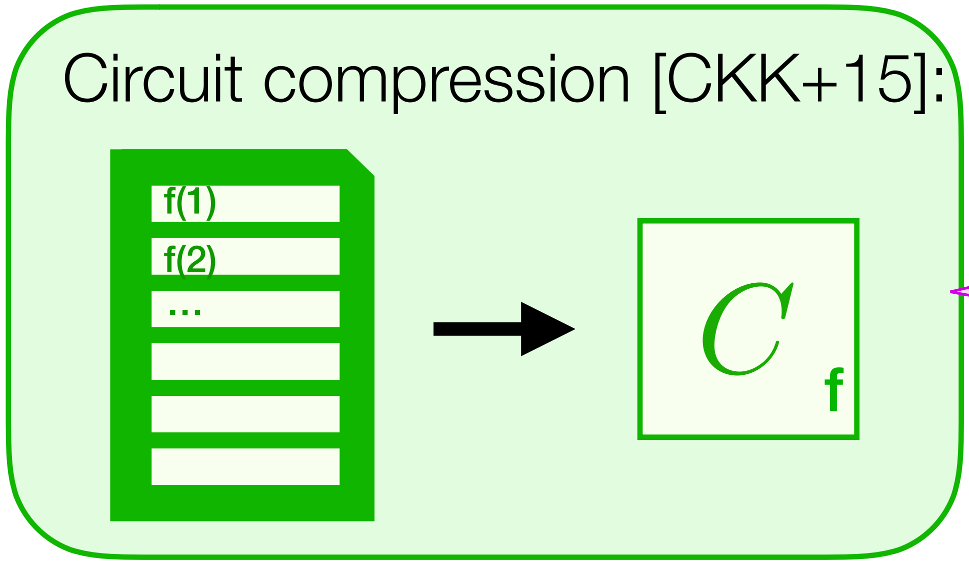

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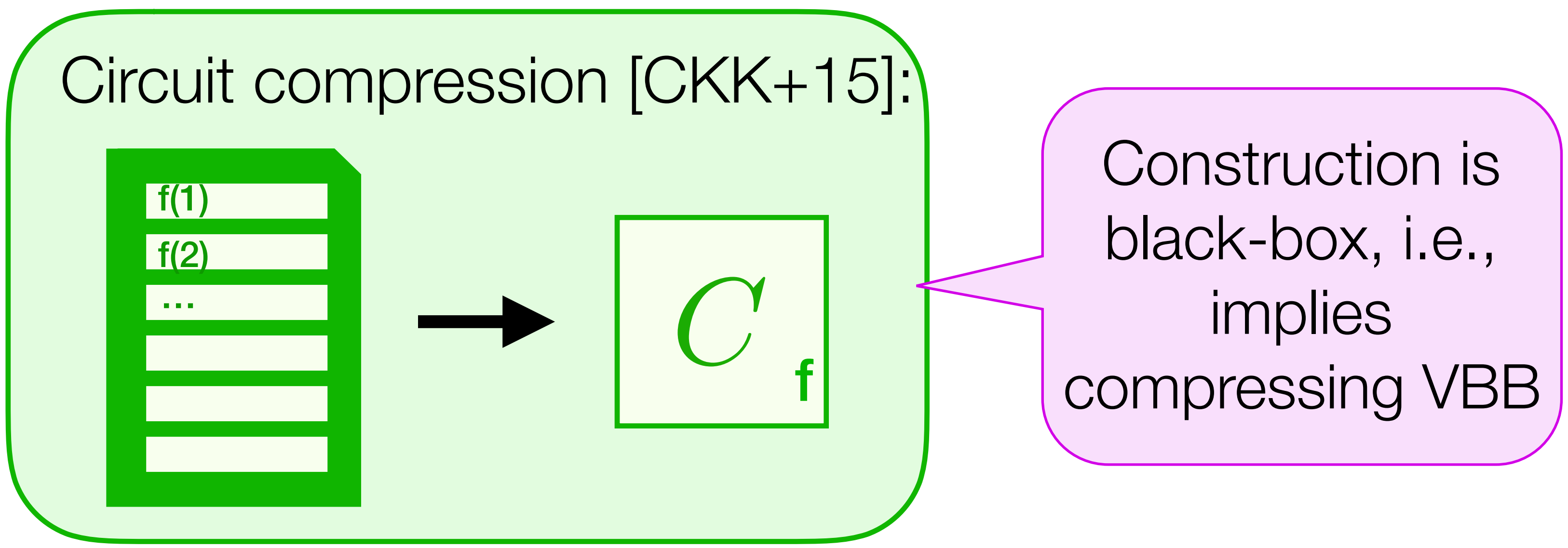

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Theorem: $(2^{\epsilon n}, 2^{\epsilon n})$-compressing obfuscation for depth 2 circuits implies UNSAT $\in$ AM$[2^{c\epsilon n}]$ for a constant c

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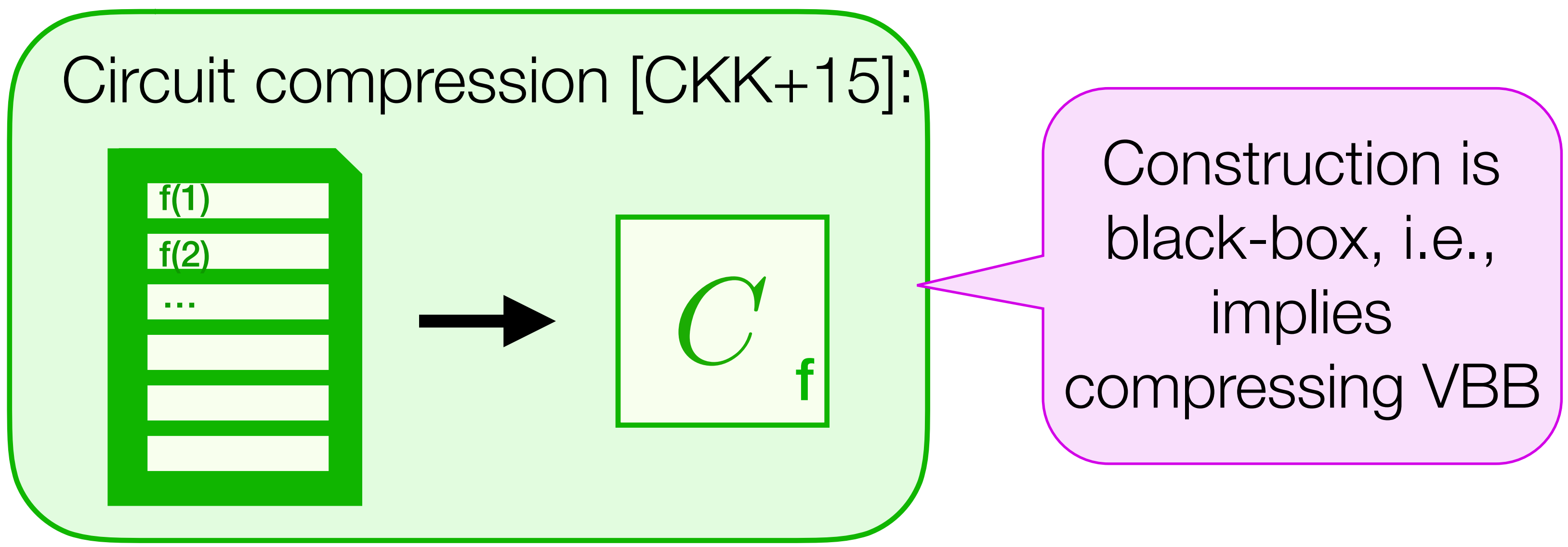

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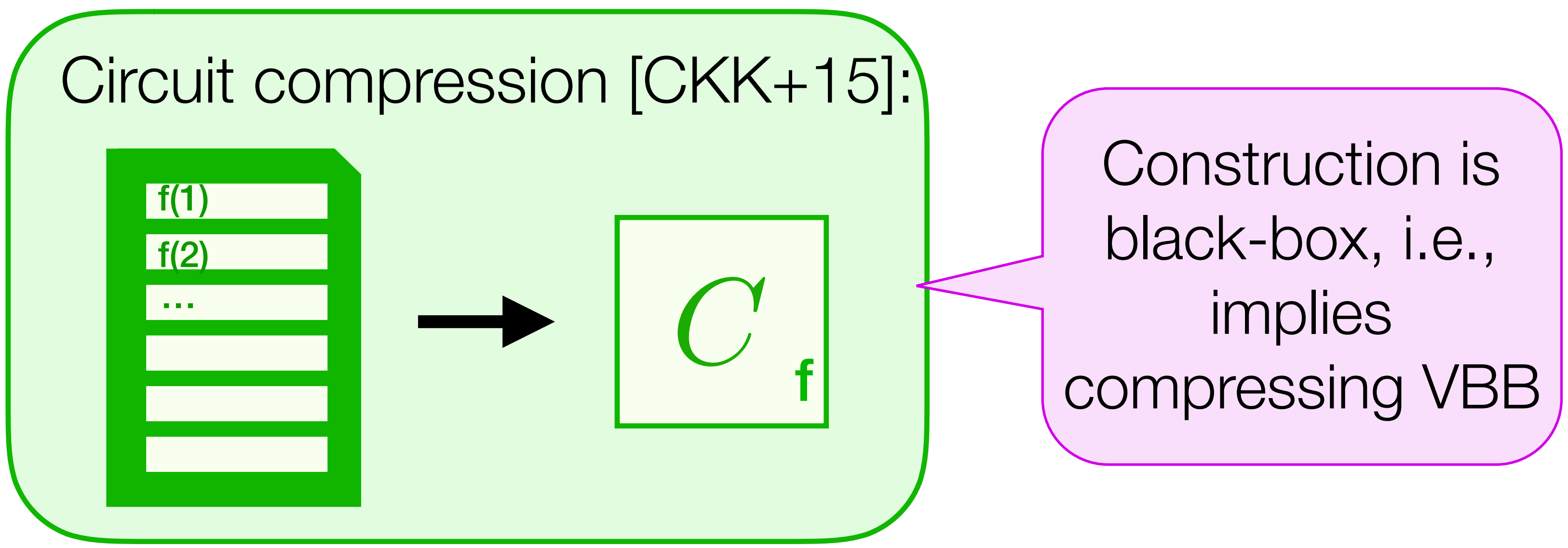

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- Conclusion is true when ϵ = 1/2 [W16]
- Unknown for smaller ϵ

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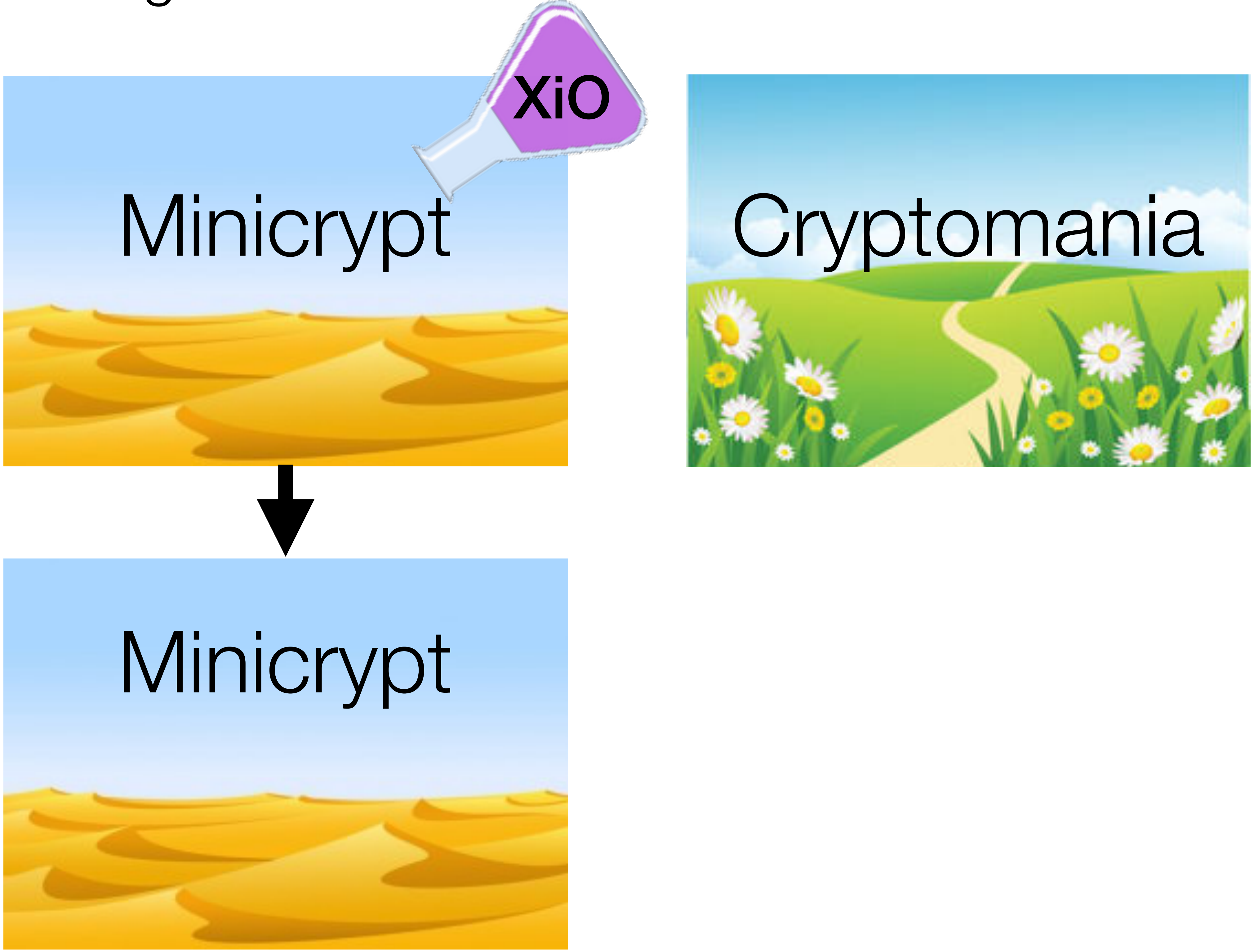

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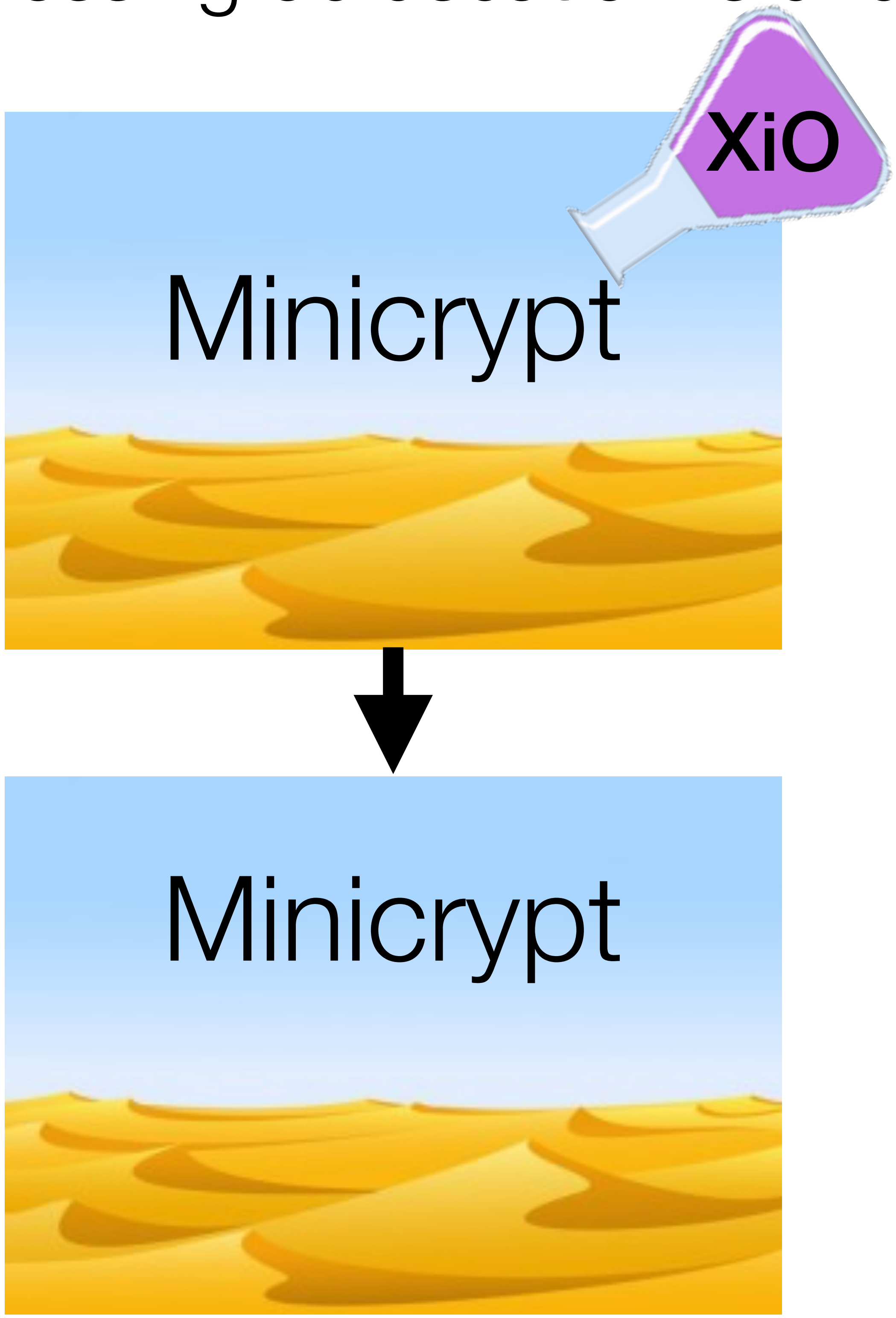

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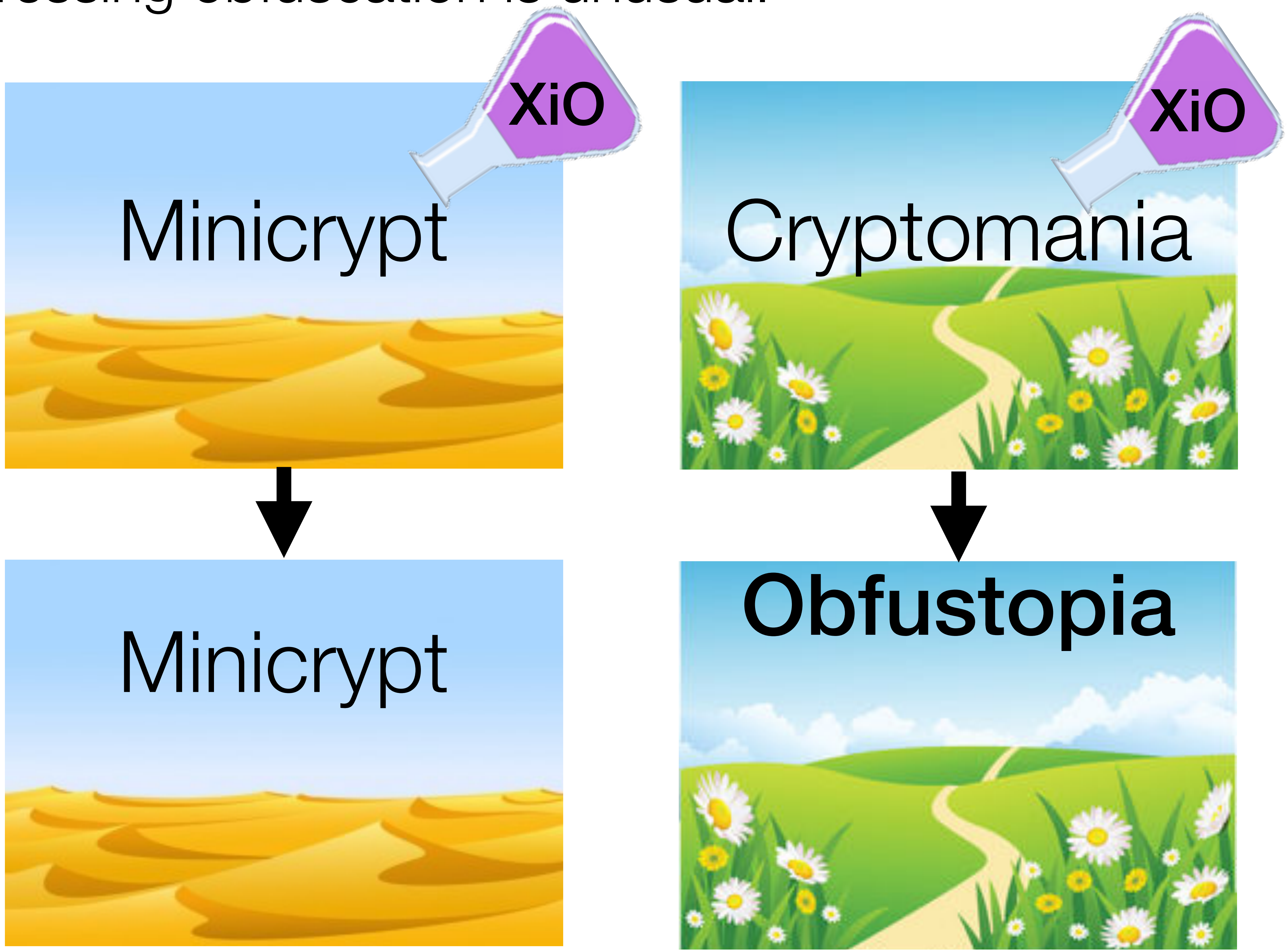

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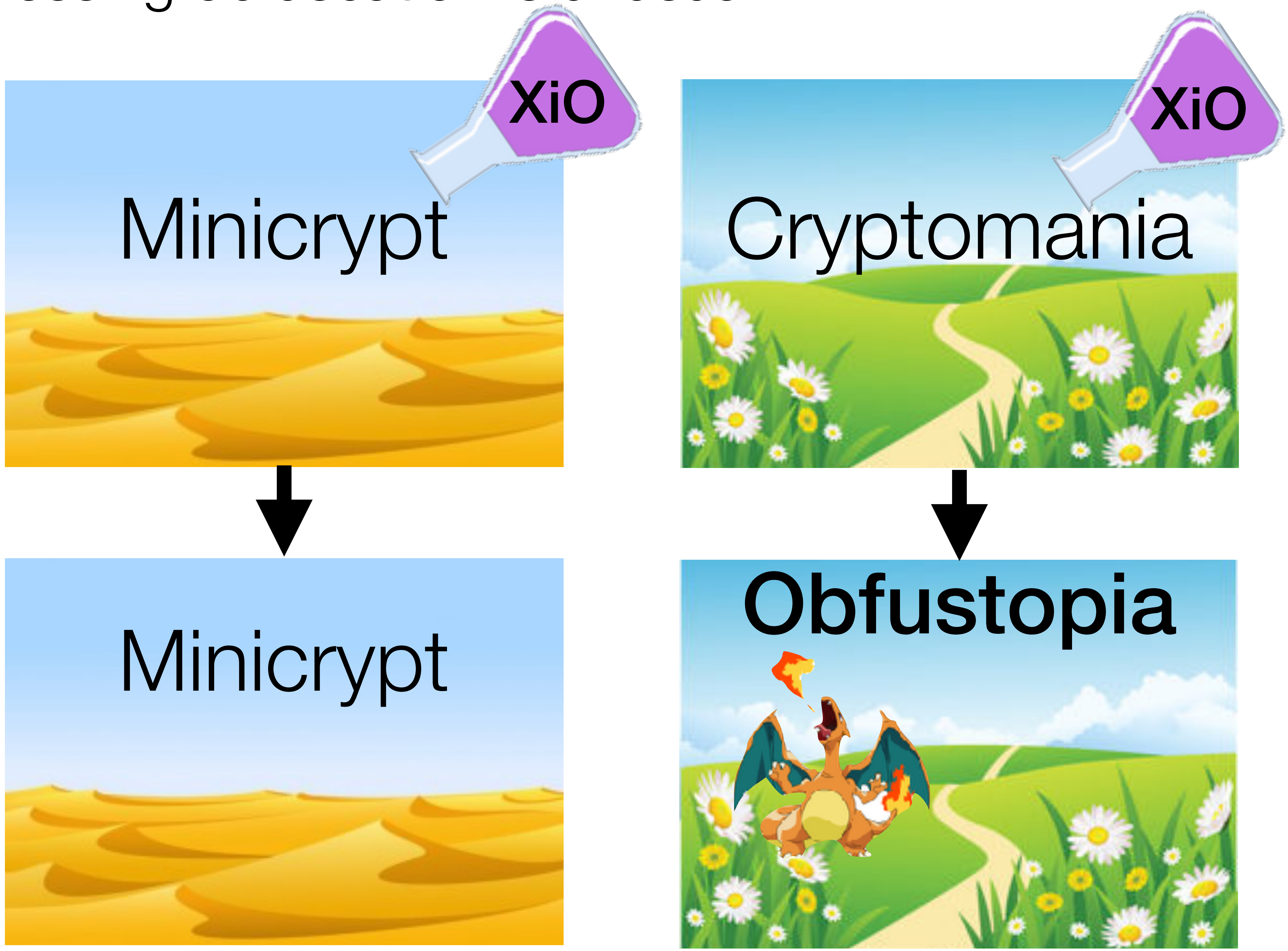

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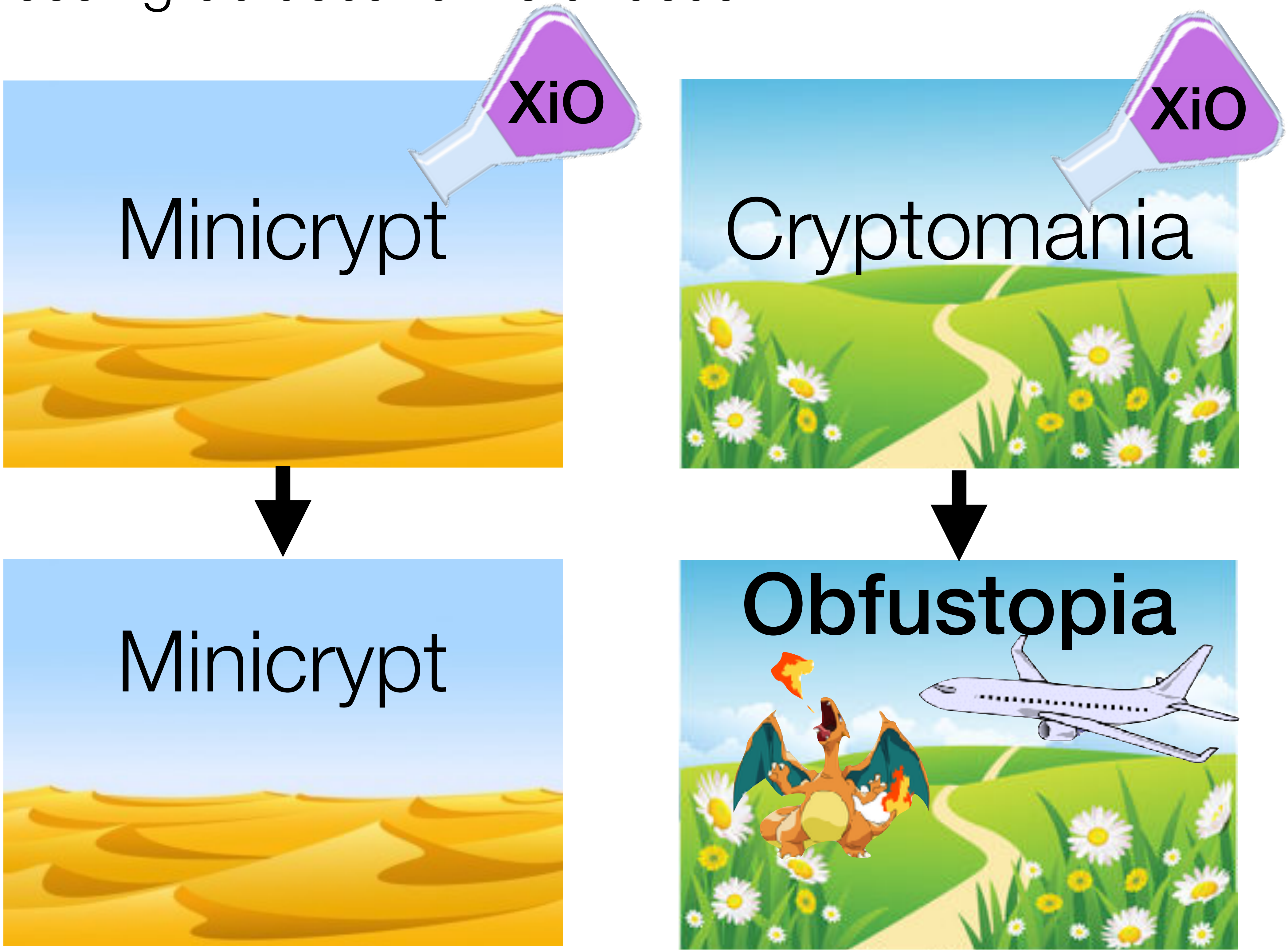

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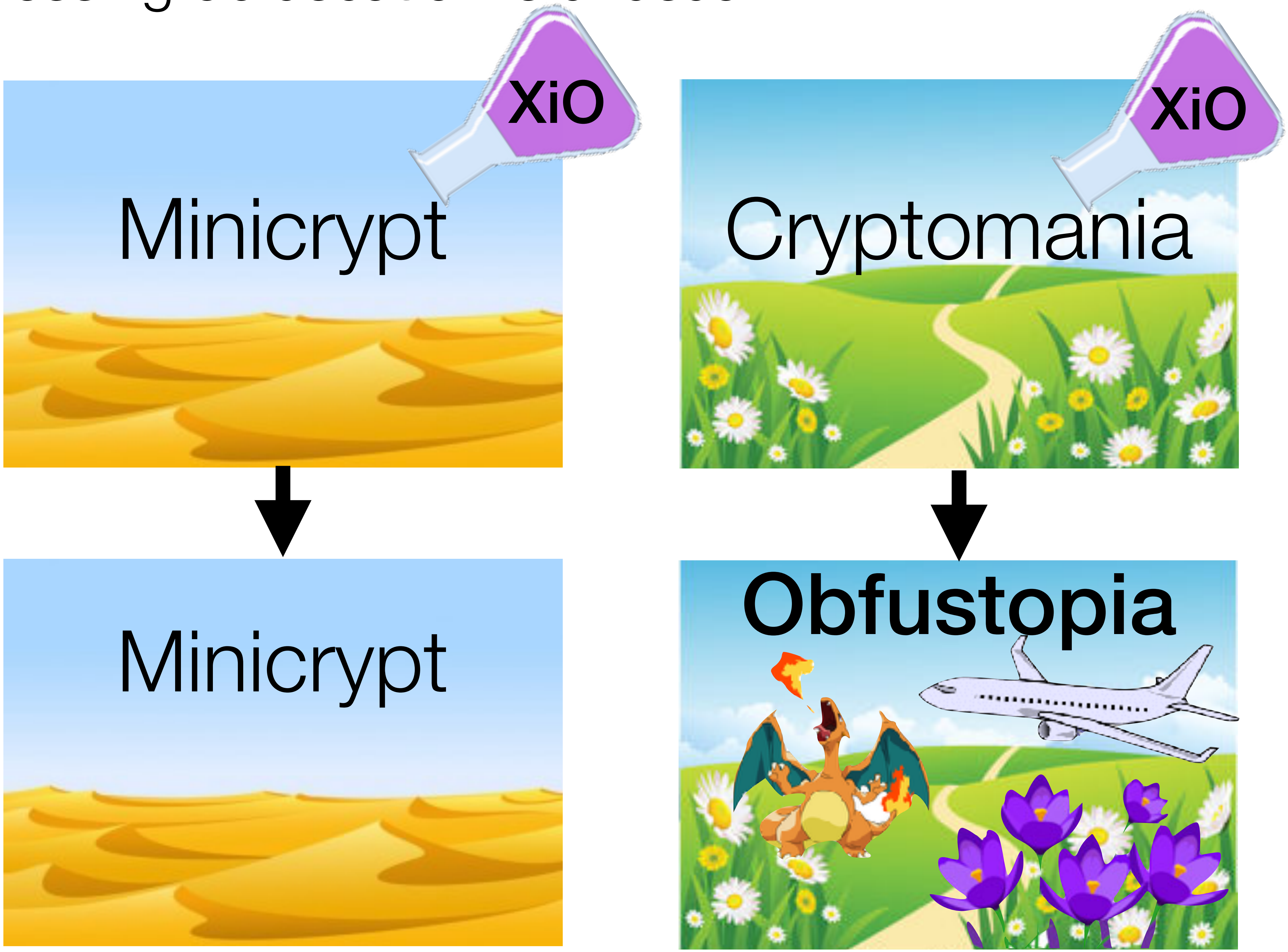

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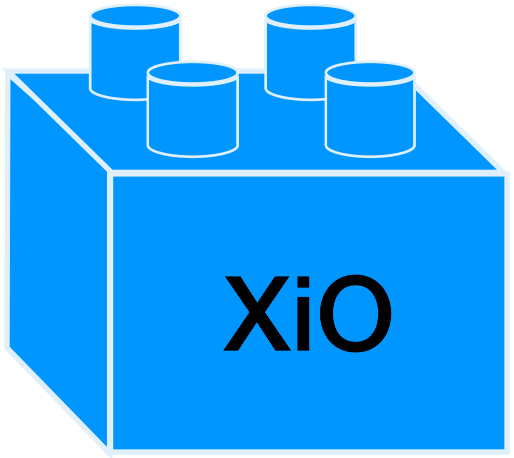

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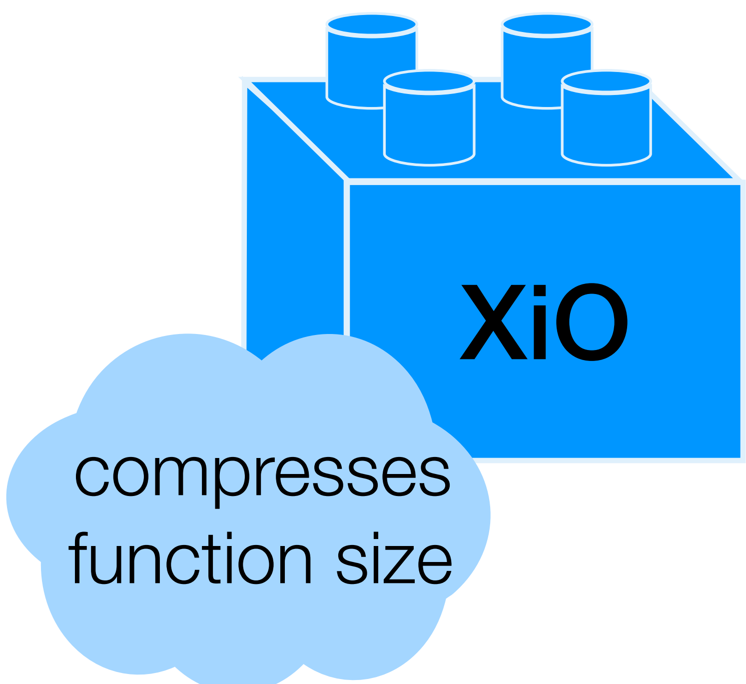

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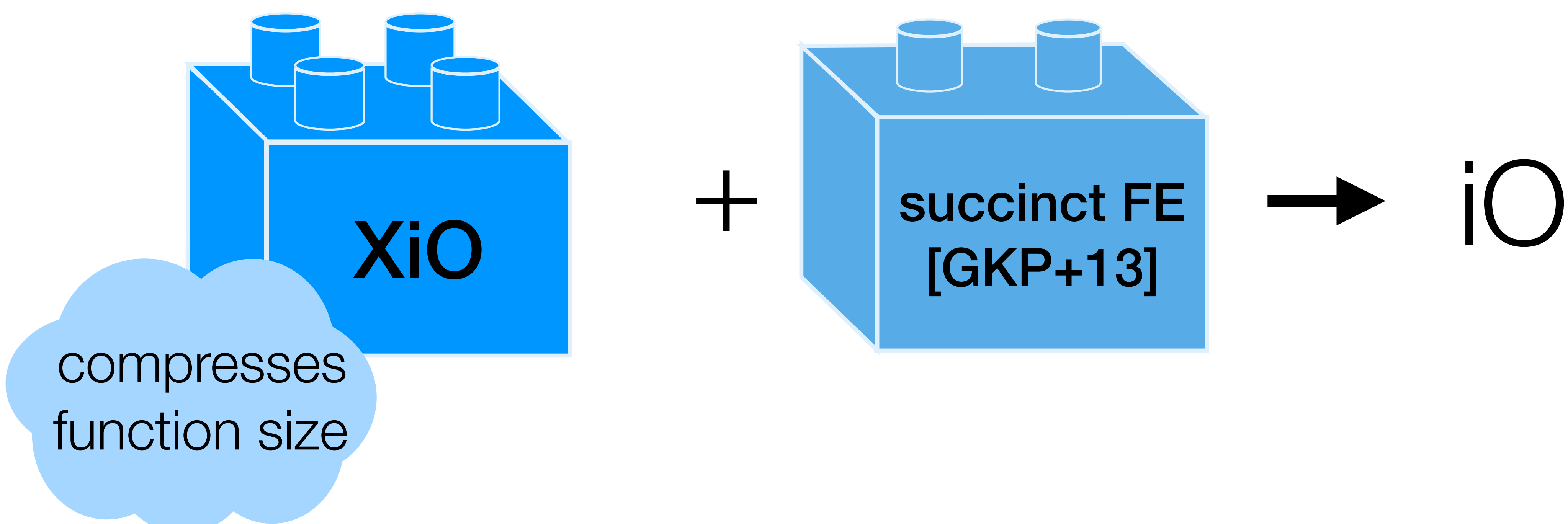

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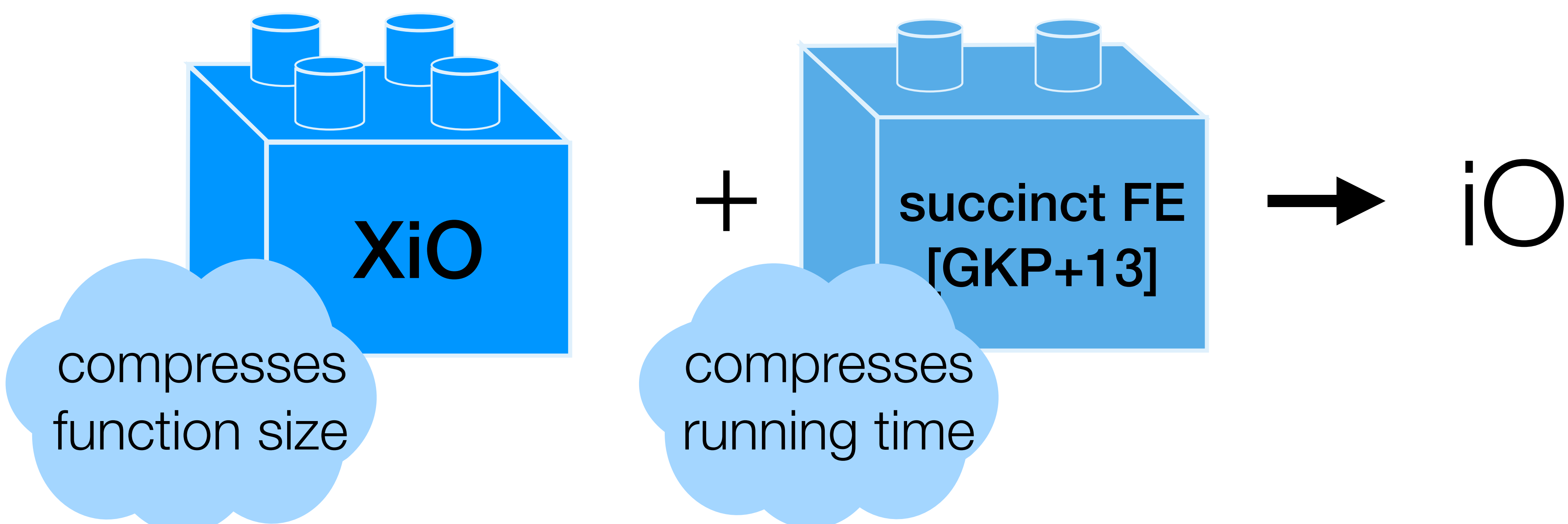

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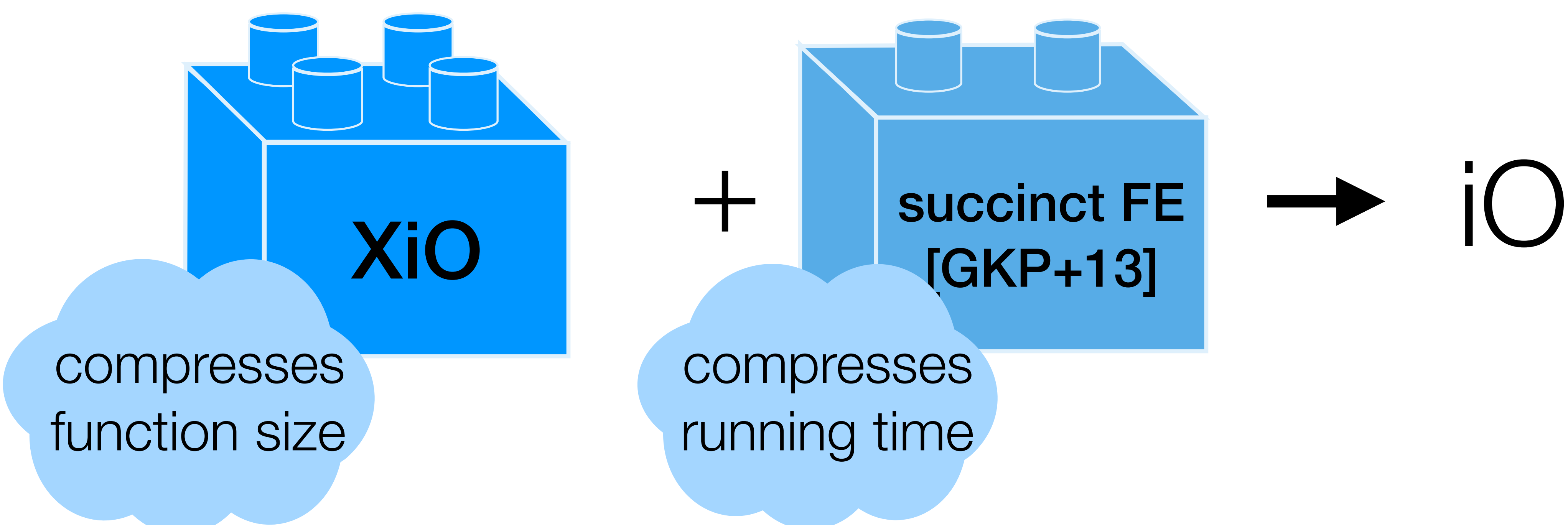

Thank you!