Practical and Tightly-Secure Digital Signatures and Authenticated Key Exchange

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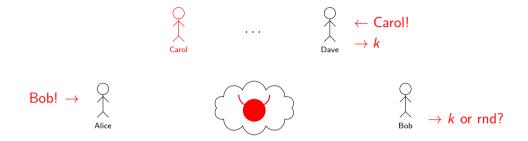




Alice and Bob want to exchange a key.

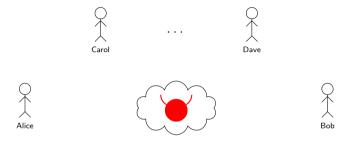


Alice and Bob and Carol and \dots want to exchange keys.



The adversary

- directs honest users' key exchanges;
- can inspect keys or test keys (real-or-random);
- controls the network;
- and can adaptively corrupt users.



Our goal: If

▶ Alice believes she has exchanged a key with Bob,

it must be true that

- ▶ Bob exchanged exactly the same key exactly once with Alice; and
- it looks random.

Why Security Proofs?

A typical security proof is a reduction from some problem to attacking our crypto:

A poly-time crypto adversary with non-negligible advantage gives us a poly-time problem solver with non-negligible advantage.

The scheme is secure if we believe no such solver exists.

Why do we want them?

- ► Why not?
- ► Ensure that our system is not trivially breakable.
- Help us choose security parameters.

Why Security Proofs?

A typical security proof is a reduction from some problem to attacking our crypto:

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A crypto adversary using time T_1 with advantage \epsilon_1 gives us a problem solver using time T_2 with advantage \epsilon_2.
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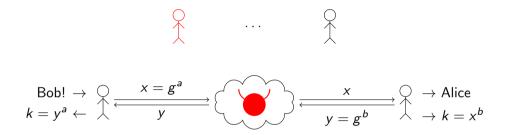
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▶ A reduction is tight if $T_1 \approx T_2$ and $\epsilon_1 \approx \epsilon_2$.

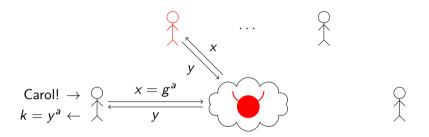
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No Tampering + Static Corruption = No Problem

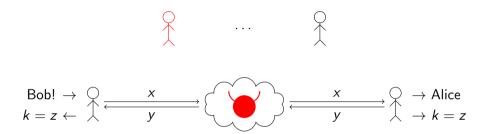


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When Alice talks to a corrupted Carol, we run Diffie-Hellman as usual.

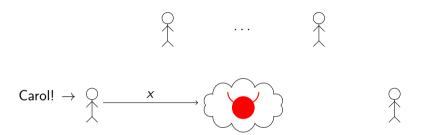
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When Alice talks to an honest Bob, we simulate the conversation using a Diffie-Hellman tuple (g, x, y, z).

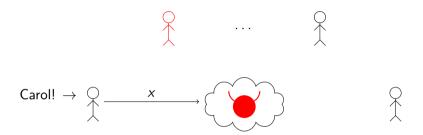
Rerandomization gives us many tuples and a tight proof.

No Tampering + Adaptive Corruption = Commitment Problem



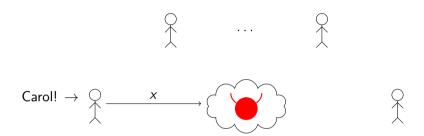
The commitment problem: the adversary may corrupt the responder after we have committed by sending the first message.

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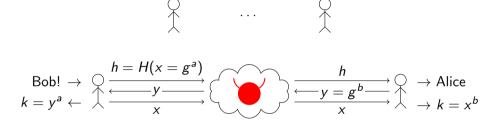
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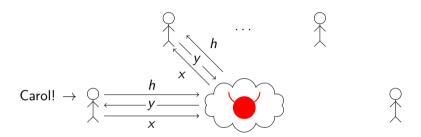
We can guess a communicating pair, but then tightness is lost.

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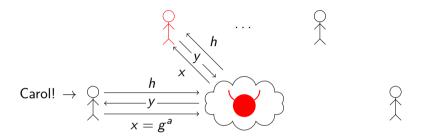
▶ We hash the first message, and send x only after receiving the response.

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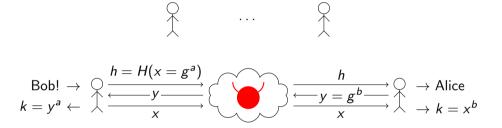
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- ▶ We hash the first message, and send x only after receiving the response.
- Note that there is an additional message flow compared to plain Diffie-Hellman. The responder also learns the key later. This is often not a problem.

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For plain Diffie-Hellman:

- ► Small-scale: 2¹⁶ users, 2¹⁶ sessions and quadratic loss 2⁶⁴: NIST P-384.
- ► Large-scale: 2³² users, 2³² sessions and quadratic loss 2¹²⁸: NIST P-521.

Exponentiation relative time cost: P-256 = 1, P-384 = 2.7, P-521 = 7.7.

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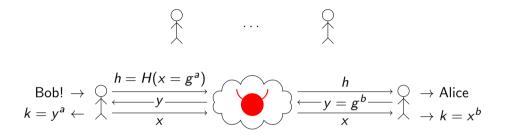
For signed Diffie-Hellman:

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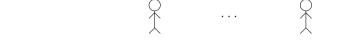
		small-scale		large-scale	
	# exps.	# bits	time	# bits	time
Our scheme	2	770	2	770	2
Plain DH	2	770	5.4	1044	15.4

No Tampering + Adaptive Corruption = A Need for Signatures



▶ If the adversary is allowed to tamper with messages, this protocol obviously fails.

No Tampering + Adaptive Corruption = A Need for Signatures



$$\begin{array}{c} \mathsf{Bob!} \to \Diamond \\ k = y^a \leftarrow \end{array} \bigwedge \xrightarrow{h = H(x = g^a)} \overbrace{\overset{h}{\leftarrow} y, \sigma_B} \xrightarrow{} \overbrace{\overset{h}{\leftarrow} y = g^b, \sigma_B} \xrightarrow{} A \mathsf{lice} \\ \xrightarrow{x, \sigma_A} \xrightarrow{} k = x^b \end{array}$$

- ▶ If the adversary is allowed to tamper with messages, this protocol obviously fails.
- ▶ The natural answer is to add the usual signatures.
- ▶ But this requires a signature scheme with a tight reduction!

Signatures with Tight Reductions

The standard security notion for signatures considers only a single key pair.

- ▶ There is a standard reduction to many key pairs, but it is non-tight.
- In fact, the loss in the standard reduction is quadratic in the number of users, because of adaptive compromise.

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Tight reductions are impossible for schemes with unique signatures or signing keys.

"Double-signature" idea

The "double-signature" idea from Bader et al. (TCC 15).

- ▶ The user has "real" and a "fake" verification key.
- ► A signature consists of a real signature for the "real" verification key, and a fake signature for the "fake" verification key.
- We embed our hard problem in the "fake" verification key.
- ▶ If "real" and "fake" keys and signatures are indistinguishable, the adversary will produce a forgery that applies to the "fake" verification key with probability 1/2.

This "double-signature" idea does not work for most signature schemes. The only previous construction is impractical.

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The usual AKE security models require strongly unforgeable signatures. This scheme is not strongly unforgeable, so we need to use a slightly different security model.

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Our scheme II	17	4358	17	4358	17
DH + ECDSA	5	2306	13.5	3128	38.5

Summary

- First practical tightly-secure signature scheme with adaptive corruptions.
- First practical tightly-secure AKE.
- More efficient than ordinary signed Diffie-Hellman for large-scale deployment settings

