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The Algebraic Group Model and its Applications

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Idealized Models In Cryptography

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- Best case: proof in *standard model* (no simplifying assumptions).
- Often, this is not possible (or just very hard to do).
- Must resort to *idealized models* instead.

Idealizing the Real World– But How?

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- Prove statements in simplified, idealized model.
- Intuition: If model is good, proofs are meaningful in the real world.

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- Generic Group Model (GGM): idealizes cyclic groups.

Generic Group Algorithms

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§ Given $a, b \in \mathbb{G}$, compute $c := a \circ b$

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- Given $a, b \in \mathbb{G}$, compute $c := a \circ b$
- Given $a, b \in \mathbb{G}$, check whether $a = b$.

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- Fitting abstraction for (some) *elliptic curves*.

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- Lower bounds are not ‘modular’.

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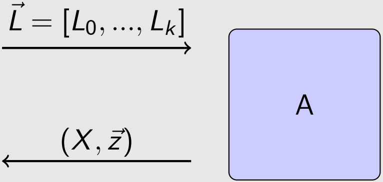
- GGM $\leq$ AGM $\leq$ Standard Model.
- Lies in between GGM and standard model.
- Provides improved abstraction of reality over GGM.
- Still allows easy proofs.

Algebraic Algorithms



- A takes as input list $\vec{L}$ of group elements.
- Outputs representation $\vec{z}$ of X, i.e., $X = \prod_i L_i^{z_i}$.

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- *Algebraic reductions* mostly used for meta-reductions.
- Some exceptions: E.g. Abdalla et al. (S&P '15) consider an algebraic *adversary* in one of their proofs

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- This gives *strictly weaker* model assumptions than the GGM.

Relation to the GGM

Lemma 1

Every generic algorithm is an algebraic algorithm.

- A generic algorithm A computes any output from elements in the list $\vec{L}$ via $\circ$.
- Thus, it must know a representation $\vec{z}$ for every output.
- We assume that it outputs $\vec{z}$ 'for free'.

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Theorem 2 (Composition)

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- *Suppose that:*
  - $\texttt{true} \overset{\texttt{GGM}}{\Longrightarrow} S$
  - $S \overset{\texttt{AGM}}{\Longrightarrow} T$
- *Then* $\texttt{true} \overset{\texttt{GGM}}{\Longrightarrow} T$*, if reduction in AGM is a generic algorithm.*

Proofs in AGM vs. Proofs in GGM

- GGM: Lower bounds for algorithms via combinatorics.
- AGM: Reductions.

Using the AGM

- How do we prove reductions in the AGM?
- Want to make use of representation vector $\vec{z}$.

Using the AGM: Example

- CDH: Given g, g^x, g^y, compute g^{xy}.
- DLP: Given g^u, compute u.
- **Lemma** : DLP $\overset{\texttt{AGM}}{\Leftrightarrow}$ CDH
- Breaking CDH algebraically is as hard as solving DLP.

CDH Game

Challenger: $U = g^u$ Adversary

$$\xrightarrow{g, g^x, g^y}$$

$$\xleftarrow{g^{xy}, z = (a, b, c)}$$

§ $g^{xy} = (g^x)^a (g^y)^b g^c$ is equivalent to $xy \equiv_p xa + yb + c$.

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- $x \equiv_p \frac{yb+c}{y-a} \implies$: Can either solve for x or $a \equiv_p y$.
- Idea: Set $U = g^x$ OR $U = g^y$ and choose the other randomly.
- Succeeds with probability $1/2$.

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Equivalence to DLP in AGM: LRSW

Challenger		Adversary
Oracle	$\xrightarrow{(X = g^x, Y = g^y)}$	
	$\xleftarrow{m_i}$	$\updownarrow$ Q times
	$\xrightarrow{(a_i, a_i^y, a_i^{xym_i+x})}$	
	$\xleftarrow{m, (a, a^y, a^{xym+x})}$	

- Basis of Camenisch-Lysyanskaya signature.
- Used for RFID Tags, Anonymous Credentials, etc.
- **Lemma** : DLP $\overset{\mathtt{AGM}}{\Leftrightarrow}$ LRSW

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- q-type variant of DDH $\stackrel{\mathtt{AGM}}{\Longrightarrow}$ ElGamal CCA1
- q-type variant of DLP $\stackrel{\mathtt{AGM}}{\Longrightarrow}$ Groth's ZK-SNARK (EC16)

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- Examples: Structure preserving signatures, ZK-SNARKS.

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- Captures a broad class of important algorithms.
- Circumvents impossibility results for black box reductions.
- Results from AGM carry over to GGM.

Open Questions

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- Automated proof tools in AGM?
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- Extend to Decisional Assumptions.

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Many thanks for your attention!

QUESTIONS?

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